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**Preface**

Here are a set of problems for my Algebra notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.
Outline

Here is a list of sections for which problems have been written.

**Preliminaries**
- Integer Exponents
- Rational Exponents
- Radicals
- Polynomials
- Factoring Polynomials
- Rational Expressions
- Complex Numbers

**Solving Equations and Inequalities**
- Solutions and Solution Sets
- Linear Equations
- Applications of Linear Equations
- Equations With More Than One Variable
- Quadratic Equations, Part I
- Quadratic Equations, Part II
- Quadratic Equations : A Summary
- Applications of Quadratic Equations
- Equations Reducible to Quadratic Form
- Equations with Radicals
- Linear Inequalities
- Polynomial Inequalities
- Rational Inequalities
- Absolute Value Equations
- Absolute Value Inequalities

**Graphing and Functions**
- Graphing
- Lines
- Circles
- The Definition of a Function
- Graphing Functions
- Combining functions
- Inverse Functions

**Common Graphs**
- Lines, Circles and Piecewise Functions
- Parabolas
- Ellipses
- Hyperbolas
- Miscellaneous Functions
- Transformations
- Symmetry
Rational Functions

Polynomial Functions
  Dividing Polynomials
  Zeros/Roots of Polynomials
  Graphing Polynomials
  Finding Zeros of Polynomials
  Partial Fractions

Exponential and Logarithm Functions
  Exponential Functions
  Logarithm Functions
  Solving Exponential Equations
  Solving Logarithm Equations
  Applications

Systems of Equations
  Linear Systems with Two Variables
  Linear Systems with Three Variables
  Augmented Matrices
  More on the Augmented Matrix
  Nonlinear Systems
Preliminaries

Introduction

Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.

- Integer Exponents
- Rational Exponents
- Radicals
- Polynomials
- Factoring Polynomials
- Rational Expressions
- Complex Numbers

Integer Exponents

For problems 1 – 10 evaluate the given expression and write the answer as a single number with no exponents.

1. \(2 \cdot 5^2 + (-4)^2\)
2. \(6^0 - 3^5\)
3. \(3 \cdot 4^3 + 2 \cdot 3^2\)
4. \((-1)^4 + 2(-3)^4\)
5. \(7^0 \left(4^2 \cdot 3^2\right)^2\)
6. \(-4^3 + (-4)^3\)
7. \(8 \cdot 2^{-3} + 16^0\)
8. \( \left( 2^{-1} + 3^{-1} \right)^{-1} \)

9. \( \frac{3^2 \cdot (-2)^3}{6^{-2}} \)

10. \( \frac{4^{-2} \cdot 5^3}{3^{-4}} \)

For problems 11 – 18 simplify the given expression and write the answer with only positive exponents.

11. \( \left( 3x^{-2}y^{-4} \right)^{-1} \)

12. \( \left( (2a^2)^{-3}b^4 \right)^{-3} \)

13. \( \frac{c^{-6}b^{10}}{b^8c^{-4}} \)

14. \( \frac{4a^3(b^2a)^{-4}}{c^{-6}a^2b^{-7}} \)

15. \( \frac{(6v^2)^{-1}w^{-4}}{(2v)^{-3}w^{10}} \)

16. \( \left( \frac{8x^{21}y^{-3}x^8}{y^{-9}x^{-1}} \right)^6 \)

17. \( \left( \frac{a^7b^{-4}c^{-1}}{b^{-9}c^8a^{-4}} \right)^2 \)

18. \( \left( \frac{p^{-6}q^7(p^2q)^{-3}}{(p^{-1}q^{-4})^2p^{10}} \right)^3 \)

For problems 19 – 23 determine if the statement is true or false. If it is false explain why it is false and give a corrected version of the statement.
19. \( \frac{1}{6x} = 6x^{-1} \)

20. \( (x^3)^7 = x^{21} \)

21. \( (m^3n^4)^2 = m^{12}n^8 \)

22. \( (z^2)^3 = z^6 \)

23. \( (x + y)^3 = x^3 + y^3 \)

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**Rational Exponents**

For problems 1 – 15 evaluate the given expression and write the answer as a single number with no exponents.

1. \( 64^{\frac{1}{3}} \)

2. \( -64^{\frac{1}{2}} \)

3. \( 16^{\frac{1}{2}} \)

4. \( 16^{\frac{1}{4}} \)

5. \( (-243)^{\frac{1}{5}} \)

6. \( 121^{\frac{1}{2}} \)

7. \( (-64)^{\frac{1}{3}} \)

8. \( \left( \frac{625}{256} \right)^{\frac{1}{4}} \)

9. \( \left( -\frac{27}{8} \right)^{\frac{1}{3}} \)
10. \( 49^2 \)

11. \( 64^{\frac{5}{6}} \)

12. \((-729)^{\frac{4}{3}}\)

13. \(\left(\frac{121}{36}\right)^{\frac{3}{2}}\)

14. \(\left(-\frac{32}{243}\right)^{\frac{2}{3}}\)

15. \(\left(\frac{81}{625}\right)^{\frac{3}{4}}\)

For problems 16 – 23 simplify the given expression and write the answer with only positive exponents.

16. \(\left(p^{-2}q^{-4}\right)^{\frac{3}{2}}\)

17. \(x^\frac{3}{4}\left(x^2x^{-\frac{1}{3}}\right)^{\frac{3}{2}}\)

18. \(a^2a^{-\frac{1}{3}}a^\frac{1}{4}\)

19. \(\left(m^{-\frac{7}{2}}n^{\frac{3}{4}}\right)^{\frac{8}{9}}\)

20. \(\left(\frac{a^{-\frac{1}{3}}b^{\frac{2}{3}}}{b^{\frac{2}{3}}a^{-\frac{5}{4}}}\right)^{\frac{1}{3}}\)

21. \(\left(\frac{p^{\frac{1}{2}}q^{\frac{1}{3}}}{p^{-\frac{1}{2}}q^{-\frac{1}{4}}}\right)^{-3}\)
22. \( \left( x^\frac{3}{4} y^\frac{-2}{3} \right)^\frac{7}{8} \)

23. \( \left( \frac{b^3 c^{-\frac{1}{2}} a^{-\frac{1}{3}}}{b^\frac{1}{2} a^{-\frac{2}{3}} c^\frac{1}{3}} \right)^\frac{2}{3} \)

For problems 24 & 25 determine if the statement is true or false. If it is false explain why it is false and give a corrected version of the statement.

24. \( a^{-\frac{3}{2}} = a^{\frac{2}{3}} \)

25. \( x^{-n} = x^{\frac{1}{n}} \)

**Radicals**

For problems 1 – 6 write the expression in exponential form.

1. \( \sqrt[3]{n} \)
2. \( \sqrt[4]{2y} \)
3. \( \sqrt[5]{7x^3} \)
4. \( \sqrt[4]{xyz} \)
5. \( \sqrt{x + y} \)
6. \( \sqrt[3]{a^3 + b^3} \)

For problems 7 – 12 evaluate the radical.

7. \( \sqrt{256} \)
8. \( \sqrt[4]{256} \)
9. \( \sqrt[3]{256} \)
10. \( \sqrt[3]{-1024} \)

11. \( \sqrt[2]{-216} \)

12. \( \sqrt[3]{343} \)

For problems 13 – 22 simplify each of the following. Assume that \( x, y \) and \( z \) are all positive.

13. \( \sqrt{z^5} \)

14. \( \sqrt[3]{z^5} \)

15. \( \sqrt[6]{16x^{17}} \)

16. \( \sqrt[2]{128}y^{11} \)

17. \( \sqrt{x^3y^{17}z^4} \)

18. \( \sqrt[3]{x^3y^{20}z^5} \)

19. \( \sqrt[4]{729}x^2yz^{13} \)

20. \( \sqrt[4]{4x^5y} \sqrt[4]{10x^5y^2} \)

21. \( \sqrt{3x} \sqrt[4]{6x} \sqrt[4]{14x} \)

22. \( \sqrt[6]{2xy^3} \sqrt[6]{32x^2y^5} \)

For problems 23 – 26 multiply each of the following. Assume that \( x \) is positive.

23. \( (2\sqrt{x} + 4)(\sqrt{x} - 7) \)

24. \( \sqrt{x} (\sqrt{x} + 2\sqrt{x^4}) \)

25. \( (\sqrt{x} + \sqrt[4]{2y})(\sqrt{x} - \sqrt[4]{2y}) \)

26. \( (\sqrt{x} + \sqrt{x^2})^2 \)
For problems 27 – 35 rationalize the denominator. Assume that $x$ and $y$ are both positive.

27. \( \frac{9}{\sqrt{y}} \)

28. \( \frac{3}{\sqrt{7x}} \)

29. \( \frac{1}{\sqrt{x}} \)

30. \( \frac{12}{\sqrt[3]{3x^2}} \)

31. \( \frac{2}{4-\sqrt{x}} \)

32. \( \frac{9}{\sqrt{3y} + 2} \)

33. \( \frac{4}{\sqrt{7} - 6\sqrt{x}} \)

34. \( \frac{-6}{\sqrt{5x} + 10\sqrt{3y}} \)

35. \( \frac{4 + x}{x - \sqrt{x}} \)

For problems 36 – 38 determine if the statement is true or false. If it is false explain why it is false.

36. \( \frac{1}{3}x^2 = \sqrt{3x} \)

37. \( \frac{\sqrt[3]{x} + 6}{\sqrt[3]{x} + 3\sqrt[3]{6}} \)

38. \( \frac{\sqrt[3]{x^2}}{\sqrt{x}} \)

39. For problems 13 – 35 above we always added the instruction to assume that the variables were positive. Why was this instruction added? How would the answers to the problems change if we did not have that instruction?
Polynomials

For problems 1 – 18 perform the indicated operation and identify the degree of the result.

