Preface

Here are a set of problems for my Calculus I notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.

Limits

Introduction

Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.

Tangent Lines and Rates of Change
The Limit
1. For the function \( f(x) = x^3 - 3x^2 \) and the point \( P \) given by \( x = 3 \) answer each of the following questions.

(a) For the points \( Q \) given by the following values of \( x \) compute (accurate to at least 8 decimal places) the slope, \( m_{PQ} \), of the secant line through points \( P \) and \( Q \).

(i) 3.5  
(ii) 3.1  
(iii) 3.01  
(iv) 3.001  
(v) 3.0001  
(vi) 2.5  
(vii) 2.9  
(viii) 2.99  
(ix) 2.999  
(x) 2.9999

(b) Use the information from (a) to estimate the slope of the tangent line to \( f(x) \) at \( x = 3 \) and write down the equation of the tangent line.

2. For the function \( g(x) = \frac{x}{x^2 + 4} \) and the point \( P \) given by \( x = 0 \) answer each of the following questions.

(a) For the points \( Q \) given by the following values of \( x \) compute (accurate to at least 8 decimal places) the slope, \( m_{PQ} \), of the secant line through points \( P \) and \( Q \).

(i) 1  
(ii) 0.5  
(iii) 0.1  
(iv) 0.01  
(v) 0.001  
(vi) -1  
(vii) -0.5  
(viii) -0.1  
(ix) -0.01  
(x) -0.001

(b) Use the information from (a) to estimate the slope of the tangent line to \( g(x) \) at \( x = 0 \) and write down the equation of the tangent line.

3. For the function \( h(x) = 2 - (x + 2)^2 \) and the point \( P \) given by \( x = -2 \) answer each of the following questions.
(a) For the points $Q$ given by the following values of $x$ compute (accurate to at least 8 decimal places) the slope, $m_{PQ}$, of the secant line through points $P$ and $Q$.

(i) -2.5  (ii) -2.1  (iii) -2.01  (iv) -2.001  (v) -2.0001  
(vi) -1.5  (vii) -1.9  (viii) -1.99  (ix) -1.999  (x) -1.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $h(x)$ at $x = -2$ and write down the equation of the tangent line.

4. For the function $P(x) = e^{2-8x^2}$ and the point $P$ given by $x = 0.5$ answer each of the following questions.

(a) For the points $Q$ given by the following values of $x$ compute (accurate to at least 8 decimal places) the slope, $m_{PQ}$, of the secant line through points $P$ and $Q$.

(i) 1  (ii) 0.51  (iii) 0.501  (iv) 0.5001  (v) 0.50001  
(vi) 0  (vii) 0.49  (viii) 0.499  (ix) 0.4999  (x) 0.49999

(b) Use the information from (a) to estimate the slope of the tangent line to $h(x)$ at $x = 0.5$ and write down the equation of the tangent line.

5. The amount of grain in a bin is given by $V(t) = \frac{11t + 4}{t + 4}$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the amount of grain in the bin between $t = 6$ and the following values of $t$.

(i) 6.5  (ii) 6.1  (iii) 6.01  (iv) 6.001  (v) 6.0001  
(vi) 5.5  (vii) 5.9  (viii) 5.99  (ix) 5.999  (x) 5.9999

(b) Use the information from (a) to estimate the instantaneous rate of change of the volume of air in the balloon at $t = 6$.

6. The population (in thousands) of insects is given by $P(t) = 2 - \frac{1}{\pi} \cos(3\pi t) \sin\left(\frac{\pi t}{2}\right)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the population of insects between $t = 4$ and the following values of $t$. Make sure your calculator is set to radians for the computations.

(i) 4.5  (ii) 4.1  (iii) 4.01  (iv) 4.001  (v) 4.0001  
(vi) 3.5  (vii) 3.9  (viii) 3.99  (ix) 3.999  (x) 3.9999
(b) Use the information from (a) to estimate the instantaneous rate of change of the population of the insects at \( t = 4 \).

7. The amount of water in a holding tank is given by \( V(t) = 8t^3 - t^2 + 7 \) answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the amount of grain in the bin between \( t = 0.25 \) and the following values of \( t \).

(i) 1  
(ii) 0.5  
(iii) 0.251  
(iv) 0.2501  
(v) 0.25001

(vi) 0  
(vii) 0.1  
(viii) 0.249  
(ix) 0.2499  
(x) 0.24999

(b) Use the information from (a) to estimate the instantaneous rate of change of the volume of water in the tank at \( t = 0.25 \).

