Here are a set of problems for my Algebra notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.
Preliminaries

Introduction

Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.

- Integer Exponents
- Rational Exponents
- Real Exponents
- Radicals
- Polynomials
- Factoring Polynomials
- Rational Expressions
- Complex Numbers

Integer Exponents

For problems 1 – 10 evaluate the given expression and write the answer as a single number with no exponents.

1. \(2 \cdot 5^2 + (-4)^2\)
2. \(6^0 - 3^5\)
3. \(3 \cdot 4^3 + 2 \cdot 3^2\)
4. \((-1)^4 + 2(-3)^4\)
5. \(7^0 \left(4^2 \cdot 3^1\right)^2\)
6. \(-4^3 + (-4)^3\)
7. \(8 \cdot 2^{-3} + 16^0\)
8. \((2^{-1} + 3^{-1})^{-1}\)

9. \(\frac{3^2 \cdot (-2)^3}{6^2}\)

10. \(\frac{4^{-2} \cdot 5^3}{3^4}\)

For problems 11 – 18 simplify the given expression and write the answer with only positive exponents.

11. \((3x^{-2}y^{-4})^{-1}\)

12. \(((2a^2)^{-3}b^4)^3\)

13. \(\frac{c^{-6}b^{10}}{b^n c^{-11}}\)

14. \(\frac{4a^3(b^2a)^{-4}}{c^{-6}a^2b^{-7}}\)

15. \(\frac{(6v^2)^{-1}w^{-4}}{(2v)^{-3}w^{10}}\)

16. \(\left(\frac{8x^{21}y^{-3}x^8}{y^{-9}x^{-1}}\right)^6\)

17. \(\left(\frac{a^2b^4c^{-1}}{b^{-9}c^8a^{-4}}\right)^{-2}\)

18. \(\frac{p^{-6}q^7(p^2q)^{-3}}{(p^{-1}q^{-4})^2p^{10}}\)

For problems 19 – 23 determine if the statement is true or false. If it is false explain why it is false and give a corrected version of the statement.

19. \(\frac{1}{6x} = 6x^{-1}\)
20. \( \left( x^3 \right)^7 = x^{10} \)

21. \( (m^3 n^4)^2 = m^{12} n^8 \)

22. \( (z^2)^3 \) \( \overline{4} \) = \( z^{24} \)

23. \( (x + y)^3 = x^3 + y^3 \)

**Rational Exponents**

For problems 1 – 15 evaluate the given expression and write the answer as a single number with no exponents.

1. \( 64^{\frac{1}{2}} \)

2. \(-64^{\frac{1}{3}} \)

3. \( 16^{\frac{1}{2}} \)

4. \( 16^{\frac{1}{4}} \)

5. \((−243)^{\frac{1}{3}} \)

6. \(121^{\frac{1}{2}} \)

7. \((−64)^{\frac{1}{3}} \)

8. \( \left( \frac{625}{256} \right)^{\frac{1}{2}} \)

9. \( \left( −\frac{27}{8} \right)^{\frac{1}{3}} \)

10. \( 49^{\frac{5}{2}} \)
11. \(64^{-\frac{5}{6}}\)

12. \((-729)^{\frac{4}{3}}\)

13. \(\left(\frac{121}{36}\right)^{\frac{3}{2}}\)

14. \(\left(-\frac{32}{243}\right)^{\frac{2}{3}}\)

15. \(\left(\frac{81}{625}\right)^{\frac{3}{4}}\)

For problems 16 – 23 simplify the given expression and write the answer with only positive exponents.

16. \((p^{-2} q^{-4})^{\frac{3}{2}}\)

17. \(x^{\frac{3}{4}}\left(x^{\frac{2}{3}} x^{-\frac{1}{3}}\right)^{\frac{3}{2}}\)

18. \(a^{-\frac{1}{2}} a^{-\frac{1}{3}} a^{\frac{1}{4}}\)

19. \((m^{-\frac{7}{3}} n^{\frac{5}{4}})^{-\frac{8}{9}}\)

20. \(\left\{\frac{a^{-\frac{1}{3}} b^{\frac{2}{3}}}{b^{\frac{1}{3}} a^{-\frac{1}{4}}}\right\}^{\frac{1}{3}}\)

21. \(\left\{\frac{p^{\frac{1}{2}} q^{\frac{1}{4}}}{p^{-\frac{1}{2}} q^{-\frac{1}{4}}}\right\}^{-3}\)
22. \( \left( \frac{x^\frac{3}{4} y^{-\frac{2}{3}}}{y^\frac{7}{8}} \right)^7 \)

23. \( \left( \frac{b^3 c^{-\frac{1}{4}} a^{-\frac{3}{2}}}{b^2 a^{-\frac{2}{3}} c^{-\frac{3}{2}}} \right)^2 \)

For problems 24 & 25 determine if the statement is true or false. If it is false explain why it is false and give a corrected version of the statement.

