Preface

Here are a set of problems for my Algebra notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.
Polynomial Functions

Introduction
Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.
- Dividing Polynomials
- Zeroes/Roots of Polynomials
- Graphing Polynomials
- Finding Zeroes of Polynomials
- Partial Fractions

Dividing Polynomials
For problems 1 – 6 use long division to perform the indicated division.

1. Divide $7x^2 + 4x - 9$ by $x - 1$
2. Divide $8x^3 - 4x + 1$ by $x + 6$
3. Divide $x^4 - 2x^2 + 7x$ by $x - 4$
4. Divide $2x^4 - 9x^3 + 2x + 8$ by $x + 3$
5. Divide $8x^4 + x^3 - 3x^2 + 1$ by $x^2 - 2$
6. Divide $4x^5 - 7x^3 + x^2 - 4x + 2$ by $2x^2 - 3x - 6$

For problems 7 – 11 use synthetic division to perform the indicated division.

7. Divide $-x^3 - 8x^2 + x + 10$ by $x + 2$
8. Divide $10x^3 - 9x$ by $x - 10$
9. Divide $3x^4 + 5x^3 + x - 2$ by $x + 7$

10. Divide $x^4 + 2x^3 - 9x + 11$ by $x + 3$

11. Divide $5x^4 - 4x^3 + 3x^2 - 2x + 1$ by $x - 1$

**Zeroes/Roots of Polynomials**

For problems 1 – 6 list all of the zeros of the polynomial and give their multiplicities.

1. \( f(x) = x^2 + 2x - 120 \)

2. \( R(x) = x^2 + 12x + 32 \)

3. \( h(x) = 4x^3 + x^2 - 3x \)

4. \( A(x) = x^3 + 2x^4 - 35x^3 + 92x^2 - 92x + 32 = (x - 1)^2 (x + 8)(x - 2)^2 \)

5. \( Q(x) = x^{10} + 17x^9 + 115x^8 + 387x^7 + 648x^6 + 432x^5 = x^5 (x + 3)^3 (x + 4)^2 \)

6. \( g(x) = x^8 + 2x^7 - 14x^6 - 16x^5 + 49x^4 + 62x^3 - 44x^2 - 88x - 32 = (x + 4)(x + 1)^4 (x - 2)^3 \)

For problems 7 – 11 \( x = r \) is a root of the given polynomial. Find the other two roots and write the polynomial in fully factored form.

7. \( P(x) = x^4 - 3x^3 - 18x^2 \); \( r = 6 \)

8. \( P(x) = x^3 + x^2 - 46x + 80 \); \( r = -8 \)

9. \( P(x) = x^3 - 9x^2 + 26x - 24 \); \( r = 3 \)

10. \( P(x) = 12x^3 - 13x^2 - 1 \); \( r = -1 \)

11. \( P(x) = 4x^3 + 11x^2 - 134x - 105 \); \( r = 5 \)

For problems 12 – 14 determine the smallest possible degree for a polynomial with the given zeros and their multiplicities.

12. \( r_1 = -2 \) (multiplicity 1), \( r_2 = 1 \) (multiplicity 1), \( r_3 = 4 \) (multiplicity 1)

13. \( r_1 = 3 \) (multiplicity 4), \( r_2 = -5 \) (multiplicity 1)
14. $r_1 = 7$ (multiplicity 2), $r_2 = 4$ (multiplicity 7), $r_3 = -10$ (multiplicity 5)

15. A 7th degree polynomial has roots $r_1 = -9$ (multiplicity 2) and $r_2 = 3$ (multiplicity 1). What is the maximum number of remaining roots for the polynomial?

**Graphing Polynomials**

Sketch the graph of each of the following polynomials.

1. $f(x) = -x^3 - x^2 + 17x - 15 = -(x - 1)(x - 3)(x + 5)$
2. $A(x) = x^3 + 2x^2 - 3x$
3. $h(x) = x^4 + 2x^3 - 3x^2$
4. $g(x) = x^4 + 14x^3 + 68x^2 + 136x + 96 = (x + 2)^2 (x + 4)(x + 6)$
5. $Q(x) = -x^5 + 8x^4 - 13x^3 - 22x^2 + 32x + 32 = -(x - 4)^2 (x + 1)^2 (x - 2)$
6. $P(x) = -x^4 + 5x^3 - 6x^2 - 4x + 8 = -(x - 2)^3 (x + 1)$
7. $h(x) = x^5 + 5x^4 - 18x^3 - 58x^2 + 145x - 75 = (x - 1)^2 (x + 5)^2 (x - 3)$
8. $R(x) = x^6 - 2x^5 - 11x^4 + 12x^3 + 36x^2 = x^2 (x + 2)^2 (x - 3)^2$

**Finding Zeroes of Polynomials**

Find all the zeroes of the following polynomials.

1. $h(x) = x^3 - 2x^2 - 11x + 12$
2. $f(x) = x^3 + 10x^2 + 29x + 20$
3. $h(x) = 2x^3 - 15x^2 + 34x - 24$
4. $g(x) = x^4 - 6x^3 + 22x + 15$
5. \( f(x) = x^4 - 3x^3 - 7x^2 + 15x + 18 \)

6. \( Q(x) = 4x^4 + x^3 - 35x^2 - 24x + 36 \)

7. \( h(x) = 9x^4 + 15x^3 - 11x^2 - 11x - 2 \)

8. \( A(x) = 2x^5 + 19x^4 + 68x^3 + 114x^2 + 90x + 27 \)

9. \( P(x) = 16x^3 - 48x^2 + 24x^3 + 40x^2 - 39x + 9 \)

**Partial Fractions**

Determine the partial fraction decomposition of each of the following expressions.

1. \[ \frac{22 + 7x}{x^2 + 5x + 4} \]

2. \[ \frac{7x - 44}{4x^2 + 25x - 21} \]

3. \[ \frac{-x - 47}{x^2 - 11x + 24} \]

4. \[ \frac{5 - 38x}{8x^2 + 2x - 1} \]

5. \[ \frac{6x^2 + 50x + 16}{(x-1)(x+2)(x+7)} \]

6. \[ \frac{32x^2 + 39x - 8}{(x+1)(x+2)(2x-3)} \]

7. \[ \frac{36 + 115x - 19x^2}{(x+3)(x-5)(4x-3)} \]

8. \[ \frac{3 - 5x}{(x-3)^2} \]

9. \[ \frac{24x + 41}{(3x+5)^2} \]
10. \( \frac{10x + 93}{(x + 10)^2} \)

11. \( \frac{7x^2 + 31x + 107}{(x - 4)(x + 3)^2} \)

12. \( \frac{9x^2 - 58x - 37}{(x + 7)(x - 2)^2} \)

13. \( \frac{21x^2 - 43x + 20}{(3x - 2)(x - 1)^2} \)

14. \( \frac{-7x^2 + 108x - 11}{x(x^2 - 9x + 1)} \)

15. \( \frac{24x^2 + 2x + 117}{x(2x^2 + x + 13)} \)

16. \( \frac{2 - 11x + x^2 - 7x^3}{(x^2 + 2)^3} \)

17. \( \frac{4x^3 - 3x^2 - 5x - 5}{(x^2 + 1)^3} \)