8. The position of an object is given by \( s(t) = x^2 + \frac{72}{x+1} \) answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between \( t = 5 \) and the following values of \( t \).

(i) 5.5  
(ii) 5.1  
(iii) 5.01  
(iv) 5.001  
(v) 5.0001

(vi) 4.5  
(vii) 4.9  
(viii) 4.99  
(ix) 4.999  
(x) 4.9999

(b) Use the information from (a) to estimate the instantaneous velocity of the object at \( t = 5 \) and determine if the object is moving to the right (i.e. the instantaneous velocity is positive), moving to the left (i.e. the instantaneous velocity is negative), or not moving (i.e. the instantaneous velocity is zero).

9. The position of an object is given by \( s(t) = 2\cos(4t-8) - 7\sin(t-2) \). Note that a negative position here simply means that the position is to the left of the “zero position” and is perfectly acceptable. Answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between \( t = 2 \) and the following values of \( t \). Make sure your calculator is set to radians for the computations.

(i) 2.5  
(ii) 2.1  
(iii) 2.01  
(iv) 2.001  
(v) 2.0001

(vi) 1.5  
(vii) 1.9  
(viii) 1.99  
(ix) 1.999  
(x) 1.9999

(b) Use the information from (a) to estimate the instantaneous velocity of the object at \( t = 2 \) and determine if the object is moving to the right (i.e. the instantaneous velocity is positive), moving to the left (i.e. the instantaneous velocity is negative), or not moving (i.e. the instantaneous velocity is zero).
10. The position of an object is given by \( s(t) = t^2 - 10t + 11 \). Note that a negative position here simply means that the position is to the left of the “zero position” and is perfectly acceptable. Answer each of the following questions.

(a) Determine the time(s) in which the position of the object is at \( s = -5 \)

(b) Estimate the instantaneous velocity of the object at each of the time(s) found in part (a) using the method discussed in this section.

The Limit

1. For the function \( g(x) = \frac{x^2 + 6x + 9}{x^2 + 3x} \) answer each of the following questions.

(a) Evaluate the function the following values of \( x \) compute (accurate to at least 8 decimal places).

\[
\begin{align*}
\text{(i)} & \quad -2.5 \\
\text{(ii)} & \quad -2.9 \\
\text{(iii)} & \quad -2.99 \\
\text{(iv)} & \quad -2.999 \\
\text{(v)} & \quad -2.9999 \\
\text{(vi)} & \quad -3.5 \\
\text{(vii)} & \quad -3.1 \\
\text{(viii)} & \quad -3.01 \\
\text{(ix)} & \quad -3.001 \\
\text{(x)} & \quad -3.0001
\end{align*}
\]

(b) Use the information from (a) to estimate the value of \( \lim_{x \to -3} \frac{x^2 + 6x + 9}{x^2 + 3x} \).

2. For the function \( f(z) = \frac{10z - 9 - z^2}{z^2 - 1} \) answer each of the following questions.

(a) Evaluate the function the following values of \( z \) compute (accurate to at least 8 decimal places).

\[
\begin{align*}
\text{(i)} & \quad 1.5 \\
\text{(ii)} & \quad 1.1 \\
\text{(iii)} & \quad 1.01 \\
\text{(iv)} & \quad 1.001 \\
\text{(v)} & \quad 1.0001 \\
\text{(vi)} & \quad 0.5 \\
\text{(vii)} & \quad 0.9 \\
\text{(viii)} & \quad 0.99 \\
\text{(ix)} & \quad 0.999 \\
\text{(x)} & \quad 0.9999
\end{align*}
\]

(b) Use the information from (a) to estimate the value of \( \lim_{z \to 1} \frac{10z - 9 - z^2}{z^2 - 1} \).

3. For the function \( h(t) = \frac{2 - \sqrt{4 + 2t}}{t} \) answer each of the following questions.
(a) Evaluate the function the following values of $\theta$ compute (accurate to at least 8 decimal places). Make sure your calculator is set to radians for the computations.

(i) 0.5  (ii) 0.1  (iii) 0.01  (iv) 0.001  (v) 0.0001  
(vi) -0.5  (vii) -0.1  (viii) -0.01  (ix) -0.001  (x) -0.0001

(b) Use the information from (a) to estimate the value of $\lim_{t \to 0} \frac{2 - \sqrt{4 + 2t}}{t}$.