1. Add $10x^5 + 2x^3 - 1$ to $8x^4 - x^3 + 16x^2$

2. Add $7t^2 - 13t + 4$ to $-6t^2 + 13t - 4$

3. Subtract $-12z^2 + 9z - 3$ from $z^3 + 2z^2 - 15z + 7$

4. Subtract $100x^4 - 19x^2 - 7x$ from $150x^3 + 8x - 14$

5. Subtract $w^4 + w^3 + w^2 + w + 1$ from $w^5$

6. $6y^2 (3 - y^2 + 2y^3)$

7. $x^3 (x^2 + 7x - 4)$

8. $(7x - 5)(4 - 10x)$

9. $(4 + 9t^2)(t^3 - 3t)$

10. $(1 + 8y)(y^3 - 4y^2 + 7)$

11. $7(x - 9)(2x + 3)$

12. $z^2 (1 - z^2)(1 + z^2)$

13. $(2 - x + 4x^2)(6x + 7)$

14. $(10w^2 - 4w + 9)(w^3 + 5w^2 + 2)$

15. $10(x + 3x^2)^2$

16. $(1 - 5y)(4 + y)^2$

17. Subtract $(3 - x)(3 + x)$ from $x^2 - 7x + 10$

18. Subtract $(4x^2 - 1)^2$ from $(x + 9x^3)^2$
19. If we multiply a polynomial with degree $n$ and a polynomial of degree $m$ what is the degree of the result?

20. If we add 2 polynomials of degree $n$ and $m$ with $n < m$ what is the degree of the result?

21. If we subtract 2 polynomials of degree $n$ and $m$ with $n < m$ what is the degree of the result?

22. If we add two polynomials, both of degree $n$, is it possible for the result to not be degree $n$? If it is not possible can you give an example of two polynomials, both of degree $n$, whose sum is not degree $n$?

23. If we subtract two polynomials, both of degree $n$, is it possible for the result to not be degree $n$? If it is not possible can you give an example of two polynomials, both of degree $n$, whose difference is not degree $n$?

### Factoring Polynomials

For problems 1 – 8 factor out the greatest common factor from each polynomial.

1. $x^3 - 6x^8 + 10x^{10}$
2. $25u^6 - 15u^5 + 30u^8$
3. $2y^6z - y^4z^{10} + 3y^2z^2$
4. $7a^{10}b^7 + 14a^8b^9 - 35a^6b^{12}$
5. $3(9 + 7x) - (2 - x)(9 + 7x)$
6. $z^2(4z - z^3) + 7(z^3 - 4z)$
7. $8y(2y + 7)^4 - 2y^3(2y + 7)^9$
8. $w^2(1 + w^2)(8w - 1)^{10} + 9w(1 + w^2)^4(8w - 1)^7$

For problems 9 – 13 factor each of the following by grouping.

9. $18x - 2x^3 + 9 - x^2$
10. $6w^4 + 3w^3 - 14w^2 - 7w$
11. $y^4 + y^3 + 9y^3 + 9y^2$
12. $21x - 56x^4 - 12x^3 + 32x^6$
13. $6t^3 + 3t^4 - 2t^5 - t^6$

For problems 14 – 32 factor each of the following.

14. $x^2 - 10x + 9$

15. $t^2 + 11t + 24$

16. $z^2 - 9z - 10$

17. $x^2 - 3x - 28$

18. $x^2 + 10x - 24$

19. $w^2 - 8w + 16$

20. $z^2 + 6z + 9$

21. $x^2 - 144$

22. $36 - x^2$

23. $4z^2 - 23z - 6$

24. $2y^2 - 9y + 10$

25. $12x^2 + 31x + 7$

26. $6z^2 - 35z + 36$

27. $8t^2 + 29t - 12$

28. $21 - w - 2w^2$

29. $36y^2 - 49$

30. $100x^2 + 20x + 1$

31. $25z^2 - 40z + 16$

32. $9y^2 - 121$

For problems 33 – 38 factor each of the following.
33. \(4x^3 - 20x^2 - 144x\)
34. \(t^4 + 15t^3 + 14t^2\)
35. \(6u^8 - 3u^6 - 3u^4\)
36. \(t^8 + 5t^4 - 24\)
37. \(2z^4 - 5z^2 - 12\)
38. \(4x^6 + x^3 - 5\)

For problems 39 & 40 determine the possible values of \(a\) for which the polynomial will factor.

39. \(x^2 + ax - 16\)
40. \(x^2 + ax + 20\)

For problems 41 – 44 use the knowledge of factoring that you’ve learned in this section to factor the following expressions.

41. \(x^2 + 1 - 6x^{-2}\)
42. \(x^2 - 2 + \frac{1}{x^2}\)
43. \(x^4 - \frac{49}{x^2}\)
44. \(x - 7\sqrt{x} - 18\)

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**Rational Expressions**

For problems 1 – 6 reduce each of the following to lowest terms.

1. \(\frac{x^3 + 10x^2}{x^2 + 6x - 40}\)
2. \(\frac{x^2 + 18x + 72}{2x^2 + 11x - 6}\)
3. \(\frac{x^2 - 3x - 28}{49 - x^2}\)
4. \[\frac{6x^2 + 13x + 5}{3x^2 + 26x + 35}\]

5. \[\frac{-x^2 + 10x - 9}{-x^2 + 6x + 27}\]

6. \[\frac{x^3 + x^2 - 20x}{x^4 - 12x^3 + 36x^2}\]

For problems 7 – 13 perform the indicated operation and reduce the answer to lowest terms.

7. \[\frac{x^2 + 14x + 40}{x^2 + 2x - 8} \cdot \frac{x^2 + 5x - 14}{x^2 + 7x - 30}\]

8. \[\frac{4x^3 - x^2 - 3x}{x^2 - 10x + 25} \cdot \frac{10 + 3x - x^2}{x^4 - x^3}\]

9. \[\frac{x^2 + 5x - 24}{x^3 - 5x + 4} + \frac{x^2 + x - 12}{x - 1}\]

10. \[\frac{6x^2 + x^3 - x^4}{x^2 - 4} \div \frac{3x^3 - 9x^2}{x^2 + 6x - 16}\]

11. \[\frac{3x^2 + 23x + 14}{x^2 + 4x + 3} \div \frac{6x^2 + 13x + 6}{x^2 + 2x + 1}\]

12. \[\frac{5x^2 - 18x - 8}{x - 4} \div \frac{x - 4}{x + 6}\]

13. \[\frac{2}{6x^3 + 17x^2} \div \frac{x + 4}{x^2 + 3x - 4}\]

For problems 14 – 22 perform the indicated operations.

14. \[\frac{2}{3x^2} - \frac{1}{4x^2} + \frac{7}{6x^3}\]

15. \[\frac{2x}{x + 9} - \frac{x - 1}{x}\]
16. \( \frac{x+1}{x-1} + \frac{6}{x-7} \)

17. \( \frac{9}{x^2 - 4} - \frac{7x}{x^2 - 4x + 4} \)

18. \( \frac{2x+1}{4x^2 - 3x - 7} - \frac{x+3}{x+1} + \frac{x}{4x - 7} \)

19. \( \frac{3}{6x - x^2} - \frac{x}{x^2 - 5x - 6} \)

20. \( \frac{2}{x^2 - 4x - 12} + \frac{8x}{x^2 + 12x + 20} \)

21. \( \frac{3}{x^2} + \frac{x + 9}{x^2 + 5x} - \frac{2}{x^2 + 10x + 25} \)

22. \( \frac{1}{x + 1} - \frac{2}{(x + 1)^2} - \frac{3}{(x + 1)^3} \)

**Complex Numbers**

Perform the indicated operation and write your answer in standard form.

1. \( 2i + (-8 - 15i) \)

2. \( (12 + i) + (9 + 2i) \)

3. \( 4 - (3 - 20i) \)

4. \( \left( \frac{3}{2} - \frac{1}{3} i \right) - \left( \frac{5}{4} + \frac{7}{5} i \right) \)

5. \( (3 + 2i) + (3 - 8i) - (-4 - 7i) \)

6. \( -2i(9 + i) \)

7. \( (10 + 3i)(-1 + 7i) \)

8. \( (6 + 2i)^2 \)
9. \((2 - 14i)(2 + 14i)\)

10. \((2 - \frac{1}{2}i)(-\frac{1}{3} + 5i)\)

11. \((9 + 2i)(1 - 3i)(5 + 4i)\)

12. \(\frac{1+i}{7-i}\)

13. \(\frac{2+4i}{-9+3i}\)

14. \(\frac{6i}{-4-7i}\)

15. \(\frac{12-2i}{9i}\)

16. \(\frac{4+5i}{4-5i}\)

17. \(\frac{i(10-12i)}{(2+i)(-1+4i)}\)

---

**Solving Equations and Inequalities**

**Introduction**

Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.

- Solutions and Solution Sets
- Linear Equations
- Applications of Linear Equations
- Equations With More Than One Variable
**Quadratic Equations, Part I**
**Quadratic Equations, Part II**

**Quadratic Equations : A Summary** – No problems written yet.

Applications of Quadratic Equations

Equations Reducible to Quadratic Form

Equations with Radicals

Linear Inequalities

Polynomial Inequalities

Rational Inequalities

Absolute Value Equations

Absolute Value Inequalities

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**Solutions and Solution Sets**

For each of the following determine if the given number is a solution to the given equation or inequality.

1. Is $u = -1$ a solution to $4u^2 - 40 = 10(2u - 1) - 6$?

2. Is $t = 7$ a solution to $7(t + 2) = 5(t + 4) + 2$?

3. Is $z = -\frac{1}{3}$ a solution to $6(z - 1) + 5 = 9z$?

4. Is $x = -6$ a solution to $x^2 = -10x - 24$?

5. Is $t = \frac{1}{4}$ a solution to $3t^2 + 8t = 3(1 - t)$?

6. Is $w = -3$ a solution to $2w^2 - 10 = w^2 - 7w + 8$?

7. Is $x = \frac{1}{2}$ a solution to $\frac{3}{x} - \frac{1}{x^2} = 2$?

8. Is $v = -2$ a solution to $\frac{v^2 + v - 2}{v - 1} = 0$?

9. Is $v = 1$ a solution to $\frac{v^2 + v - 2}{v - 1} = 0$?

10. Is $x = -1$ a solution to $\frac{3x + 1}{x^2} - \frac{6}{x + 2} = \frac{x - 7}{3x + 4}$?

11. Is $y = 4$ a solution to $4y^2 - y^3 \leq 5y + 2$?
12. Is \( w = 0 \) a solution to \( 3(w - 7) + 2(w + 1) > 10w \) ?

