Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Preliminaries

Integer Exponents

1. Evaluate the following expression and write the answer as a single number without exponents.

\[-6^2 + 4 \cdot 3^2\]

Solution
There is not really a whole lot to this problem. All we need to do is the evaluations recalling the proper order of operations.

\[-6^2 + 4 \cdot 3^2 = -36 + 4 \cdot (9) = -36 + 36 = 0\]

Be careful with the first term and recall that,

\[-6^2 = -(6^2) = -(36) = -36\]

If we’d wanted the minus sign to also get squared we’d have written,

\[(-6)^2 = 36\]

Always remember to be careful with exponents. The only thing that gets the exponent is the number/term immediately to the left of the exponent. If we want to include minus signs on numbers with exponents then we need to add in parenthesis.

2. Evaluate the following expression and write the answer as a single number without exponents.

\[
\frac{(-2)^4}{(3^2 + 2^2)^3}
\]

Solution
There is not really a whole lot to this problem. All we need to do is the evaluations recalling the proper order of operations.

\[
\frac{(-2)^4}{(3^2 + 2^2)^3} = \frac{16}{(9 + 4)^3} = \frac{16}{(13)^3} = \frac{16}{169}
\]

Remember that we need to do the evaluations inside the parenthesis in the denominator before we deal with the overall exponent that is on the parenthesis.
3. Evaluate the following expression and write the answer as a single number without exponents.

\[ \frac{4^0 \cdot 2^{-2}}{3^{-1} \cdot 4^{-2}} \]

Solution

There is not really a whole lot to this problem. All we need to do is the evaluations recalling the proper order of operations.

\[
\frac{4^0 \cdot 2^{-2}}{3^{-1} \cdot 4^{-2}} = \frac{4^0 \cdot 3^1 \cdot 4^2}{2^2 \cdot 4} = \frac{(1) \cdot (3) \cdot (16)}{4} = 12
\]

It is almost always going to be best to first get rid of negative exponents prior to doing any of the rest of the evaluation work. Also, don’t forget to reduce any resultant fractions down as much as possible.

4. Evaluate the following expression and write the answer as a single number without exponents.

\[ 2^{-1} + 4^{-1} \]

Solution

There is not really a whole lot to this problem. All we need to do is the evaluations recalling the proper order of operations.

\[
2^{-1} + 4^{-1} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}
\]

It is almost always going to be best to first get rid of negative exponents prior to doing any of the rest of the evaluation work. Also, make sure you can add/subtract fractions! We’re going to be running into a lot of fractions here and you need to be able to work with those.

5. Simplify the following expression and write the answer with only positive exponents.

\[ \left(2w^4v^{-5}\right)^{-2} \]

Solution

There is not really a whole lot to this problem. All we need to do is use the properties from this section to do the simplification.

\[
\left(2w^4v^{-5}\right)^{-2} = 2^{-2}w^{-8}v^{10} = \frac{v^{10}}{2^2w^8} = \frac{v^{10}}{4w^8}
\]
Note that there are several “paths” (i.e. the order in which you chose to use the properties) you can take to do the simplification. Each will end up with the same answer however and so you don’t need to get excited if you chose a different order in which to use the properties than we did here.

6. Simplify the following expression and write the answer with only positive exponents.

\[
\frac{2x^4y^{-1}}{x^{-6}y^3}
\]

Solution
There is not really a whole lot to this problem. All we need to do is use the properties from this section to do the simplification.

\[
\frac{2x^4y^{-1}}{x^{-6}y^3} = \frac{2x^4x^6}{y^3y^1} = \frac{2x^{10}}{y^4}
\]

Note that there are several “paths” (i.e. the order in which you chose to use the properties) you can take to do the simplification. Each will end up with the same answer however and so you don’t need to get excited if you chose a different order in which to use the properties than we did here.