4. For the function $g(\theta) = \frac{\cos(\theta - 4) - 1}{2\theta - 8}$ answer each of the following questions.

(a) Evaluate the function the following values of $\theta$ compute (accurate to at least 8 decimal places). Make sure your calculator is set to radians for the computations.

(i) 4.5  (ii) 4.1  (iii) 4.01  (iv) 4.001  (v) 4.0001  
(vi) 3.5  (vii) 3.9  (viii) 3.99  (ix) 3.999  (x) 3.9999

(b) Use the information from (a) to estimate the value of $\lim_{\theta \to 0} \frac{\cos(\theta - 4) - 1}{2\theta - 8}$.

5. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -2$  (b) $a = -1$  (c) $a = 2$  (d) $a = 3$

6. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -3$  (b) $a = -1$  (c) $a = 1$  (d) $a = 3$
7. Below is the graph of \( f(x) \). For each of the given points determine the value of \( f(a) \) and \( \lim_{x \to a} f(x) \). If any of the quantities do not exist clearly explain why.

(a) \( a = -4 \)  
(b) \( a = -2 \)  
(c) \( a = 1 \)  
(d) \( a = 4 \)

8. Explain in your own words what the following equation means.
\[
\lim_{x \to 12} f(x) = 6
\]

9. Suppose we know that \( \lim_{x \to -7} f(x) = 18 \). If possible, determine the value of \( f(-7) \). If it is not possible to determine the value explain why not.

10. Is it possible to have \( \lim_{x \to 1} f(x) = -23 \) and \( f(1) = 107 \)? Explain your answer.
One-Sided Limits

1. Below is the graph of \( f(x) \). For each of the given points determine the value of \( f(a) \), \( \lim_{x \to a^-} f(x) \), and \( \lim_{x \to a^+} f(x) \). If any of the quantities do not exist clearly explain why.

   (a) \( a = -5 \)  
   (b) \( a = -2 \)  
   (c) \( a = 1 \)  
   (d) \( a = 4 \)

2. Below is the graph of \( f(x) \). For each of the given points determine the value of \( f(a) \), \( \lim_{x \to a^-} f(x) \), and \( \lim_{x \to a^+} f(x) \). If any of the quantities do not exist clearly explain why.

   (a) \( a = -1 \)  
   (b) \( a = 1 \)  
   (c) \( a = 3 \)

3. Below is the graph of \( f(x) \). For each of the given points determine the value of \( f(a) \), \( \lim_{x \to a^-} f(x) \), and \( \lim_{x \to a^+} f(x) \). If any of the quantities do not exist clearly explain why.

   (a) \( a = -3 \)  
   (b) \( a = -1 \)  
   (c) \( a = 1 \)  
   (d) \( a = 2 \)
4. Sketch a graph of a function that satisfies each of the following conditions.

\[ \lim_{{x \to 1}} f(x) = -2 \quad \lim_{{x \to 1}^+} f(x) = 3 \quad f(1) = 6 \]

5. Sketch a graph of a function that satisfies each of the following conditions.

\[ \lim_{{x \to -3}} f(x) = 1 \quad \lim_{{x \to -3}^+} f(x) = 1 \quad f(-3) = 4 \]

6. Sketch a graph of a function that satisfies each of the following conditions.

\[ \lim_{{x \to -5}^-} f(x) = -1 \quad \lim_{{x \to -5}^+} f(x) = 7 \quad f(-5) = 4 \]
\[ \lim_{{x \to 4}} f(x) = 6 \quad f(4) \text{ does not exist} \]

7. Explain in your own words what each of the following equations mean.

\[ \lim_{{x \to 8}^-} f(x) = 3 \quad \lim_{{x \to 8}^+} f(x) = -1 \]

8. Suppose we know that \( \lim_{{x \to -7}^-} f(x) = 18 \). If possible, determine the value of \( \lim_{{x \to -7}^+} f(x) \) and the value of \( \lim_{{x \to -7}^+} f(x) \). If it is not possible to determine one or both of these values explain why not.