13. Is \( x = 7 \) a solution to \( 3 + 4x < x + 24 \) ?

**Linear Equations**

Solve each of the following equations and check your answer.

1. \( 13 + 2(1-u) = 8u - 5(u + 7) \)

2. \( 8(2 + 3z) + 1 = z - 10(z + 1) \)

3. \( 8 - (4 - 12t) + 2 = 3t + 2(7 - 3t) \)

4. \( 2x(6x - 1) + 21 = 8x - x(3 - 12x) \)

5. \( \frac{3w - 1}{5} + 1 = \frac{7w + 2}{15} \)

6. \( \frac{10y}{9} + \frac{1}{3} = \frac{2y - 1}{9} \)

7. \( 2\left(3 - \frac{x}{4}\right) = \frac{2x + 5}{3} - \frac{1}{3} \)

8. \( \frac{6x + 24}{x + 4} = 5 \)

9. \( \frac{3}{v + 7} - \frac{2 - 7v}{v^2 + 5v - 14} = \frac{4}{v - 2} \)

10. \( \frac{6t - 1}{t^2 + 5t + 4} = -\frac{19}{t + 1} \)

11. \( \frac{8 - 4z}{3z - 2} = 2 - \frac{10z}{3z - 2} \)

12. \( \frac{4w - 1}{w - 2} + \frac{8w}{w^2 - 6w + 8} = \frac{4w + 3}{w - 4} \)
1. In a clearance bin everything has been reduced by 75%. One item is listed in the bin for $32.40. How much was the price of the item before it was put into the clearance bin?

2. A piece of electronics has been marked up 20% and is selling for $21.50. How much did the store pay for the item?

3. A widget is on sale for $715.80 and has been marked down by 11%. What was the original price of the widget?

4. Two cars start at the same point and move in the same direction. One car travels 5mph faster than the twice the speed of other car. After 10 hours the distance separating the two cars is 60 miles. What was the speed of each car?

5. Two people start out 100 meters apart from each other and start moving towards each other at the same time. One person is moving at half the speed of the other person and they meet after 25 seconds of travel time. What was the speed of each person?

6. Two boats start at the same point. One boat starts traveling to the east at 45 mph and two hours later the second boat starts traveling to the east at 60 mph. At some point in time the faster boat will be 145 miles in front of the slower boat. How long has each boat been traveling when this happens?

7. One machine can complete a production run at a factory in 46 hours. Two machines can complete the production run in 25 hours if they work together. How long would it take the second machine to complete the production run if it had to do the job by itself?

8. One person can mow a field in 52 minutes and a second can mow the same field in 40 minutes. How long would it take the two of them to mow the field together?

9. One pump can fill a pool in 11 hours and a second pump can empty the same pool in 4 hours. While the pool is full both pumps are accidentally both turned on at the same time. How long will it take to empty the pool?

10. How much pure acid should we add to a 32% acid solution to get 10 liters of a 60% acid solution.

11. We have 80 liters of a 2% saline solution. How much of a 10% saline solution should we add to this to increase the salinity to 4%?

12. We have 10 gallons of a 26% alcohol solution and we need 15 gallons of an 18% alcohol solution. What % alcohol solution should we add to the 26% solution to get the solution we want?

13. There is a field whose width is 6 meters less than its length. If both the length and width are doubled the perimeter will be 120 meters. What are the dimensions of the field?
14. A triangular piece of glass has been cut for a stained glass window. Two of the sides are the same length and the third side is 1 inch shorter than \( \frac{1}{2} \) the length of the other two sides. If the perimeter is 23 inches what are the lengths of the sides?

**Equations With More Than One Variable**

1. Solve \( A = 3p\left(4 - 2r\right) \) for \( p \).

2. Solve \( A = 3p\left(4 - 2r\right) \) for \( r \).

3. Solve \( T = \frac{c}{3}\left(6p + \frac{3q}{c}\right) - 7p \) for \( p \).

4. Solve \( T = \frac{c}{3}\left(6p + \frac{3q}{c}\right) - 7p \) for \( c \).

5. Solve \( \frac{1}{n} = \frac{2}{m} - \frac{3}{q} \) for \( n \).

6. Solve \( \frac{1}{n} = \frac{2}{m} - \frac{3}{q} \) for \( q \).

7. Solve \( 3A + 6C = 4A\left(B - 7C\right) \) for \( C \).

8. Solve \( 3A + 6C = 4A\left(B - 7C\right) \) for \( A \).

9. Solve \( y = \frac{4 - 9x}{3} \) for \( x \).

10. Solve \( y = \frac{12}{1 - x} \) for \( x \).

11. Solve \( y = \frac{7}{10x + 9} \) for \( x \).

12. Solve \( y = \frac{8 - 5x}{9 - 7x} \) for \( x \).

13. Solve \( y = \frac{2 + 11x}{1 + 4x} \) for \( x \).
14. Solve \( y = \frac{9 + 2x}{4 - x} \) for \( x \).

**Quadratic Equations – Part I**

For problems 1 – 15 solve the quadratic equation by factoring.

1. \( z^2 - 11z + 24 = 0 \)
2. \( w^2 + 13w + 12 = 0 \)
3. \( x^2 + 32 = 12x \)
4. \( y^2 = 6y + 27 \)
5. \( u^2 - 4u - 20 = 3u + 24 \)
6. \( z^2 - 36 = 0 \)
7. \( 144x^2 - 25 = 0 \)
8. \( 7x^2 + 19x = 6 \)
9. \( 4y^2 + 15y + 6 = 4y \)
10. \( 6z^2 - 11z + 15 = 12z - 5 \)
11. \( 20v^2 + 3v = 5v^2 + 5v + 1 \)
12. \( x^2 - 4x + 16 = 4x \)
13. \( 9y^2 + 17y + 20 = 4 - 7y \)
14. \( 7u^2 + 9u = 0 \)
15. \( 14x = 3x^2 \)

For problems 16 – 18 use factoring to solve the equation.

16. \( 3v^3 - 19v^2 - 14v = 0 \)
17. \( y^6 + y^5 = 20y^4 \)
18. \( z^4 + 2z^3 + z^2 = 0 \)
For problems 19 – 22 use factoring to solve the equation.

19. \(1 + \frac{2}{x-2} = \frac{12-x}{x^2 + x - 6}\)

20. \(\frac{t+1}{t+2} = \frac{4(t-5)}{t^2 + 2t} + \frac{4}{t}\)

21. \(\frac{w^2 - 1}{w+6} = \frac{5 - 5w}{w+6} - w\)

22. \(\frac{y - 2}{y - 9} + \frac{y^2 - 19y + 34}{y^2 - 10y + 9} = \frac{y - 3}{y - 1}\)

For problems 23 – 31 use the Square Root Property to solve the equation.

23. \(v^2 - 144 = 0\)

24. \(81x^2 - 25 = 0\)

25. \(4t^2 + 1 = 0\)

26. \(7y^2 - 3 = 0\)

27. \(14 + 2x^2 = 0\)

28. \((3t - 8)^2 - 16 = 0\)

29. \((u + 1)^2 + 6 = 0\)

30. \(4(2x - 1)^2 - 36 = 0\)

31. \((4 - z)^2 - 121 = 0\)

### Quadratic Equations – Part II

For problems 1 – 6 complete the square.

1. \(w^2 + 3w\)

2. \(x^2 - 10x\)
3. \( y^2 + 14y \)

4. \( 3u^2 - 36u \)

5. \( 2t^2 - 9t \)

6. \( 18x - x^2 \)

For problems 7 – 16 solve the quadratic equation by completing the square.

7. \( x^2 + 3x - 10 = 0 \)

8. \( z^2 - 12z + 40 = 0 \)

9. \( t^2 - 7t + 2 = 0 \)

10. \( u^2 + 5u + 9 = 0 \)

11. \( 4x^2 - 4x + 5 = 0 \)

12. \( 16w^2 + 8w + 1 = 0 \)

13. \( 4y^2 - 24y + 29 = 0 \)

14. \( 81z^2 + 54z + 10 = 0 \)

15. \( 9t^2 - 12t - 14 = 0 \)

16. \( 5v^2 - 14v + 11 = 0 \)

For problems 17 – 26 use the quadratic formula to solve the quadratic equation.

17. \( w^2 - 14w + 245 = 0 \)

18. \( 3t^2 + 20t + 31 = 0 \)

19. \( 6x + 61 + 18x^2 = 0 \)

20. \( x^2 = 4x - 23 \)

21. \( y^2 + 20y = 4y - 64 \)

22. \( 33 = 8z + z^2 \)
23. \(2t^2 + 49 = 32t - 2t^2\)

24. \(40u + 25u^2 = 10u - 11\)

25. \(10x^2 - 10x = 4x^2 - 3x + 10\)

26. \(16z^2 + 4z - 40 = 140z + 19\)

**Solving Quadratic Equations : A Summary**

For problems 1 – 7 use the discriminant to determine the type of roots for the equation. Do not find any roots.

1. \(25x^2 - 120x + 619 = 0\)

2. \(104x^2 - 75x - 14 = 0\)

3. \(2x^2 + 60x + 450 = 0\)

4. \(\frac{1}{6}x^2 - 43 = 0\)

5. \(97 + 136x + 289x^2 = 0\)

6. \(10x^2 - 7x = 0\)

7. \(\frac{49}{9}x^2 + \frac{14}{15}x + \frac{1}{25} = 0\)

**Application of Quadratic Equations**

1. The length of a rectangle is 4 feet more than the width. If the area of the rectangle is 136 ft\(^2\) what are the dimensions of the rectangle?