7. Simplify the following expression and write the answer with only positive exponents.

\[
\frac{m^{-2}n^{-10}}{m^{-7}n^{-3}}
\]

Solution
There is not really a whole lot to this problem. All we need to do is use the properties from this section to do the simplification.

\[
\frac{m^{-2}n^{-10}}{m^{-7}n^{-3}} = \frac{m^7n^3}{m^2n^{10}} = \frac{m^5}{n^7}
\]

8. Simplify the following expression and write the answer with only positive exponents.

\[
\frac{(2p^2)^{-3}q^4}{(6q)^{-1}p^{-7}}
\]

Solution
College Algebra

There is not really a whole lot to this problem. All we need to do is use the properties from this section to do the simplification.

\[
\frac{(2p^3)^3q^4}{6q^{-1}} \cdot \frac{p^{-7}}{2^{-1}q^{-1}p^{-7}} = \frac{6^1p^7q^4q^1}{2^1p^5} = \frac{6pq^5}{8} = \frac{3pq^5}{4}
\]

Don’t try to do use too many properties all at once. Sometimes it is very easy to use too many properties all in one step and make a mistake. There’s nothing wrong with using only a single property or two with each step.

9. Simplify the following expression and write the answer with only positive exponents.

\[
\left( \frac{z^2y^{-1}x^{-3}}{x^{-8}z^6y^4} \right)^{-4}
\]

Solution
There is not really a whole lot to this problem. All we need to do is use the properties from this section to do the simplification.

\[
\left( \frac{z^2y^{-1}x^{-3}}{x^{-8}z^6y^4} \right)^{-4} = \left( \frac{z^2x^8}{x^3z^6y^4} \right)^{-4} = \left( \frac{x^5}{z^4y^2} \right)^{-4} = \left( \frac{z^4y^5}{x^5} \right) = \frac{z^{16}y^{20}}{x^{20}}
\]

In this case since there was a fair amount of simplification that could be done on the fraction inside the parenthesis so we decided to do that simplification prior to dealing with the exponent on the parenthesis.

---

**Rational Exponents**

1. Evaluate the following expression and write the answer as a single number without exponents.

\[
\frac{1}{36^2}
\]

Hint : Recall that \(b^n\) is really asking what number did we raise to the \(n\) to get \(b\). Or in other words,

\[
b^{-n} = ? \quad \text{is equivalent to} \quad ?^n = b
\]

Solution
For this problem we know that \(6^2 = 36\) and so we also know that,
\[
\frac{1}{36^2} = \boxed{6}
\]

Note that if you aren’t sure of the answer to these kinds of problems all you really need to do is set up 
\[?^2 = 36\]
and start trying integers until you get the one you need.

2. Evaluate the following expression and write the answer as a single number without exponents.
\[
(-125)^{\frac{1}{3}}
\]

Hint: Recall that \(b^{\frac{1}{n}}\) is really asking what number did we raise to the \(n\) to get \(b\). Or in other words,
\[
\frac{1}{b^{n}} \text{ is equivalent to } ?^{n} = b
\]

Solution
For this problem we know that \(5^3 = 125\). Therefore we also know that \((-5)^3 = -125\) and so we further know that,
\[
(-125)^{\frac{1}{3}} = \boxed{-5}
\]

Note that if you aren’t sure of the answer to these kinds of problems all you really need to do is set up 
\[?^3 = -125\]
and start trying integers until you get the one you need. We also know that because the result is a negative number we had to have a negative number to start off with since we can’t turn a positive number into a negative number simply by raising it to an integer.

3. Evaluate the following expression and write the answer as a single number without exponents.
\[
-16^{\frac{3}{2}}
\]

Hint: Don’t forget your basic exponent rules and how the first two practice problems worked. Also, be careful with minus signs in this problem.