9. Suppose we know that \( f(6) = -53 \). If possible, determine the value of \( \lim_{{x \to 6}^-} f(x) \) and the value of \( \lim_{{x \to 6}^+} f(x) \). If it is not possible to determine one or both of these values explain why not.
Limit Properties

1. Given \( \lim_{x \to 0} f(x) = 5 \), \( \lim_{x \to 0} g(x) = -1 \) and \( \lim_{x \to 0} h(x) = -3 \) use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) \( \lim_{x \to 0} \left[ 11 + 7f(x) \right] \)
(b) \( \lim_{x \to 0} \left[ 6 - 4g(x) - 10h(x) \right] \)
(c) \( \lim_{x \to 0} \left[ 4g(x) - 12f(x) + 3h(x) \right] \)
(d) \( \lim_{x \to 0} \left[ g(x)(1 + 2f(x)) \right] \)

2. Given \( \lim_{x \to 2} f(x) = 2 \), \( \lim_{x \to 2} g(x) = 6 \) and \( \lim_{x \to 2} h(x) = 9 \) use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) \( \lim_{x \to 2} \left[ h(x)f(x) + \frac{1 + g(x)}{g(x)} \right] \)
(b) \( \lim_{x \to 2} \left[ (3 - f(x))(1 + 2g(x)) \right] \)
(c) \( \lim_{x \to 2} \left( \frac{f(x) + 1}{3g(x) - 2h(x)} \right) \)
(d) \( \lim_{x \to 2} \left( \frac{f(x) - 2g(x)}{7 + h(x)f(x)} \right) \)

3. Given \( \lim_{x \to 1} f(x) = 0 \), \( \lim_{x \to 1} g(x) = 9 \) and \( \lim_{x \to 1} h(x) = -7 \) use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) \( \lim_{x \to 1} \left[ (g(x))^2 - (h(x))^3 \right] \)
(b) \( \lim_{x \to 1} \sqrt{3 + 6f(x) - h(x)} \)
(c) \( \lim_{x \to 1} \sqrt{f(x) - g(x)h(x)} \)
(d) \( \lim_{x \to 1} \sqrt{\frac{2 + g(x)}{1 - 10h(x)}} \)

For each of the following limits use the limit properties given in this section to compute the limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

4. \( \lim_{x \to 4} \left( 3x^2 - 9x + 2 \right) \)
5. \( \lim_{w \to -1} \left(w - \left(w^2 + 3\right)^2\right) \)

6. \( \lim_{t \to 0} \left(t^4 - 4t^2 + 12t - 8\right) \)

7. \( \lim_{z \to 2} \frac{10 + z^2}{3 - 4z} \)

8. \( \lim_{x \to 7} \frac{8x}{x^2 - 14x + 49} \)

9. \( \lim_{y \to 3} \frac{y^3 - 20y + 4}{y^2 + 8y - 1} \)

10. \( \lim_{w \to -6} \sqrt[3]{8 + 7w} \)

11. \( \lim_{t \to 1} \left(4t^2 - \sqrt{8t + 1}\right) \)

12. \( \lim_{x \to 8} \left(\sqrt[3]{3x - 8} + \sqrt{9 + 2x}\right) \)

**Computing Limits**

For problems 1 – 20 evaluate the limit, if it exists.

1. \( \lim_{x \to -4} \left(1 - 4x^3\right) \)

2. \( \lim_{y \to -1} \left(6y^4 - 7y^3 + 12y + 25\right) \)

3. \( \lim_{t \to 0} \frac{t^2 + 6}{t^2 - 3} \)

4. \( \lim_{z \to -4} \frac{6z}{2 + 3z^2} \)

5. \( \lim_{w \to -2} \frac{w + 2}{w^2 - 6w - 16} \)

© 2007 Paul Dawkins

http://tutorial.math.lamar.edu/terms.aspx
6. \( \lim_{t \to -5} \frac{t^2 + 6t + 5}{t^2 + 2t - 15} \)

7. \( \lim_{x \to 3} \frac{5x^2 - 16x + 3}{9 - x^2} \)

8. \( \lim_{z \to 1} \frac{10 - 9z - z^2}{3z^2 + 4z - 7} \)

9. \( \lim_{x \to -2} \frac{x^3 + 8}{x^2 + 8x + 12} \)

10. \( \lim_{t \to 8} \frac{t(t - 5) - 24}{t^2 - 8t} \)

11. \( \lim_{w \to -4} \frac{w^2 - 16}{(w - 2)(w + 3) - 6} \)

12. \( \lim_{h \to 0} \frac{(2 + h)^3 - 8}{h} \)

13. \( \lim_{h \to 0} \frac{(1 + h)^4 - 1}{h} \)

14. \( \lim_{t \to 25} \frac{5 - \sqrt{t}}{t - 25} \)

15. \( \lim_{x \to 2} \frac{x - 2}{\sqrt{2} - \sqrt{x}} \)