24. \( a^{\frac{3}{2}} = a^3 \)

25. \( x^{-n} = x^n \)

**Real Exponents**

For problems 1 – 5 simplify the given expression and write the answer with only positive exponents.

1. \( (a^{-1.9} b^{5.2})^{0.7} \)

2. \( (x^{8.1} y^{-0.3} z^{-3.5})^{-0.1} \)

3. \( \left( \frac{m^{1.1} n^{-2.2}}{n^{3.3} m^{-4.4}} \right)^{0.2} \)

4. \( \left( \frac{p^{-2.6} q^{-0.4}}{p^{-10.1} q^{-1.6}} \right)^2 \)

5. \( \left( \frac{a^2 b^{-3.4} c^{0.7}}{c^{2.1} a^{-1.9}} \right)^{-6.2} \)

**Radicals**

For problems 1 – 6 write the expression in exponential form.

1. \( \sqrt[3]{n} \)
2. $\sqrt[3]{2}y$

3. $\sqrt[5]{7x^3}$

4. $\sqrt[4]{xyz}$

5. $\sqrt{x+y}$

6. $\sqrt[3]{a^3+b^3}$

For problems 7 – 12 evaluate the radical.

7. $\sqrt{256}$

8. $\sqrt{256}$

9. $\sqrt{256}$

10. $\sqrt[3]{-1024}$

11. $\sqrt[3]{-216}$

12. $\sqrt[3]{343}$

For problems 13 – 22 simplify each of the following. Assume that $x$, $y$ and $z$ are all positive.

13. $\sqrt{z^5}$

14. $\sqrt[3]{z^5}$

15. $\sqrt[3]{16x^{17}}$

16. $\sqrt[6]{128y^{11}}$

17. $\sqrt{x^3y^{17}z^4}$

18. $\sqrt[4]{x^3y^{20}x^5}$

19. $\sqrt[4]{729x^7y^3z^{13}}$
20. \( \sqrt[3]{4x^2y} \sqrt[3]{10x^5y^2} \)

21. \( \sqrt[3]{3x} \sqrt[6]{6x} \sqrt[4]{14x} \)

22. \( \sqrt[4]{2xy^3} \sqrt[4]{32x^2y^2} \)

For problems 23 – 26 multiply each of the following. Assume that \( x \) is positive.

23. \( (2\sqrt{x} + 4)(\sqrt{x} - 7) \)

24. \( 3\sqrt{x} \left( \frac{2}{\sqrt{x}} + 2\sqrt[4]{x^4} \right) \)

25. \( (\sqrt{x} + \sqrt{2y})(\sqrt{x} - \sqrt{2y}) \)

26. \( (\sqrt{x} + \sqrt[4]{x^2})^2 \)

For problems 27 – 35 rationalize the denominator. Assume that \( x \) and \( y \) are both positive.

27. \( \frac{9}{\sqrt{y}} \)

28. \( \frac{3}{\sqrt[6]{7x}} \)

29. \( \frac{1}{\sqrt[4]{x}} \)

30. \( \frac{12}{\sqrt[3]{3x^2}} \)

31. \( \frac{2}{4 - \sqrt{x}} \)

32. \( \frac{9}{\sqrt[3]{3y + 2}} \)

33. \( \frac{4}{\sqrt{7} - 6\sqrt{x}} \)
34. \( \frac{-6}{\sqrt{5x} + 10y} \)

35. \( \frac{4 + x}{x - \sqrt{x}} \)

For problems 36 – 38 determine if the statement is true or false. If it is false explain why it is false.

36. \( 3x^2 = \sqrt{3x} \)

37. \( 3\sqrt{x} + 6 = 3\sqrt{x} + \sqrt{6} \)

38. \( 4\sqrt{x^2} = \sqrt{x} \)

39. For problems 13 – 35 above we always added the instruction to assume that the variables were positive. Why was this instruction added? How would the answers to the problems change if we did not have that instruction?

**Polynomials**

For problems 1 – 18 perform the indicated operation and identify the degree of the result.