2. The area of some rectangle is 35 in\(^2\). Four times the width of this rectangle is the same as 3 inches more than twice the length. What are the dimensions of the rectangle?

3. The area of a triangle is 28 m\(^2\) and the height of the triangle is 2 meters less than 5 times the base. What are the height and base of this triangle?

4. Two cars start out at the same spot. One car starts to drive north at 18 mph 5 hours before the second car starts driving to the east at 32 mph. How long after the first car starts driving does it take for the two cars to be 350 miles apart?
5. Two cars start out at the same point and at the same time one starts driving north while the other starts driving east at a speed that is 4 mph faster than the car driving north. Twelve hours after the cars start driving they are 600 miles apart. What was the speed of each car?

6. Two people can paint a house in 21 hours. Working individually one of the people can paint the house in 6 hours more than it takes the other person to paint the house. How long would it take each person working individually to paint the house?

**Equations Reducible to Quadratic Form**

Solve each of the following equations.

1. \(8x^6 + 215x^3 - 27 = 0\)

2. \(x^\frac{4}{1} - 13x^\frac{2}{1} + 36 = 0\)

3. \(32x^{-10} - 31x^{-5} - 1 = 0\)

4. \(x - 8\sqrt{x} + 15 = 0\)

5. \(x^{-\frac{1}{2}} - 13x^{-\frac{1}{4}} + 30 = 0\)

6. \(x^{-6} - 3x^{-3} - 28 = 0\)

7. \(x^{10} - 1024 = 0\)

8. \(x^{4} - 8x^{2} + 5 = 0\)

9. \(\frac{1}{x^{4}} + \frac{10}{x^{2}} + 22 = 0\)

**Equations with Radicals**

Solve each of the following equations.

1. \(x = \sqrt{4x - 3}\)

2. \(2x = -\sqrt{3 - x}\)

3. \(4 - \sqrt{x + 6} = -x\)
Linear Inequalities

For problems 1 – 6 solve each of the following inequalities. Give the solution in both inequality and interval notations.

1. \(7x + 2(4 - x) < 12 - 3(5 + 6x)\)

2. \(10(3 + w) \geq 9(2 - 4w)\)

3. \(2(4 + 5y) \leq 12y - 6(1 - 3y)\)

4. \(2\left(\frac{1}{3} - \frac{1}{6}z\right) > \frac{1}{9}z + 4\left(2 - \frac{7}{18}z\right)\)

5. \(2 \leq 2 + 4(3 - x) \leq 6\)

6. \(-4 < 7x + 8 \leq 1\)

7. \(\frac{1}{2} < 2\left(\frac{1}{4} + \frac{1}{8}t\right) < \frac{3}{4}\)

8. \(-12 \leq 4 - 11m \leq 3\)

9. \(0 \leq \frac{3}{7} - \frac{5}{14}x < \frac{1}{2}\)

10. \(-8 < 2(3 + 4x) - 4(1 + 3x) \leq 3\)

11. If \(-7 < x \leq 6\) determine \(a\) and \(b\) for the inequality: \(a \leq 3x + 8 < b\)
12. If $-3 \leq x \leq -1$ determine $a$ and $b$ for the inequality: $a \leq 6 - 2x < b$

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**Polynomial Inequalities**

Solve each of the following inequalities.

1. $z^2 - 11z + 24 < 0$
2. $2x^2 - 3 \geq 5x$
3. $t^2 > 30 - 7t$
4. $m^2 - 7m \leq 8$
5. $x^2 + 6x \geq -9$
6. $u^2 + u \leq 1$
7. $w^2 + 4w - 12 > 0$
8. $x^2 + 49 > 14x$
9. $t^2 \leq t$
10. $x^3 - 8x > -14$
11. $9u^2 - 6u + 1 < 0$
12. $z^6 + 8z^5 + 12z^4 \geq 0$
13. $2w^3 - 3w^2 > 14w$

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**Rational Inequalities**

Solve each of the following inequalities.

1. $\frac{t + 6}{t - 1} < 0$
2. $\frac{4x + 2}{3 - x} \geq 0$
3. \( \frac{2u + 3}{u + 6} > 0 \)

4. \( \frac{3 - z}{z + 1} < -2 \)

5. \( \frac{w + 9}{w + 2} \leq 3 \)

6. \( \frac{x^2 + 9x + 14}{x - 1} > 0 \)

7. \( \frac{4z^2 + 3z - 10}{z} \geq 0 \)

8. \( \frac{t^2 + 10t + 16}{t^2 - 4t + 3} \leq 0 \)

9. \( \frac{z^2 - 6z + 4}{z - 5} < 4 \)

10. \( w - 5 \geq \frac{3 - w^2}{w} \)

11. \( \frac{x^2 + 8x + 16}{x} > 0 \)

12. \( \frac{u - 8}{3u^4 - u^5} \leq 0 \)

13. \( \frac{2}{x^2 - 2x + 1} \geq 0 \)

**Absolute Value Equations**

For problems 1 – 10 solve each of the equation.

1. \( |2x + 9| = 7 \)

2. \( |5w - 2| = 3 \)

3. \( |6 - 7t| = 10 \)
4. \[2 = \left| \frac{1}{4}m - \frac{1}{3} \right|\]

5. \[|8u + 9| = 9\]

6. \[|x + 3| = 4x + 1\]

7. \[|2z - 7| = 3z - 10\]

8. \[|3y + 9| = 10 - y\]

9. \[|6w + 12| = 1 + w\]

10. \[|8x + 3| = 0\]

For problems 11 – 13 find all the real valued solutions to the equation.

11. \[|x^2 + 1| = -4\]

12. \[|u^2 - 7u| = 12\]

13. \[|z^2 - 6| = z\]

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**Absolute Value Inequalities**

Solve each of the following inequalities.

1. \[|3x + 1| \leq 9\]

2. \[|10w - 4| < 2\]

3. \[|8t - 5| \leq 0\]

4. \[|9 - z| < 14\]

5. \[|2 - 7u| \leq 20\]

6. \[|4x + 2| < -1\]
7. $|1 - 4z| > 1$

8. $|3w + 15| \geq 4$

9. $|6t - 10| > 12$

10. $|8 - 2x| \geq 5$

11. $|4u - 1| > -1$

**Graphing and Functions**

**Introduction**

Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.

- Graphing
- Lines
- Circles
- The Definition of a Function
- Graphing Functions
- Combining functions
- Inverse Functions
Graphing

For problems 1 – 7 construct a table of at least 4 ordered pairs of points on the graph of the equation and use the ordered pairs from the table to sketch the graph of the equation.

1. \( y = \frac{1}{2} x + \frac{3}{2} \)
2. \( y = 4 - x \)
3. \( y = 3x^2 \)
4. \( y = (x + 3)^2 \)
5. \( y = \sqrt{x + 2} \)
6. \( y = |x| \)
7. \( y = x^3 \)

For problems 8 – 18 determine the \( x \)-intercepts and \( y \)-intercepts for the equation. Do not sketch the graph.

8. \( y = \frac{7}{3} x + 2 \)
9. \( 6y + 11x = -2 \)
10. \( y = 10x^2 \)
11. \( y = x^2 - 10x + 25 \)
12. \( y = 16x^2 - 8x + 17 \)
13. \( y = -x^2 - 25x - 24 \)
14. \( y = 2x^2 - 6x + 7 \)
15. \( y = -4x^2 - 3 \)
16. \( y = 6x^3 + 48 \)
17. \( y = |x + 4| - 7 \)

18. \( y = 4 - \sqrt{x - 2} \)

**Lines**

For problems 1 – 5 determine the slope of the line containing the two points and sketch the graph of the line.

1. \((2,10), (2,14)\)

2. \((-6,0), (-1,3)\)

3. \((2,12), (6,10)\)

4. \((-5,7), (1,-11)\)

5. \((-1,-6), (4,-6)\)

For problems 6 – 12 write down the equation of the line that passes through the two points. Give your answer in point-slope form and slope-intercept form.

6. \((2,10), (4,14)\)

7. \((-6,0), (-1,3)\)

8. \((2,12), (6,10)\)

9. \((-5,7), (1,-11)\)

10. \((-1,-6), (4,-6)\)

11. \((0,10), (4,2)\)

12. \((-9,2), (3,24)\)

For problems 13 – 17 determine the slope of the line and sketch the graph of the line.

13. \(6x - y = 8\)
14. \( y + 2x = -3 \)
15. \( 3x - y = 1 \)
16. \( 5y + 4x = 7 \)
17. \( 6y - 13x = -4 \)

For problems 18 - 20 determine if the two given lines are parallel, perpendicular or neither.

18. The line containing the two points \((0,0), (3, 18)\) and the line containing the two points \((-1, -5), (1, 7)\).
19. \( y - 4x = 9 \) and \( 4y - x = -3 \)
20. \( y = \frac{2}{3}x - 4 \) and the line containing the two points \((-4, 7), (2, -2)\)

21. Find the equation of the line through \((6, -1)\) and is parallel to the line \(9x + 2y = 1\).
22. Find the equation of the line through \((6, -1)\) and is perpendicular to the line \(9x + 2y = 1\).
23. Find the equation of the line through \((-4, -9)\) and is parallel to the line \(-8y - x = 43\).
24. Find the equation of the line through \((-4, -9)\) and is perpendicular to the line \(-8y - x = 43\).

**Circles**

1. Write the equation of the circle with radius 1 and center \((11, 4)\).
2. Write the equation of the circle with radius 10 and center \((-6, 0)\).
3. Write the equation of the circle with radius \(\sqrt{19}\) and center \((7, -2)\).
4. Write the equation of the circle with radius \(\frac{7}{3}\) and center \(\left(-\frac{1}{2}, \frac{3}{4}\right)\).