16. \( \lim_{z \to 6} \frac{z - 6}{\sqrt{3z - 2} - 4} \)

17. \( \lim_{z \to -2} \frac{3 - \sqrt{1 - 4z}}{2z + 4} \)
18. \( \lim_{{t \to -1}} \frac{3-t}{\sqrt{t+1} - \sqrt{5t-11}} \)

19. \( \lim_{{x \to 7}} \frac{1}{7} - \frac{1}{x} \)

20. \( \lim_{{y \to -1}} \frac{1}{4+3y} + \frac{1}{y+1} \)

21. Given the function

\[ f(x) = \begin{cases} 
15 & x < -4 \\
6 - 2x & x \geq -4 
\end{cases} \]

Evaluate the following limits, if they exist.

(a) \( \lim_{{x \to -7}} f(x) \)  
(b) \( \lim_{{x \to -4}} f(x) \)

22. Given the function

\[ g(t) = \begin{cases} 
t^2 - t^3 & t < 2 \\
5t - 14 & t \geq 2 
\end{cases} \]

Evaluate the following limits, if they exist.

(a) \( \lim_{{t \to -3}} g(t) \)  
(b) \( \lim_{{t \to 2}} g(t) \)

23. Given the function

\[ h(w) = \begin{cases} 
2w^2 & w \leq 6 \\
w - 8 & w > 6 
\end{cases} \]

Evaluate the following limits, if they exist.

(a) \( \lim_{{w \to 6}} h(w) \)  
(b) \( \lim_{{w \to 2}} h(w) \)

24. Given the function

\[ g(x) = \begin{cases} 
5x + 24 & x < -3 \\
x^2 & -3 \leq x < 4 \\
1 - 2x & x \geq 4 
\end{cases} \]

Evaluate the following limits, if they exist.
Calculus I

(a) \( \lim_{x \to -3} g(x) \)  
(b) \( \lim_{x \to 0} g(x) \)  
(c) \( \lim_{x \to 4} g(x) \)  
(d) \( \lim_{x \to 12} g(x) \)

For problems 25 – 30 evaluate the limit, if it exists.

25. \( \lim_{x \to -10} (|t + 10| + 3) \)

26. \( \lim_{x \to 4} (9 + |8 - 2x|) \)

27. \( \lim_{h \to 0} \frac{|h|}{h} \)

28. \( \lim_{t \to 2} \frac{2 - t}{|t - 2|} \)

29. \( \lim_{w \to -5} \frac{2w + 10}{w + 5} \)

30. \( \lim_{x \to 4} \frac{|x - 4|}{x^2 - 16} \)

31. Given that \( 3 + 2x \leq f(x) \leq x - 1 \) for all \( x \) determine the value of \( \lim_{x \to 4} f(x) \).

32. Given that \( \sqrt{x + 7} \leq f(x) \leq \frac{x - 1}{2} \) for all \( x \) determine the value of \( \lim_{x \to 9} f(x) \).

33. Use the Squeeze Theorem to determine the value of \( \lim_{x \to 0} x^4 \cos \left( \frac{3}{x} \right) \).

34. Use the Squeeze Theorem to determine the value of \( \lim_{x \to 0} x \cos \left( \frac{1}{x} \right) \).

35. Use the Squeeze Theorem to determine the value of \( \lim_{x \to 1} (x - 1)^2 \cos \left( \frac{1}{x - 1} \right) \).
**Infinite Limits**

For problems 1 – 8 evaluate the indicated limits, if they exist.

1. For \( g(x) = \frac{-4}{(x-1)^2} \) evaluate,

(a) \( \lim_{x \to 1^-} g(x) \)  \hspace{1cm} (b) \( \lim_{x \to 1^+} g(x) \)  \hspace{1cm} (c) \( \lim_{x \to 1} g(x) \)

2. For \( h(z) = \frac{17}{(4-z)^3} \) evaluate,

(a) \( \lim_{z \to 4^-} h(z) \)  \hspace{1cm} (b) \( \lim_{z \to 4^+} h(z) \)  \hspace{1cm} (c) \( \lim_{z \to 4} h(z) \)

3. For \( g(t) = \frac{4t^2}{(t+3)^7} \) evaluate,

(a) \( \lim_{t \to -3} g(t) \)  \hspace{1cm} (b) \( \lim_{t \to -3^-} g(t) \)  \hspace{1cm} (c) \( \lim_{t \to -3^+} g(t) \)