Step 1
First, let’s write the problem as,
\[
-\left(16^{\frac{3}{2}}\right)
\]
so we aren’t tempted to bring the minus sign into the exponent.
Now, let’s recall our basic exponent rules and note that we can easily write this as,

\[ -\left(16^{\frac{3}{2}}\right) = -\left(\left(16^{\frac{1}{2}}\right)^3\right) \]

Step 2
Now, recalling how the first two practice problems worked we can see that,

\[ \frac{1}{16^2} = 4 \]

because \(4^2 = 16\).

Therefore,

\[ -16^{\frac{3}{2}} = -\left(16^{\frac{3}{2}}\right) = -\left(\left(16^{\frac{1}{2}}\right)^3\right) = -\left((4)^3\right) = -(64) = -64 \]

Sometimes the easiest way to do these kinds of problems when you first run into them is to break them up into manageable steps as we did here.

4. Evaluate the following expression and write the answer as a single number without exponents.

\[ 27^{\frac{5}{3}} \]

Hint: Don’t forget your basic exponent rules and how the first two practice problems worked.

Step 1
Let’s first recall our basic exponent rules and note that we can easily write this as,

\[ 27^{\frac{5}{3}} = \frac{1}{27^{\frac{3}{5}}} = \frac{1}{\left(27^{\frac{1}{3}}\right)^5} \]

Step 2
Now, recalling how the first two practice problems worked we can see that,

\[ 27^{\frac{1}{3}} = 3 \]

because \(3^3 = 27\).

Therefore,
Sometimes the easiest way to do these kinds of problems when you first run into them is to break them up into manageable steps as we did here.

5. Evaluate the following expression and write the answer as a single number without exponents.

\[
\left( \frac{9}{4} \right)^{\frac{1}{2}}
\]

Hint: Don’t forget your basic exponent rules and how the first two practice problems worked.

Step 1
Let’s first recall our basic exponent rules and note that we can easily write this as,

\[
\left( \frac{9}{4} \right)^{\frac{1}{2}} = \frac{9^{\frac{1}{2}}}{4^{\frac{1}{2}}}
\]

Step 2
Now, recalling how the first two practice problems worked we can see that,

\[
\frac{9}{4} = 3 \quad \quad \frac{1}{4} = 2
\]

Therefore,

\[
\left( \frac{9}{4} \right)^{\frac{1}{2}} = \frac{9^{\frac{1}{2}}}{4^{\frac{1}{2}}} = \frac{3}{2}
\]

6. Evaluate the following expression and write the answer as a single number without exponents.

\[
\left( \frac{8}{343} \right)^{\frac{2}{3}}
\]

Hint: Don’t forget your basic exponent rules and how the first two practice problems worked.

Step 1
Let’s first recall our basic exponent rules and note that we can easily write this as,
\[
\left( \frac{8}{343} \right)^{\frac{2}{3}} = \left( \frac{343}{8} \right)^{\frac{2}{3}} = \frac{343^2}{8^2} \left( \frac{1}{8^3} \right)^2
\]

Step 2
Now, recalling how the first two practice problems worked we can see that,

\[
343^\frac{1}{3} = 7 \quad \text{and} \quad 8^\frac{1}{3} = 2
\]

Therefore,

\[
\left( \frac{8}{343} \right)^{\frac{2}{3}} = \left( \frac{343}{8} \right)^{\frac{2}{3}} = \frac{343^2}{8^2} \left( \frac{1}{8^3} \right)^2 = \frac{7^2}{2^2} = \frac{49}{4}
\]
9. Simplify the following expression and write the answer with only positive exponents.

\[
\left( \frac{q^3 p^{\frac{1}{2}}}{q^{\frac{1}{3}} p^{\frac{1}{3}}} \right)^{\frac{3}{7}}
\]

Solution
There isn’t really a lot to do here other than to use the exponent properties from the previous section to do the simplification.