For problems 5 – 10 determine the radius and center of the circle and sketch the graph of the circle.
5. \((x+8)^2 + y^2 = 36\)

6. \((x-1)^2 + (y-7)^2 = 16\)

7. \((x+10)^2 + (y-6)^2 = 25\)

8. \(x^2 + (y+4)^2 = \frac{49}{144}\)

9. \((x+2)^2 + (y-1)^2 = 3\)

10. \((x-5)^2 + (y-3)^2 = 11\)

For problems 11 – 17 determine the radius and center of the circle. If the equation is not the equation of a circle clearly explain why not.

11. \(x^2 + y^2 - 8y = 0\)

12. \(x^2 + y^2 - 6x - 4y - 12 = 0\)

13. \(x^2 + y^2 + 12x + 2y + 28 = 0\)

14. \(16x^2 + 16y^2 - 16x + 8y - 11 = 0\)

15. \(2x^2 + 2y^2 - 3x + 1 = 0\)

16. \(x^2 + y^2 + 2x - 2y + 11 = 0\)

17. \(x^2 + y^2 - 10x + 4y + 29 = 0\)

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**The Definition of a Function**

For problems 1 – 6 determine if the given relation is a function.

1. \(\{(0,1),(2,6),(9,4),(7,2),(12,3)\}\)

2. \(\{(-4,1),(-2,1),(0,1),(3,1)\}\)

3. \(\{(0,4),(0,6),(0,8)\}\)
4. \((1, 6), (-3, 4), (7, 6), (2, -10)\)

5. \((0, 1), (2, 3), (4, 5), (6, 7), (8, 9), (10, 11), (12, 13)\)

6. \((-7, 0), (4, 2), (4, 1), (-2, 3), (6, 0)\)

For problems 7 – 13 determine if the given equation is a function.

7. \(y = \frac{2}{5}x + \frac{7}{5}\)

8. \(y = 3x^2 + 4x + 1\)

9. \(y = 2 - x^4\)

10. \(y^2 = 10 - 3x\)

11. \(y^2 = x^2 + 1\)

12. \(y^4 + x^3 = 1\)

13. \(y^3 + x^4 = 1\)

14. Given \(A(t) = 7t + 2\) determine each of the following.
   (a) \(A(-9)\)  (b) \(A(0)\)  (c) \(A(2)\)  (d) \(A(6x)\)  (e) \(A(t^2 + 1)\)

15. Given \(f(x) = \frac{3}{x}\) determine each of the following.
   (a) \(f(-4)\)  (b) \(f\left(\frac{1}{3}\right)\)  (c) \(f\left(\frac{6}{7}\right)\)  (d) \(f(4t + 2)\)  (e) \(f\left(\frac{6}{x}\right)\)

16. Given \(h(w) = \sqrt{2w + 10}\) determine each of the following.
   (a) \(h(-1)\)  (b) \(h(0)\)  (c) \(h(3)\)  (d) \(h(-2t)\)  (e) \(h(w + 4)\)

17. Given \(P(x) = 3 - 2x - x^2\) determine each of the following.
   (a) \(P(-6)\)  (b) \(P(0)\)  (c) \(P(3)\)  (d) \(P(z^2)\)  (e) \(P(4 - x)\)

18. Given \(f(z) = 2z^3 - z^2\) determine each of the following.
(a) $f(-1)$  (b) $f(0)$  (c) $f(4)$  (d) $f\left(\frac{1}{2}t\right)$  (e) $f(z-1)$

19. Given $g(t)=\begin{cases}2+t & \text{if } t \geq 10 \\ t-7 & \text{if } t < 10 \end{cases}$ determine each of the following.

(a) $g(14)$  (b) $g(10)$  (c) $g(-1)$

20. Given $f(x)=\begin{cases}4x^2 & \text{if } x < -4 \\ 6x & \text{if } x \geq -4 \end{cases}$ determine each of the following.

(a) $f(-6)$  (b) $f(-4)$  (c) $f(3)$

21. Given $g(x)=\begin{cases}\frac{1}{2}x & \text{if } x \leq 7 \\ x^2+1 & \text{if } 7 < x < 11 \\ 3-x & \text{if } x \geq 11 \end{cases}$ determine each of the following.

(a) $g(2)$  (b) $g(7)$  (c) $g(8)$  (d) $g(11)$  (e) $g(14)$

22. Given $A(w)=\begin{cases}12 & \text{if } w > -8 \\ 2+3w & \text{if } -10 \leq w \leq -8 \\ -1 & \text{if } w < -10 \end{cases}$ determine each of the following.

(a) $A(-12)$  (b) $A(-10)$  (c) $A(-9)$  (d) $A(-8)$  (e) $A(0)$

23. Given $f(x)=\begin{cases}2x & \text{if } x < 6 \\ 4+x & \text{if } x = 6 \\ x^2 & \text{if } x > 6 \end{cases}$ determine each of the following.

(a) $f(0)$  (b) $f(2)$  (c) $f(6)$  (d) $f(8)$  (e) $f(10)$

For problems 24 – 28 compute the difference quotient for the given function. The difference quotient for the function $f(x)$ is defined to be,

$$\frac{f(x+h)-f(x)}{h}$$

24. $f(x)=8x-1$

25. $f(x)=3x^2$

26. $f(x)=7-x^2$
27. \( f(x) = 3x^2 + 7x - 4 \)

28. \( f(x) = \frac{2}{x} \)

For problems 29 – 39 determine the domain of the function.

29. \( f(x) = 9 - x \)

30. \( P(z) = z^2 - 4 \)

31. \( h(x) = \frac{2 + x}{8x - 1} \)

32. \( A(t) = \frac{t^2 - 4}{t^2 + 6t - 7} \)

33. \( h(w) = \frac{w^2 + 3w + 2}{w^2 + 12w + 36} \)

34. \( g(x) = \sqrt{10x - 15} \)

35. \( f(t) = \frac{10t}{\sqrt{6 - 4t}} \)

36. \( f(w) = \frac{\sqrt{w + 7}}{\sqrt{2 - w}} \)

37. \( A(z) = \sqrt{z^2 - 9z} \)

38. \( h(z) = \sqrt{z^2 - z - 20} \)

39. \( g(t) = \sqrt{\frac{6 + t}{5t - 10}} \)

**Graphing Functions**

For problems 1 – 13 construct a table of at least 4 ordered pairs of points on the graph of the function and use the ordered pairs from the table to sketch the graph of the function.
1. \( f(x) = 6x - 1 \)
2. \( f(x) = 3 - 5x \)
3. \( f(x) = 2x^2 \)
4. \( f(x) = x^2 + 7 \)
5. \( f(x) = \sqrt{x + 3} \)
6. \( f(x) = \sqrt{6 - x} \)
7. \( f(x) = \frac{1}{x} \), use only positive \( x \)'s
8. \( f(x) = \frac{1}{x} \), use only negative \( x \)'s
9. \( f(x) = \begin{cases} 3 & \text{if } x \geq 0 \\ 4 - x & \text{if } x < 0 \end{cases} \)
10. \( f(x) = \begin{cases} 4x & \text{if } x \leq -2 \\ 3 - 2x & \text{if } x > -2 \end{cases} \)
11. \( f(x) = \begin{cases} 2 - x^2 & \text{if } x < 1 \\ (x - 2)^2 & \text{if } x \geq 1 \end{cases} \)
12. \( f(x) = \begin{cases} x^2 & \text{if } x > 3 \\ 4 & \text{if } -2 \leq x \leq 3 \\ 1 - x & \text{if } x < -2 \end{cases} \)
13. \( f(x) = \begin{cases} 1 - x & \text{if } x \geq 1 \\ x^2 - 1 & \text{if } -1 < x < 1 \\ -1 - x & \text{if } x \leq -1 \end{cases} \)

**Combining Functions**
1. Given \( f(x) = x + 12 \) and \( g(x) = 9 + 4x \) compute each of the following.
(a) \( f + g \)  
(b) \( f - g \)(1)  
(c) \( f g \)(x)  
(d) \( \frac{f}{g} \)

2. Given \( h(w) = w^2 - 4w \) and \( f(w) = 2 + w^2 \) compute each of the following.
(a) \( h - f \)(w)  
(b) \( f + h \)(-4)  
(c) \( f h \)  
(d) \( \left( \frac{h}{f} \right)(w) \)

3. Given \( A(x) = 6x - 1 \) and \( P(x) = \frac{1}{4 - x} \) compute each of the following.
(a) \( (A + P)(0) \)  
(b) \( (P - A)(-2) \)  
(c) \( A P \)  
(d) \( \left( \frac{A}{P} \right)(x) \)

4. Given \( f(t) = 2t + 9 \) and \( g(t) = 2t - 1 \) compute each of the following.
(a) \( f g \)(t)  
(b) \( (g \circ f)(t) \)  
(c) \( (g \circ f)(t) \)  
(d) \( (g \circ g)(t) \)

5. Given \( h(x) = x^2 + 1 \) and \( g(x) = 6 - 4x \) compute each of the following.
(a) \( g h \)(x)  
(b) \( (g \circ h)(x) \)  
(c) \( (h \circ g)(x) \)  
(d) \( (h \circ h)(x) \)

6. Given \( A(w) = 2w^2 + 9 \) and \( R(w) = 1 - 2w - w^2 \) compute each of the following.
(a) \( (A R)(w) \)  
(b) \( (A \circ R)(w) \)  
(c) \( (R \circ A)(w) \)  
(d) \( (A \circ A)(w) \)

7. Given \( f(x) = 9x^2 + 10x + 12 \) and \( g(x) = 2 \) compute each of the following.
(a) \( f g \)(x)  
(b) \( (g \circ f)(x) \)  
(c) \( (f \circ g)(x) \)  
(d) \( (g \circ g)(x) \)