4. For \( f(x) = \frac{1+x}{x^3 + 8} \) evaluate,

(a) \( \lim_{x \to -2^-} f(x) \)  \hspace{1cm} (b) \( \lim_{x \to -2^+} f(x) \)  \hspace{1cm} (c) \( \lim_{x \to -2} f(x) \)

5. For \( f(x) = \frac{x-1}{(x^2 - 9)^2} \) evaluate,

(a) \( \lim_{x \to 3^-} f(x) \)  \hspace{1cm} (b) \( \lim_{x \to 3^+} f(x) \)  \hspace{1cm} (c) \( \lim_{x \to 3} f(x) \)

6. For \( W(t) = \ln(t+8) \) evaluate,

(a) \( \lim_{w \to 8^-} W(t) \)  \hspace{1cm} (b) \( \lim_{w \to 8^+} W(t) \)  \hspace{1cm} (c) \( \lim_{w \to 8} W(t) \)

7. For \( h(z) = \ln|z| \) evaluate,

(a) \( \lim_{z \to 0^-} h(z) \)  \hspace{1cm} (b) \( \lim_{z \to 0^+} h(z) \)  \hspace{1cm} (c) \( \lim_{z \to 0} h(z) \)

8. For \( R(y) = \cot(y) \) evaluate,

(a) \( \lim_{y \to \pi^-} R(y) \)  \hspace{1cm} (b) \( \lim_{y \to \pi^+} R(y) \)  \hspace{1cm} (c) \( \lim_{y \to \pi} R(y) \)
For problems 9 – 12 find all the vertical asymptotes of the given function.

9. \( h(x) = \frac{-6}{9-x} \)

10. \( f(x) = \frac{x+8}{x^2(5-2x)^3} \)

11. \( g(t) = \frac{5t}{x(x+7)(x-12)} \)

12. \( g(z) = \frac{z^2+1}{(z^2-1)^5(z+15)^6} \)

**Limits At Infinity, Part I**

1. For \( f(x) = 8x + 9x^3 - 11x^5 \) evaluate each of the following limits.
   (a) \( \lim_{x \to -\infty} f(x) \)  
   (b) \( \lim_{x \to \infty} f(x) \)

2. For \( h(t) = 10t^2 + t^4 + 6t - 2 \) evaluate each of the following limits.
   (a) \( \lim_{t \to -\infty} h(t) \)  
   (b) \( \lim_{t \to \infty} h(t) \)

3. For \( g(z) = 7 + 8z + \sqrt{z^4} \) evaluate each of the following limits.
   (a) \( \lim_{z \to -\infty} g(z) \)  
   (b) \( \lim_{z \to \infty} g(z) \)

For problems 4 – 17 answer each of the following questions.

(a) Evaluate \( \lim_{x \to -\infty} f(x) \)

(b) Evaluate \( \lim_{x \to \infty} f(x) \)

(c) Write down the equation(s) of any horizontal asymptotes for the function.

4. \( f(x) = \frac{10x^3 - 6x}{7x^3 + 9} \)
5. \( f(x) = \frac{12 + x}{3x^2 - 8x + 23} \)

6. \( f(x) = \frac{5x^8 - 9}{x^3 + 10x^5 - 3x^8} \)

7. \( f(x) = \frac{2 - 6x - 9x^2}{15x^2 + x - 4} \)

8. \( f(x) = \frac{5x + 7x^4}{4 - x^3} \)

9. \( f(x) = \frac{4x^3 - 3x^2 + 2x - 1}{10 - 5x + x^3} \)

10. \( f(x) = \frac{5 - x^8}{2x^3 - 7x + 1} \)

11. \( f(x) = \frac{1 + 4\sqrt[3]{x^2}}{9 + 10x} \)

12. \( f(x) = \frac{25x + 7}{\sqrt{5x^2 + 2}} \)

13. \( f(x) = \frac{\sqrt{8 + 11x^2}}{-9 - x} \)

14. \( f(x) = \frac{\sqrt{9x^4 + 2x^2 + 3}}{5x - 2x^2} \)

15. \( f(x) = \frac{6 + x^3}{\sqrt{8 + 4x^6}} \)

16. \( f(x) = \frac{\sqrt[3]{2 - 8x^3}}{4 + 7x} \)
17. \( f(x) = \frac{1 + x}{\sqrt[4]{5 + 2x^4}} \)

**Limits At Infinity, Part II**

For problems 1 – 11 evaluate (a) \( \lim_{x \to -\infty} f(x) \) and (b) \( \lim_{x \to \infty} f(x) \).