\[
\left( q^3 p^{\frac{1}{2}} \right)^{\frac{3}{7}} \left( q^{\frac{1}{3}} p^{\frac{1}{3}} \right)^{\frac{3}{7}} = \left( q^3 p^{\frac{1}{2}} \right)^{\frac{3}{7}} \left( q^{\frac{1}{3}} p^{\frac{1}{3}} \right)^{\frac{3}{7}} = \frac{q^\frac{9}{7}}{p^{\frac{14}{7}}}
\]

10. Simplify the following expression and write the answer with only positive exponents.

\[
\left( \frac{m^{\frac{1}{6}} n^{\frac{1}{3}}}{n^{-\frac{7}{4}} m^{-\frac{7}{4}}} \right)
\]

Solution
There isn’t really a lot to do here other than to use the exponent properties from the previous section to do the simplification.

\[
\left( m^{\frac{1}{6}} n^{\frac{1}{3}} \right)^{\frac{1}{6}} \left( n^{-\frac{7}{4}} m^{-\frac{7}{4}} \right)^{\frac{1}{6}} = \left( m^{\frac{1}{6}} n^{\frac{1}{3}} \right)^{\frac{1}{6}} \left( n^{-\frac{7}{4}} m^{-\frac{7}{4}} \right)^{\frac{1}{6}} = \frac{n^\frac{9}{6}}{m^\frac{3}{8}}
\]

**Real Exponents**

1. Simplify the following expression and write the answer with only positive exponents.

\[
\left( x^{0.1} y^{-0.3} \right)^{-2.4}
\]

Solution
There is not really a whole lot to this problem. Do not get excited about the fact that the exponents aren’t integers or rational numbers. The properties from the integer exponent section still work! So, all we need to do is use them to do the simplification.

\[
\left( x^{-0.1} y^{-0.3} \right)^{2.4} = x^{(-0.1)(2.4)} y^{(-0.3)(2.4)} = x^{-0.24} y^{0.72} = \frac{y^{0.72}}{x^{0.24}}
\]

2. Simplify the following expression and write the answer with only positive exponents.

\[
\left( x^{-0.15} \right)^3 \left( y^4 \right)^{-1.8}
\]

Solution
There is not really a whole lot to this problem. Do not get excited about the fact that some of the exponents aren’t integers or rational numbers. The properties from the integer exponent section still work! So, all we need to do is use them to do the simplification.

\[
\left( x^{-0.15} \right)^3 \left( y^4 \right)^{-1.8} = x^{-0.15(3)} y^{4(-1.8)} = x^{-0.45} y^{7.2} = \frac{1}{x^{0.45} y^{7.2}}
\]

3. Simplify the following expression and write the answer with only positive exponents.

\[
\left( \frac{p^{3.2} q^{-0.7}}{q^{-6.4} p^{-1.9}} \right)^{-1.5}
\]

Solution
There is not really a whole lot to this problem. Do not get excited about the fact that the exponents aren’t integers or rational numbers. The properties from the integer exponent section still work! So, all we need to do is use them to do the simplification.

\[
\left( \frac{p^{3.2} q^{-0.7}}{q^{-6.4} p^{-1.9}} \right)^{-1.5} = \left( \frac{p^{3.2} q^{6.4} p^{1.9}}{q^{0.7}} \right)^{-1.5} = \left( p^{5.1} q^{5.7} \right)^{-1.5} = p^{-7.65} q^{-8.55} = \frac{1}{p^{7.65} q^{8.55}}
\]

**Radicals**

1. Write the following expression in exponential form.
\[ \sqrt[3]{y^2} \]

**Solution**
All this problem is asking us to do is basically use the definition of the radical notation and write this in exponential form instead of radical form.

\[ \frac{1}{y^2} \]

---

2. Write the following expression in exponential form.

\[ \sqrt[x^2]{y^2} \]

**Solution**
All this problem is asking us to do is basically use the definition of the radical notation and write this in exponential form instead of radical form.

\[ (x^2)^{\frac{1}{3}} \]

---

3. Write the following expression in exponential form.

\[ \sqrt[ab]{a^b} \]