8. Given \( g(t) = t + 1 \) and \( h(t) = \frac{2}{t - 3} \) compute each of the following.
(a) \( g h \)(t)  
(b) \( (g \circ h)(t) \)  
(c) \( (h \circ g)(t) \)  
(d) \( (h \circ h)(t) \)

9. Given \( f(x) = \frac{1}{2} x - 3 \) and \( g(x) = 2x + 6 \), \( t \geq 0 \) compute each of the following.
(a) \( (f \circ g)(x) \)  
(b) \( (g \circ f)(x) \)

10. Given \( h(w) = \frac{1}{w - 3} \) and \( f(w) = \frac{1 + 3w}{w} \) compute each of the following.
(a) \( (h \circ f)(w) \)  
(b) \( (f \circ h)(w) \)
Inverse Functions

1. Given \( P(x) = 12x - 7 \) find \( P^{-1}(x) \).

2. Given \( g(x) = 7x \) find \( g^{-1}(x) \).

3. Given \( h(x) = \frac{3}{4} - \frac{9}{7}x \) find \( h^{-1}(x) \).

4. Given \( A(x) = 4 - (x + 3)^5 \) find \( A^{-1}(x) \).

5. Given \( f(x) = 2(1 - 4x)^3 + 1 \) find \( f^{-1}(x) \).

6. Given \( P(x) = \sqrt[5]{\frac{5}{x} - 8x} \) find \( P^{-1}(x) \).

7. Given \( g(x) = 1 + \sqrt[3]{3x + 4} \) find \( g^{-1}(x) \).

8. Given \( f(x) = \frac{10 - 3x}{8x} \) find \( f^{-1}(x) \).

9. Given \( g(x) = \frac{6x - 7}{4 + x} \) find \( g^{-1}(x) \).

10. Given \( f(x) = \frac{3 - x}{9 - 7x} \) find \( f^{-1}(x) \).

Common Graphs

Introduction

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Here is a list of topics in this chapter that have problems written for them.
Lines, Circles and Piecewise Functions
Parabolas
Ellipses
Hyperbolas
Miscellaneous Functions
Transformations
Symmetry
Rational Functions

Lines, Circles and Piecewise Functions

We looked at these topics in the previous chapter. Problems for these topics can be found in the following sections.

Lines : Graphing and Functions – Lines

Circles : Graphing and Functions – Circles

Piecewise Functions : Graphing and Functions – Graphing Functions

Parabolas

For problems 1 – 18 sketch the graph of the following parabolas. The graph should contain the vertex, the y-intercept, x-intercepts (if any) and at least one point on either side of the vertex.

1. \( f(x) = -4x^2 \)

2. \( f(x) = (x - 6)^2 + 1 \)

3. \( f(x) = (x + 2)^2 - 4 \)

4. \( f(x) = 3(x - 1)^2 + 12 \)

5. \( f(x) = -6(x + 5)^2 + 54 \)

6. \( f(x) = -(x - 7)^2 - 3 \)

7. \( f(x) = 2(x + 3)^2 - 6 \)

8. \( f(x) = x^2 - 8 \)
9. \( f(x) = -4x^2 - 1 \)

10. \( f(x) = x^2 - 16x + 55 \)

11. \( f(x) = x^2 - 2x + 5 \)

12. \( f(x) = 4x^2 + 16x \)

13. \( f(x) = x^3 + 10x + 25 \)

14. \( f(x) = -2x^2 + 24x - 64 \)

15. \( f(x) = 3x^2 + 6x - 12 \)

16. \( f(x) = -4x^2 + 12x - 9 \)

17. \( f(x) = -x^2 + 6x - 16 \)

18. \( f(x) = x^2 + 8x + 5 \)

For problems 19 – 25 convert the following equations into the form \( y = a(x - h)^2 + k \).

19. \( f(x) = x^2 + 4x \)

20. \( f(x) = x^2 - 6x + 19 \)

21. \( f(x) = -x^2 + 2x + 6 \)

22. \( f(x) = 7x^2 + 56x + 111 \)

23. \( f(x) = 3x^2 - 60x + 306 \)

24. \( f(x) = 25x^2 + 10x + 1 \)

25. \( f(x) = -2x^2 - 16x - 18 \)
Ellipses

For problems 1 – 7 sketch the ellipse.

1. \( \frac{(x + 5)^2}{4} + \frac{(y - 2)^2}{9} = 1 \)
2. \( (x - 4)^2 + \frac{y^2}{16} = 1 \)
3. \( \frac{(x + 1)^2}{25} + \frac{(y + 6)^2}{4} = 1 \)
4. \( \frac{(x - 3)^2}{5} + \frac{(y + 1)^2}{12} = 1 \)
5. \( 9(x - 2)^2 + 4(y - 3)^2 = 1 \)
6. \( \frac{(x - 3)^2}{9} + 2(y + 4)^2 = 1 \)
7. \( \frac{(x - 4)^2}{9} + \frac{(y - 1)^2}{9} = 1 \)

For problems 8 – 10 complete the square on the \( x \) and \( y \) portions of the equation and write the equation into the standard form of the equation of the ellipse.

8. \( 4x^2 - 16x + y^2 + 2y + 13 = 0 \)
9. \( x^2 + 6x + 4y^2 + 16y + 9 = 0 \)
10. \( 5x^2 + 10x + 3y^2 - 6y - 7 = 0 \)

Hyperbolas

For problems 1 – 5 sketch the hyperbola.

1. \( \frac{x^2}{9} - \frac{y^2}{4} = 1 \)
2. \( \frac{(y + 3)^2}{36} - \frac{(x + 2)^2}{16} = 1 \)

3. \( \frac{(y - 5)^2}{49} - \frac{x^2}{64} = 1 \)

4. \( 9(x - 4)^2 - \frac{(y - 1)^2}{4} = 1 \)

5. \( \frac{1}{25}(y + 1)^2 - 15(x - 3)^2 = 1 \)

For problems 6 – 8 complete the square on the \( x \) and \( y \) portions of the equation and write the equation into the standard form of the equation of the hyperbola.

6. \( 9x^2 - 4y^2 + 48y - 180 = 0 \)

7. \( y^2 - 6y - 4x^2 - 8x - 11 = 0 \)

8. \( 7x^2 - 28x - 4y^2 + 40y - 100 = 0 \)

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**Miscellaneous Functions**

The sole purpose of this section was to get you familiar with the basic shape of some miscellaneous functions for the next section. As such there are no problems for this section. You will see quite a few problems utilizing these functions in the Transformsations section.

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**Transformations**

Use transformations to sketch the graph of the following functions.

1. \( f(x) = |x| - 4 \)

2. \( f(x) = \sqrt{x} - 3 \)

3. \( f(x) = x^2 + 7 \)

4. \( f(x) = \sqrt{x} + 2 \)
5. \( f(x) = (x + 3)^2 \)

6. \( f(x) = |x - 1| \)

7. \( f(x) = -|x| \)

8. \( f(x) = -\sqrt{x} \)

9. \( f(x) = (-x)^3 \)

10. \( f(x) = |-x| \)

11. \( f(x) = \sqrt{x - 2} - 3 \)

12. \( f(x) = (x + 1)^2 - 4 \)

13. \( f(x) = |x + 2| + 4 \)

14. \( f(x) = (x - 5)^3 + 2 \)

**Symmetry**

Determine the symmetry of each of the following equations.

1. \( x^5 + 5y^3 = 2y \)

2. \( y + 4y^2 = 5x^3 + 1 \)

3. \( y^2 = 8x^4 + \frac{x^2}{y^2} - 1 \)

4. \( y = 4x^2 - 7x + 1 \)

5. \( y = 5|x| + 8 \)

6. \( x = 9 - 4y^2 \)

7. \( y^4 + 8y^2 = 5x - 1 \)
8. \( x^2 - 4xy + y^2 = 1 \)

9. \( y = \frac{x^2}{x^2 + 1} \)

**Rational Functions**

Sketch the graph of each of the following functions. Clearly identify all intercepts and asymptotes.

1. \( f(x) = \frac{7}{5x + 10} \)

2. \( f(x) = \frac{6 - x}{x - 3} \)

3. \( f(x) = \frac{8x + 6}{4 - 2x} \)

4. \( f(x) = \frac{-2}{x^2 - 5x} \)

5. \( f(x) = \frac{x + 3}{x^2 + 4x - 5} \)

6. \( f(x) = \frac{2}{x^2 - x - 12} \)

7. \( f(x) = \frac{5x^2 + 1}{2x^2 - 32} \)

8. \( f(x) = \frac{x^2 - 5x + 4}{x^2 + 2x - 15} \)

**Polynomial Functions**
Introduction
Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

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Here is a list of topics in this chapter that have problems written for them.

Dividing Polynomials
Zeroes/Roots of Polynomials
Graphing Polynomials
Finding Zeroes of Polynomials
Partial Fractions

Dividing Polynomials
For problems 1 – 6 use long division to perform the indicated division.

1. Divide $7x^2 + 4x - 9$ by $x - 1$
2. Divide $8x^3 - 4x + 1$ by $x + 6$
3. Divide $x^4 - 2x^2 + 7x$ by $x - 4$
4. Divide $2x^4 - 9x^3 + 2x + 8$ by $x + 3$
5. Divide $8x^4 + x^3 - 3x^2 + 1$ by $x^2 - 2$
6. Divide $4x^3 - 7x^3 + x^2 - 4x + 2$ by $2x^2 - 3x - 6$

For problems 7 – 11 use synthetic division to perform the indicated division.

7. Divide $-x^3 - 8x^2 + x + 10$ by $x + 2$
8. Divide $10x^3 - 9x$ by $x - 10$
9. Divide $3x^4 + 5x^3 + x - 2$ by $x + 7$
10. Divide $x^4 + 2x^3 - 9x + 11$ by $x + 3$
11. Divide $5x^4 - 4x^3 + 3x^2 - 2x + 1$ by $x - 1$
Zeroes/Roots of Polynomials

For problems 1 – 6 list all of the zeros of the polynomial and give their multiplicities.