1. \( f(x) = e^{x^4 + 8x} \)

2. \( f(x) = e^{2x^4 + 4x^2 + 2x^3} \)

3. \( f(x) = e^{3-x^3} \)

4. \( f(x) = e^{\frac{5-9x}{7+3x}} \)

5. \( f(x) = e^{\frac{5+2x^6}{x-8x^5}} \)

6. \( f(x) = e^x + 12e^{-3x} - 2e^{-10x} \)

7. \( f(x) = 9e^{2x} - 7e^{-14x} - e^x \)

8. \( f(x) = 20e^{-8x} - e^{-5x} + 3e^{2x} - e^{-7x} \)

9. \( f(x) = \frac{6e^{4x} + e^{-15x}}{11e^{4x} + 6e^{-15x}} \)

10. \( f(x) = \frac{e^{3x} + 9e^{-x} - 4e^{10x}}{2e^{7x} - e^{-x}} \)

11. \( f(x) = \frac{3e^{-14x} - e^{18x}}{e^{-x} - 2e^{20x} - e^{-9x}} \)

For problems 12 – 20 evaluate the given limit.
12. \( \lim_{x \to \infty} \ln\left(5x^2 + 12x - 6\right) \)

13. \( \lim_{y \to -\infty} \ln\left(5 - 7y^5\right) \)

14. \( \lim_{x \to \infty} \ln\left(\frac{3 + x}{1 + 5x^3}\right) \)

15. \( \lim_{t \to -\infty} \ln\left(\frac{2t - 5t^3}{4 + 3t^2}\right) \)

16. \( \lim_{z \to -\infty} \ln\left(\frac{10z + 8z^2}{z^2 - 1}\right) \)

17. \( \lim_{x \to -\infty} \tan^{-1}\left(7 + 4x - x^2\right) \)

18. \( \lim_{w \to \infty} \tan^{-1}\left(4w^5 - w^6\right) \)

19. \( \lim_{t \to \infty} \tan^{-1}\left(\frac{4t^3 + t^2}{1 + 3t}\right) \)

19. \( \lim_{z \to -\infty} \tan^{-1}\left(\frac{z^4 + 4}{3z^2 + 5z^3}\right) \)

**Continuity**

1. The graph of \( f(x) \) is given below. Based on this graph determine where the function is discontinuous.
2. The graph of \( f(x) \) is given below. Based on this graph determine where the function is discontinuous.

3. The graph of \( f(x) \) is given below. Based on this graph determine where the function is discontinuous.
For problems 4 – 13 using only Properties 1- 9 from the Limit Properties section, one-sided limit properties (if needed) and the definition of continuity determine if the given function is continuous or discontinuous at the indicated points.

4. \( f(x) = \frac{6 + 2x}{7x - 14} \)
   (a) \( x = -3 \), (b) \( x = 0 \), (c) \( x = 2 \)?

5. \( R(y) = \frac{2y}{y^2 - 25} \)
   (a) \( y = -5 \), (b) \( y = -1 \), (c) \( y = 3 \)?

6. \( g(z) = \frac{5z - 20}{z^2 - 12z} \)
   (a) \( z = -1 \), (b) \( z = 0 \), (c) \( z = 4 \)?

7. \( W(x) = \frac{2 + x}{x^2 + 6x - 7} \)
   (a) \( x = -7 \), (b) \( x = 0 \), (c) \( x = 1 \)?

8. \( h(z) = \begin{cases} 2z^2 & z < -1 \\ 4z + 6 & z \geq -1 \end{cases} \)
   (a) \( z = -6 \), (b) \( z = -1 \)?

9. \( g(x) = \begin{cases} x + e^x & x < 0 \\ x^2 & x \geq 0 \end{cases} \)
   (a) \( x = 0 \), (b) \( x = 4 \)?

10. \( Z(t) = \begin{cases} 8 & t < 5 \\ 1 - 6t & t \geq 5 \end{cases} \)
    (a) \( t = 0 \), (b) \( t = 5 \)?

11. \( h(z) = \begin{cases} z + 2 & z < -4 \\ 0 & z = -4 \\ 18 - z^2 & z > -4 \end{cases} \)
    (a) \( z = -4 \), (b) \( z = 2 \)?
12. \( f(x) = \begin{cases} 1-x^2 & x < 2 \\ -3 & x = 2 \\ 2x-7 & 2 < x < 7 \\ 0 & x = 7 \\ x^2 & x > 7 \end{cases} \)

(a) \( x = 2 \), (b) \( x = 7 \) ?