1. Add \( 10x^5 + 2x^3 - 1 \) to \( 8x^4 - x^3 + 16x^2 \)

2. Add \( 7t^2 - 13t + 4 \) to \( -6t^2 + 13t - 4 \)

3. Subtract \( -12z^2 + 9z - 3 \) from \( z^3 + 2z^2 - 15z + 7 \)

4. Subtract \( 100x^4 - 19x^2 - 7x \) from \( 150x^3 + 8x - 14 \)

5. Subtract \( w^4 + w^3 + w^2 + w + 1 \) from \( w^5 \)

6. \( 6y^2 \left( 3 - y^2 + 2y^3 \right) \)

7. \( x^3 \left( x^2 + 7x - 4 \right) \)

8. \( (7x - 5)(4 - 10x) \)

9. \( (4 + 9t^2)(t^3 - 3t) \)

10. \( (1 + 8y)(y^3 - 4y^2 + 7) \)
11. $7(x - 9)(2x + 3)$

12. $z^2(1 - z^2)(1 + z^2)$

13. $(2 - x + 4x^2)(6x + 7)$

14. $(10w^2 - 4w + 9)(w^3 + 5w^2 + 2)$

15. $10(x + 3x^2)^2$

16. $(1 - 5y)(4 + y)^2$

17. Subtract $(3 - x)(3 + x)$ from $x^2 - 7x + 10$

18. Subtract $(4x^2 - 1)^2$ from $(x + 9x^3)^2$

19. If we multiply a polynomial with degree $n$ and a polynomial of degree $m$ what is the degree of the result?

20. If we add 2 polynomials of degree $n$ and $m$ with $n < m$ what is the degree of the result?

21. If we subtract 2 polynomials of degree $n$ and $m$ with $n < m$ what is the degree of the result?

22. If we add two polynomials, both of degree $n$, is it possible for the result to not be degree $n$? If it is not possible can you give an example of two polynomials, both of degree $n$, whose sum is not degree $n$?

23. If we subtract two polynomials, both of degree $n$, is it possible for the result to not be degree $n$? If it is not possible can you give an example of two polynomials, both of degree $n$, whose difference is not degree $n$?

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**Factoring Polynomials**

For problems 1 – 8 factor out the greatest common factor from each polynomial.

1. $x^3 - 6x^8 + 10x^{10}$

2. $25u^6 - 15u^5 + 30u^8$

3. $2y^6z - y^4z^{10} + 3y^2z^2$

4. $7a^{10}b^7 + 14a^8b^9 - 35a^6b^{12}$

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http://tutorial.math.lamar.edu/terms.aspx
5. \(3(9 + 7x) - (2 - x)(9 + 7x)\)

6. \(z^2(4z - z^3) + 7(z^3 - 4z)\)

7. \(8y(2y^2 + 7)^4 - 2y^3(2y + 7)^9\)

8. \(w^2(1 + w^2)(8w - 1)^{10} + 9w(1 + w^2)^4(8w - 1)^7\)

For problems 9 – 13 factor each of the following by grouping.

9. \(18x - 2x^3 + 9 - x^2\)

10. \(6w^4 + 3w^3 - 14w^2 - 7w\)

11. \(y^4 + y^3 + 9y^2 + 9y^2\)

12. \(21x - 56x^4 - 12x^3 + 32x^6\)

13. \(6t^3 + 3t^4 - 2t^5 - t^6\)

For problems 14 – 32 factor each of the following.

14. \(x^2 - 10x + 9\)

15. \(t^2 + 11t + 24\)

16. \(z^2 - 9z - 10\)

17. \(x^2 - 3x - 28\)

18. \(x^2 + 10x - 24\)

19. \(w^2 - 8w + 16\)

20. \(z^2 + 6z + 9\)

21. \(x^2 - 144\)

22. \(36 - x^2\)

23. \(4z^2 - 23z - 6\)

24. \(2y^2 - 9y + 10\)
For problems 33 – 38 factor each of the following.

33. \(4x^3 - 20x^2 - 144x\)
34. \(t^4 + 15t^3 + 14t^2\)
35. \(6u^8 - 3u^6 - 3u^4\)
36. \(t^8 + 5t^4 - 24\)
37. \(2z^4 - 5z^2 - 12\)
38. \(4x^6 + x^3 - 5\)

For problems 39 & 40 determine the possible values of \(a\) for which the polynomial will factor.

39. \(x^2 + ax - 16\)
40. \(x^2 + ax + 20\)

For problems 41 – 44 use the knowledge of factoring that you’ve learned in this section to factor the following expressions.