1. \( f(x) = x^2 + 2x - 120 \)
2. \( R(x) = x^2 + 12x + 32 \)
3. \( h(x) = 4x^3 + x^2 - 3x \)
4. \( A(x) = x^5 + 2x^4 - 35x^3 + 92x^2 - 92x + 32 = (x-1)^2(x+8)(x-2)^2 \)
5. \( Q(x) = x^{10} + 17x^9 + 115x^8 + 387x^7 + 648x^6 + 432x^5 = x^5(x+3)^3(x+4)^2 \)
6. \( g(x) = x^8 + 2x^7 - 14x^6 + 16x^5 + 49x^4 + 62x^3 - 44x^2 - 88x - 32 = (x+4)(x+1)^4(x-2)^3 \)

For problems 7 – 11 \( x = r \) is a root of the given polynomial. Find the other two roots and write the polynomial in fully factored form.

7. \( P(x) = x^4 - 3x^3 - 18x^2 \); \( r = 6 \)
8. \( P(x) = x^3 + x^2 - 46x + 80 \); \( r = -8 \)
9. \( P(x) = x^3 - 9x^2 + 26x - 24 \); \( r = 3 \)
10. \( P(x) = 12x^3 - 13x^2 - 1 \); \( r = -1 \)
11. \( P(x) = 4x^3 + 11x^2 - 134x - 105 \); \( r = 5 \)

For problems 12 – 14 determine the smallest possible degree for a polynomial with the given zeros and their multiplicities.

12. \( r_1 = -2 \) (multiplicity 1), \( r_2 = 1 \) (multiplicity 1), \( r_3 = 4 \) (multiplicity 1)
13. \( r_1 = 3 \) (multiplicity 4), \( r_2 = -5 \) (multiplicity 1)
14. \( r_1 = 7 \) (multiplicity 2), \( r_2 = 4 \) (multiplicity 7), \( r_3 = -10 \) (multiplicity 5)
15. A 7th degree polynomial has roots \( r_1 = -9 \) (multiplicity 2) and \( r_2 = 3 \) (multiplicity 1). What is the maximum number of remaining roots for the polynomial?
Graphing Polynomials

Sketch the graph of each of the following polynomials.

1. \( f(x) = -x^3 - x^2 + 17x - 15 = -(x-1)(x-3)(x+5) \)

2. \( A(x) = x^3 + 2x^2 - 3x \)

3. \( h(x) = x^4 + 2x^3 - 3x^2 \)

4. \( g(x) = x^4 + 14x^3 + 68x^2 + 136x + 96 = (x+2)^2(x+4)(x+6) \)

5. \( Q(x) = -x^5 + 8x^4 - 13x^3 - 22x^2 + 32x + 32 = -(x-4)^2(x+1)^2(x-2) \)

6. \( P(x) = -x^4 + 5x^3 - 6x^2 - 4x + 8 = -(x-2)^3(x+1) \)

7. \( h(x) = x^5 + 5x^4 - 18x^3 - 58x^2 + 145x - 75 = (x-1)^2(x+5)^2(x-3) \)

8. \( R(x) = x^6 - 2x^5 - 11x^4 + 12x^3 + 36x^2 = x^2(x+2)^2(x-3)^2 \)

Finding Zeroes of Polynomials

Find all the zeroes of the following polynomials.

1. \( h(x) = x^3 - 2x^2 - 11x + 12 \)

2. \( f(x) = x^3 + 10x^2 + 29x + 20 \)

3. \( h(x) = 2x^3 - 15x^2 + 34x - 24 \)

4. \( g(x) = x^4 - 6x^3 + 22x + 15 \)

5. \( f(x) = x^4 - 3x^3 - 7x^2 + 15x + 18 \)

6. \( Q(x) = 4x^4 + x^3 - 35x^2 - 24x + 36 \)

7. \( h(x) = 9x^4 + 15x^3 - 11x^2 - 11x - 2 \)

8. \( A(x) = 2x^5 + 19x^4 + 68x^3 + 114x^2 + 90x + 27 \)
Partial Fractions

Determine the partial fraction decomposition of each of the following expressions.

1. \( \frac{22 + 7x}{x^2 + 5x + 4} \)
2. \( \frac{7x - 44}{4x^2 + 25x - 21} \)
3. \( \frac{-x - 47}{x^2 - 11x + 24} \)
4. \( \frac{5 - 38x}{8x^2 + 2x - 1} \)
5. \( \frac{6x^2 + 50x + 16}{(x-1)(x+2)(x+7)} \)
6. \( \frac{32x^2 + 39x - 8}{(x+1)(x+2)(2x-3)} \)
7. \( \frac{36 + 115x - 19x^2}{(x+3)(x-5)(4x-3)} \)
8. \( \frac{3 - 5x}{(x-3)^2} \)
9. \( \frac{24x + 41}{(3x + 5)^2} \)
10. \( \frac{10x + 93}{(x + 10)^2} \)
11. \( \frac{7x^2 + 31x + 107}{(x-4)(x+3)^2} \)

9. \( P(x) = 16x^5 - 48x^4 + 24x^3 + 40x^2 - 39x + 9 \)
12. \( \frac{9x^2 - 58x - 37}{(x + 7)(x - 2)^2} \)

13. \( \frac{21x^2 - 43x + 20}{(3x - 2)(x - 1)^2} \)

14. \( \frac{-7x^2 + 108x - 11}{x(x^2 - 9x + 1)} \)

15. \( \frac{24x^2 + 2x + 117}{x(2x^2 + x + 13)} \)

16. \( \frac{2 - 11x + x^2 - 7x^3}{(x^2 + 2)^2} \)

17. \( \frac{4x^3 - 3x^2 - 5x - 5}{(x^2 + 1)^2} \)

**Exponential and Logarithm Functions**

**Introduction**

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Here is a list of topics in this chapter that have problems written for them.

**Exponential Functions**
**Logarithm Functions**
**Solving Exponential Equations**
**Solving Logarithm Equations**
**Applications**
Exponential Functions

1. Given the function \( f(x) = 9^x \) evaluate each of the following.
   \( a \) \( f(-3) \) \( b \) \( f(-1) \) \( c \) \( f(0) \) \( d \) \( f(\frac{1}{2}) \) \( e \) \( f(\frac{1}{3}) \)

2. Given the function \( f(x) = 8^x \) evaluate each of the following.
   \( a \) \( f(-\frac{2}{3}) \) \( b \) \( f(-1) \) \( c \) \( f(0) \) \( d \) \( f(2) \) \( e \) \( f(\frac{2}{3}) \)

3. Given the function \( f(x) = \left(\frac{1}{4}\right)^x \) evaluate each of the following.
   \( a \) \( f(-2) \) \( b \) \( f(-1) \) \( c \) \( f(0) \) \( d \) \( f(2) \) \( e \) \( f(4) \)

4. Given the function \( f(x) = \left(\frac{1}{16}\right)^x \) evaluate each of the following.
   \( a \) \( f(-2) \) \( b \) \( f(-\frac{1}{2}) \) \( c \) \( f(0) \) \( d \) \( f(2) \) \( e \) \( f(\frac{1}{4}) \)

5. Sketch each of the following.
   \( a \) \( f(x) = \left(\frac{1}{4}\right)^x \) \( b \) \( g(x) = \left(\frac{1}{4}\right)^x + 2 \) \( c \) \( g(x) = \left(\frac{1}{4}\right)^{x+4} \)

6. Sketch each of the following.
   \( a \) \( f(x) = 5^x \) \( b \) \( g(x) = 5^x - 4 \) \( c \) \( g(x) = 5^{x-3} \)

7. Sketch the graph of \( f(x) = 10^{x-2} + 6 \).

8. Sketch the graph of \( f(x) = \left(\frac{1}{7}\right)^{x+4} - 1 \).

9. Sketch the graph of \( f(x) = e^{x+1} - 2 \).

10. Sketch the graph of \( f(x) = e^{x-4} - 1 \).

Logarithm Functions

For problems 1 – 5 write the expression in logarithmic form.

1. \( 11^{-3} = \frac{1}{1331} \)

2. \( 4^7 = 16384 \)
3. \( \left( \frac{2}{7} \right)^{-3} = \frac{343}{8} \)

4. \( 25^{\frac{3}{2}} = 125 \)

5. \( 27^{-\frac{5}{3}} = \frac{1}{243} \)

For problems 6 – 10 write the expression in exponential form.

6. \( \log_{\frac{1}{6}} 36 = -2 \)

7. \( \log_{12} 20736 = 4 \)

8. \( \log_{4} 243 = \frac{5}{2} \)

9. \( \log_{4} \frac{1}{128} = -\frac{7}{2} \)

10. \( \log_{8} 32768 = 5 \)

For problems 11 – 18 determine the exact value of each of the following without using a calculator.

11. \( \log_{7} 343 \)

12. \( \log_{4} 1024 \)

13. \( \log_{5} \frac{27}{512} \)

14. \( \log_{11} \frac{1}{121} \)

15. \( \log_{0.1} 0.0001 \)

16. \( \log_{16} 4 \)

17. \( \log 10000 \)
18. \( \ln \frac{1}{\sqrt[3]{e}} \)

For problems 19 – 20 write each of the following in terms of simpler logarithms

19. \( \log_7 \left( 10a^7b^3c^{-8} \right) \)

20. \( \log \left[ z^2 \left( x^2 + 4 \right)^3 \right] \)

21. \( \ln \left( \frac{w^2 \sqrt[3]{t}}{\sqrt{t + w}} \right) \)

For problems 22 – 24 combine each of the following into a single logarithm with a coefficient of one.

22. \( 7 \ln t - 6 \ln s + 5 \ln w \)

23. \( \frac{1}{2} \log (z + 1) - 2 \log x - 4 \log y - 3 \log z \)

24. \( 2 \log_3 (x + y) + 6 \log_3 x - \frac{1}{3} \)

For problems 25 & 26 use the change of base formula and a calculator to find the value of each of the following.