13. \( g(w) = \begin{cases} 3w & w < 0 \\ 0 & w = 0 \\ w+6 & 0 < w < 8 \\ 14 & w = 8 \\ 22-w & w > 8 \end{cases} \)

(a) \( w = 0 \), (b) \( w = 8 \) ?

For problems 14 – 22 determine where the given function is discontinuous.

14. \( f(x) = \frac{11-2x}{2x^2-13x-7} \)

15. \( Q(z) = \frac{3}{2z^2+3z-4} \)

16. \( h(t) = \frac{t^2-1}{t^3+6t^2+t} \)

17. \( f(z) = \frac{4z+1}{5\cos\left(\frac{z}{7}\right)+1} \)

18. \( h(x) = \frac{1-x}{x\sin(x-1)} \)

19. \( f(x) = \frac{3}{4e^{x^2-7} - 1} \)

20. \( R(w) = \frac{e^{w^2+1}}{e^w - 2e^{-w}} \)

21. \( g(x) = \cot(4x) \)
22. \( f(t) = \sec(\sqrt{t}) \)

For problems 23 – 27 use the Intermediate Value Theorem to show that the given equation has at least one solution in the indicated interval. Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.

23. \( 1 + 7x^3 - x^4 = 0 \) on \([4,8]\)

24. \( z^2 + 11z = 3 \) on \([-15, -5]\)

25. \( \frac{t^2 + t - 15}{t - 8} = 0 \) on \([-5, 1]\)

26. \( \ln(2t^2 + 1) - \ln(t^2 - 4) = 0 \) on \([-1, 2]\)

27. \( 10 = w^3 + w^2 e^{-w} - 5 \) on \([0,4]\)

For problems 28 – 33 assume that \( f(x) \) is continuous everywhere unless otherwise indicated in some way. From the given information is it possible to determine if there is a root of \( f(x) \) in the given interval?

If it is possible to determine that there is a root in the given interval clearly explain how you know that a root must exist. If it is not possible to determine if there is a root in the interval sketch a graph of two functions each of which meets the given information and one will have a root in the given interval and the other will not have a root in the given interval.

28. \( f(-5) = 12 \) and \( f(0) = -3 \) on the interval \([-5,0]\).

29. \( f(1) = 30 \) and \( f(9) = 6 \) on the interval \([1,9]\).

30. \( f(20) = -100 \) and \( f(40) = -100 \) on the interval \([20,40]\).

31. \( f(-4) = -10 \), \( f(5) = 17 \), \( \lim_{x \to 1^+} f(x) = -2 \), and \( \lim_{x \to 1^-} f(x) = 4 \) on the interval \([-4,5]\).

32. \( f(-8) = 2 \), \( f(1) = 23 \), \( \lim_{x \to -4^+} f(x) = 35 \), and \( \lim_{x \to -4^-} f(x) = 1 \) on the interval \([-8,1]\).
33. \( f(0) = -1 \), \( f(9) = 10 \), \( \lim_{x \to 2} f(x) = -12 \), and \( \lim_{x \to 2} f(x) = -3 \) on the interval \([0, 10]\).

**The Definition of the Limit**

Use the definition of the limit to prove the following limits.

1. \( \lim_{x \to 4} (2x) = 8 \)

2. \( \lim_{x \to 1} (-7x) = -7 \)

3. \( \lim_{x \to 3} (2x + 8) = 14 \)

4. \( \lim_{x \to 2} (5 - x) = 3 \)

5. \( \lim_{x \to -2} x^2 = 4 \)

6. \( \lim_{x \to 4} x^2 = 16 \)

7. \( \lim_{x \to 1} (x^2 + x + 6) = 8 \)

8. \( \lim_{x \to -2} (x^2 + 3x - 1) = -3 \)

9. \( \lim_{x \to 1} x^4 = 1 \)

10. \( \lim_{x \to 6} \frac{1}{(x + 6)^2} = \infty \)
11. \( \lim_{x \to 0^-} \frac{-3}{x^2} = -\infty \)

12. \( \lim_{x \to 0^+} \frac{1}{x} = \infty \)

13. \( \lim_{x \to 1^-} \frac{1}{x-1} = -\infty \)

14. \( \lim_{x \to -\infty} \frac{1}{x^2} = 0 \)

15. \( \lim_{x \to \infty} \frac{1}{x^3} = 0 \)