41. \(x^2 + 1 - 6x^{-2}\)
42. \(x^2 - 2 + \frac{1}{x^2}\)
43. \( x^4 - \frac{49}{x^2} \)

44. \( x - 7\sqrt{x} - 18 \)

**Rational Expressions**

For problems 1 – 6 reduce each of the following to lowest terms.

1. \( \frac{x^3 + 10x^2}{x^2 + 6x - 40} \)

2. \( \frac{x^2 + 18x + 72}{2x^2 + 11x - 6} \)

3. \( \frac{x^2 - 3x - 28}{49 - x^2} \)

4. \( \frac{6x^2 + 13x + 5}{3x^2 + 26x + 35} \)

5. \( \frac{-x^2 + 10x - 9}{-x^2 + 6x + 27} \)

6. \( \frac{x^3 + x^2 - 20x}{x^4 - 12x^3 + 36x^2} \)

For problems 7 – 13 perform the indicated operation and reduce the answer to lowest terms.

7. \( \frac{x^2 + 14x + 40}{x^2 + 2x - 8} \div \frac{x^2 + 5x - 14}{x^2 + 7x - 30} \)

8. \( \frac{4x^3 - x^2 - 3x}{x^2 - 10x + 25} \div \frac{10 + 3x - x^2}{x^4 - x^3} \)

9. \( \frac{x^2 + 5x - 24}{x^2 - 5x + 4} \div \frac{x^2 + x - 12}{x - 1} \)

10. \( \frac{6x^2 + x^3 - x^4}{x^2 - 4} \div \frac{3x^3 - 9x^2}{x^2 + 6x - 16} \)
11. \[
\frac{3x^2 + 23x + 14}{x^2 + 4x + 3} + \frac{6x^2 + 13x + 6}{x^2 + 2x + 1}
\]

12. \[
\frac{5x^2 - 18x - 8}{x - 4} \div \frac{x + 6}{x - 6}
\]

13. \[
\frac{2}{x^4 + 4x^3} \div \frac{6x^3 + 17x^2}{x^2 + 3x - 4}
\]

For problems 14 – 22 perform the indicated operations.

14. \[
\frac{2}{3x^2} - \frac{1}{4x^2} + \frac{7}{6x^3}
\]

15. \[
\frac{2x}{x + 9} - \frac{x - 1}{x}
\]

16. \[
\frac{x + 1}{x - 1} + \frac{6}{x - 7}
\]

17. \[
\frac{9}{x^2 - 4} - \frac{7x}{x^2 - 4x + 4}
\]

18. \[
\frac{2x + 1}{4x^2 - 3x - 7} = \frac{x + 3}{x + 1} + \frac{x}{4x - 7}
\]

19. \[
\frac{3}{6x - x^2} = \frac{x}{x^2 - 5x - 6}
\]

20. \[
\frac{2}{x^2 - 4x - 12} + \frac{8x}{x^2 + 12x + 20}
\]

21. \[
\frac{3}{x^2 + 5x} + \frac{x + 9}{x^2 + 10x + 25}
\]

22. \[
\frac{1}{x + 1} - \frac{2}{(x + 1)^2} - \frac{3}{(x + 1)^3}
\]
**Complex Numbers**

Perform the indicated operation and write your answer in standard form.

1. \(2i + (-8 - 15i)\)
2. \((12 + i) + (9 + 2i)\)
3. \(4 - (3 - 20i)\)
4. \(\left(\frac{3}{2} - \frac{1}{3}i\right) - \left(\frac{5}{4} + \frac{7}{2}i\right)\)
5. \((3 + 2i) + (3 - 8i) - (-4 - 7i)\)
6. \(-2i(9 + i)\)
7. \((10 + 3i)(-1 + 7i)\)
8. \((6 + 2i)^2\)
9. \((2 - 14i)(2 + 14i)\)
10. \((2 - \frac{1}{2}i)(-\frac{1}{3} + 5i)\)
11. \((9 + 2i)(1 - 3i)(5 + 4i)\)
12. \(\frac{1 + i}{7 - i}\)
13. \(\frac{2 + 4i}{-9 + 3i}\)
14. \(\frac{6i}{-4 - 7i}\)
15. \(\frac{12 - 2i}{9i}\)
16. \(\frac{4 + 5i}{4 - 5i}\)
17. \[ \frac{i(10-12i)}{(2+i)(-1+4i)} \]