25. \( \log_7 100 \)

26. \( \log_7 \frac{1}{8} \)

For problems 27 – 31 sketch each of the given functions.

27. \( g(x) = \ln (-x) \)

28. \( g(x) = \ln (x - 3) \)

29. \( g(x) = \ln (x) + 7 \)

30. \( g(x) = \ln (x + 2) - 4 \)

31. \( g(x) = \ln (x - 6) + 2 \)
**Solving Exponential Equations**

Solve each of the following equations.

1. \(11^{4+x} = 11^{7-10x}\)
2. \(3^{4x} = 3^{7x}\)
3. \(2^{1-x} = 2^{4-3x}\)
4. \(9^{x^2} = 9^{12-4x}\)
5. \(6^{x^2-3x} = 6^{20+5x}\)
6. \(6^{4+x} = \frac{1}{36^{4x+2}}\)
7. \(9^x = 27^{2+x}\)
8. \(8^{4x+1} = 1\)
9. \(3 = 14^{9-2x}\)
10. \(6^{2+x} = 8^{8+2x}\)
11. \(13^{5+7x} = 2^{3-x}\)
12. \(10^{7x} = 3\)
13. \(16 = 10^{2+3x}\)
14. \(6 = e^{4+9x}\)
15. \(9 - e^{6x} = 0\)
16. \(e^{x^2-2} = 4\)

**Solving Logarithm Equations**

Solve each of the following equations.
1. \( \log_{11} \left(x^2 + 3x\right) = \log_{11} \left(3x + 16\right) \)

2. \( \ln \left(4 - 3x\right) - \ln \left(7x\right) = \ln \left(11\right) \)

3. \( \log \left(x\right) + \log \left(x + 12\right) = \log \left(x - 10\right) \)

4. \( \ln \left(x\right) = \ln \left(15 - x\right) - \ln \left(x + 1\right) \)

5. \( \log_8 \left(4x + 1\right) = -1 \)

6. \( \log_6 \left(3x\right) - \log_6 \left(x + 5\right) = 1 \)

7. \( \log_3 \left(x\right) + \log_3 \left(x + 6\right) = 3 \)

8. \( \log_2 \left(x^2\right) = 2 + \log_2 \left(8 - x\right) \)

9. \( \log_4 \left(x\right) = 2 - \log_4 \left(x + 6\right) \)

10. \( \log \left(-x\right) + \log \left(15 - x\right) = 2 \)

11. \( \ln \left(x\right) + \ln \left(x - 2\right) = 3 \)

12. \( 2 \log \left(x\right) - \log \left(x^2 + 4x + 1\right) = 0 \)

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**Applications**

1. We have $2,500 to invest and 80 months. How much money will we have if we put the money into an account that has an annual interest rate of 9% and interest is compounded
   (a) quarterly  
   (b) monthly  
   (c) continuously

2. We are starting with $60,000 and we’re going to put it into an account that earns an annual interest rate of 7.5%. How long will it take for the money in the account to reach $100,000 if the interest is compounded
   (a) quarterly  
   (b) monthly  
   (c) continuously

3. Suppose that we put some money in an account that has an annual interest rate of 10.25%. How long will it take to triple our money if the interest is compounded
   (a) twice a year  
   (b) 8 times a year  
   (c) continuously
4. A population of bacteria initially has 90,000 present and in 2 weeks there will be 200,000 bacteria present.
   (a) Determine the exponential growth equation for this population.
   (b) How long will it take for the population to grow from its initial population of 90,000 to a population of 150,000?

5. We initially have 2 kg grams of some radioactive element and in 7250 years there will be 1.5 kg left.
   (a) Determine the exponential decay equation for this element.
   (b) How long will it take for half of the element to decay?
   (c) How long will it take until there is 250 grams of the element left?

6. For a particular radioactive element the value of \( k \) in the exponential decay equation is given by \( k = -0.000825 \).
   (a) How long will it take for a quarter of the element to decay?
   (b) How long will it take for half of the element to decay?
   (c) How long will it take 90\% of the element to decay?

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**Systems of Equations**

**Introduction**

Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.

- Linear Systems with Two Variables
- Linear Systems with Three Variables
- Augmented Matrices
- More on the Augmented Matrix
- Nonlinear Systems
Linear Systems with Two Variables

For problems 1 – 5 use the Method of Substitution to find the solution to the given system or to determine if the system is inconsistent or dependent.

1. \[ 8x + y = 13 \]
   \[ 3x + 4y = -6 \]

2. \[ x - 3y = 7 \]
   \[ -2x + 6y = 4 \]

3. \[ -12x + 6y = -12 \]
   \[ 4x + 2y = -2 \]

4. \[ 3x + 6y = 12 \]
   \[ -4x - 7y = -12 \]

5. \[ 12x - 6y = 18 \]
   \[ 4x - 2y = 6 \]

For problems 6 – 10 use the Method of Elimination to find the solution to the given system or to determine if the system is inconsistent or dependent.

6. \[ -5x + 10y = 1 \]
   \[ x - 2y = -8 \]

7. \[ 7x + 6y = 0 \]
   \[ 2x + 3y = 0 \]

8. \[ -8x + 24y = 12 \]
   \[ 10x - 30y = -15 \]

9. \[ -2x + 3y = 24 \]
   \[ 3x - 8y = -57 \]
10. \(6x + 4y = -20\)
\(7x + 3y = -35\)

**Linear Systems with Three Variables**

Find the solution to each of the following systems of equations.

1. \(-3x + 7y + 2z = -8\)
\(-2x + 5y - z = -10\)
\(8x - 2y + 3z = 38\)

2. \(6x + 4y - 8z = -56\)
\(-x - 4y + z = 5\)
\(3x + y + 9z = 10\)

3. \(2x + 6y - z = 1\)
\(-x + 2y + 9z = -19\)
\(4x + 3y - 7z = 25\)

**Augmented Matrices**

1. For the following augmented matrix perform the indicated elementary row operations.

\[
\begin{bmatrix}
9 & 0 & 7 & 4 \\
-3 & 2 & -1 & -7 \\
2 & 4 & 1 & 2
\end{bmatrix}
\]

(a) \(-4R_2\)  (b) \(R_3 \leftrightarrow R_1\)  (c) \(R_1 - 10R_3 \rightarrow R_1\)

2. For the following augmented matrix perform the indicated elementary row operations.

\[
\begin{bmatrix}
9 & 3 & 11 & 6 \\
-2 & 7 & 4 & -3 \\
1 & -1 & 1 & -1
\end{bmatrix}
\]

(a) \(5R_1\)  (b) \(R_2 \leftrightarrow R_3\)  (c) \(R_3 - 2R_2 \rightarrow R_3\)
3. For the following augmented matrix perform the indicated elementary row operations.

\[
\begin{bmatrix}
4 & 12 & -8 & | & 0 \\
-9 & -2 & 1 & | & 3 \\
1 & 5 & -1 & | & -10
\end{bmatrix}
\]

(a) \( \frac{1}{3} R_3 \)  
(b) \( R_1 \leftrightarrow R_2 \)  
(c) \( R_2 + \frac{5}{2} R_1 \rightarrow R_2 \)

4. For the following augmented matrix perform the indicated elementary row operations.

\[
\begin{bmatrix}
1 & 5 & -6 & | & -2 \\
-3 & -15 & -18 & | & 3 \\
4 & -2 & 7 & | & 1
\end{bmatrix}
\]

(a) \(-7R_3\)  
(b) \( R_1 \leftrightarrow R_3 \)  
(c) \( R_2 + 3R_1 \rightarrow R_2 \)

Note: Problems using augmented matrices to solve systems of equations are in the next section.

More on the Augmented Matrix

For each of the following systems of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

1. \( 8x + y = 13 \)  
   \( 3x + 4y = -6 \)

2. \( x - 3y = 7 \)  
   \(-2x + 6y = 4 \)

3. \(-12x + 6y = -12 \)  
   \( 4x + 2y = -2 \)

4. \( 3x + 6y = 12 \)  
   \(-4x - 7y = -12 \)
5. \[12x - 6y = 18\]
   \[4x - 2y = 6\]

6. \[-5x + 10y = 1\]
   \[x - 2y = -8\]

7. \[7x + 6y = 0\]
   \[2x + 3y = 0\]

8. \[-8x + 24y = 12\]
   \[10x - 30y = -15\]

9. \[-2x + 3y = 24\]
   \[3x - 8y = -57\]

10. \[6x + 4y = -20\]
    \[7x + 3y = -35\]

11. \[-3x + 7y + 2z = -8\]
    \[-2x + 5y - z = -10\]
    \[8x - 2y + 3z = 38\]

12. \[6x + 4y - 8z = -56\]
    \[-x - 4y + z = 5\]
    \[3x + y + 9z = 10\]

13. \[2x + 6y - z = 1\]
    \[-x + 2y + 9z = -19\]
    \[4x + 3y - 7z = 25\]
Non-Linear Systems

Find the solution to each of the following system of equations.

1. \( y = -x^2 + 5x + 16 \)
   \( y = 7x - 8 \)

2. \( y = 3 - x^2 \)
   \( y = 8x^2 + 2 \)

3. \( x^2 + \frac{y^2}{4} = 1 \)
   \( y = 4 - 4x \)

4. \( x^2 + y^2 = 9 \)
   \( y = 1 + \frac{x^2}{5} \)

5. \( x^2 + y^2 = 16 \)
   \( y^2 - \frac{x^2}{15} = 1 \)

6. \( xy = -2 \)
   \( x^2 + \frac{y^2}{25} = 1 \)

7. \( x^2 + y^2 = 1 \)
   \( \frac{x^2}{4} + y^2 = 1 \)

8. \( x^2 + y^2 = 3 \)
   \( \frac{x^2}{9} + y^2 = 1 \)