**Solution**
All this problem is asking us to do is basically use the definition of the radical notation and write this in exponential form instead of radical form.

\[ (ab)^{\frac{1}{6}} \]

Be careful with parenthesis here! Recall that the only thing that gets the exponent is the term immediately to the left of the exponent. So, if we’d dropped parenthesis we’d get,

\[ ab^{\frac{1}{6}} = a \left( b^{\frac{1}{6}} \right) = a \sqrt[6]{b} \]

which is most definitely not what we started with. The only way to make sure that we understand that both the \( a \) and the \( b \) were under the radical is to use parenthesis as we did above.
4. Write the following expression in exponential form.

\[ \sqrt[2]{w^2v^3} \]

Solution
All this problem is asking us to do is basically use the definition of the radical notation and write this in exponential form instead of radical form.

\[ \left( \frac{1}{2} \right) \left( w^2v^3 \right) \]

Recall that when no index is written on the radical it is assumed to be 2.

Also, be careful with parenthesis here! Recall that the only thing that gets the exponent is the term immediately to the left of the exponent and so we need parenthesis on the whole thing to make sure that we understand that both terms were under the root.

5. Evaluate : \( \sqrt[4]{81} \)

Hint : Recall that the easiest way to evaluate radicals is to convert to exponential form and then also recall that we evaluated exponential forms in the Rational Exponent section.

Solution
All we need to do here is to convert this to exponential form and then recall that we learned how to evaluate the exponential form in the Rational Exponent section.

\[ \sqrt[4]{81} = 81^{\frac{1}{4}} = 3 \quad \text{because} \quad 3^4 = 81 \]

6. Evaluate : \( \sqrt[3]{-512} \)

Hint : Recall that the easiest way to evaluate radicals is to convert to exponential form and then also recall that we evaluated exponential forms in the Rational Exponent section.

Solution
All we need to do here is to convert this to exponential form and then recall that we learned how to evaluate the exponential form in the Rational Exponent section.

\[ \sqrt[3]{-512} = (-512)^{\frac{1}{3}} = -8 \quad \text{because} \quad (-8)^3 = -512 \]
7. Evaluate: \( \sqrt[3]{1000} \)

Hint: Recall that the easiest way to evaluate radicals is to convert to exponential form and then also recall that we evaluated exponential forms in the Rational Exponent section.

Solution
All we need to do here is to convert this to exponential form and then recall that we learned how to evaluate the exponential form in the Rational Exponent section.

\[
\sqrt[3]{1000} = 1000^{\frac{1}{3}} = 10 \quad \text{because} \quad 10^3 = 1000
\]

8. Simplify the following expression. Assume that \( x \) is positive.

\[ \sqrt[3]{x^8} \]

Step 1
Recall that by simplify we mean we want to put the expression in simplified radical form (which we defined in the notes for this section).

To do this for this expression we’ll need to write the radicand as,

\[ x^8 = x^6x^2 = \left( x^2 \right)^3 \cdot x^2 \]

Step 2
Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.

\[
\sqrt[3]{x^8} = \sqrt[3]{\left( x^2 \right)^3 \cdot x^2} = \sqrt[3]{x^2} \cdot \sqrt[3]{x^2} = x^2 \cdot x^2
\]

9. Simplify the following expression. Assume that \( y \) is positive.

\[ \sqrt[3]{8y^3} \]

Step 1
Recall that by simplify we mean we want to put the expression in simplified radical form (which we defined in the notes for this section).

To do this for this expression we’ll need to write the radicand as,

\[ 8y^3 = \left( 4y^2 \right)(2y) \]
Step 2
Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.

\[
\sqrt{8y^3} = \sqrt{(4y^2)(2y)} = \sqrt{4y^2} \sqrt{2y} = 2y \sqrt{2y}
\]

10. Simplify the following expression. Assume that \(x, y\) and \(z\) are positive.

\[
\sqrt[4]{x^7 y^{20} z^{11}}
\]

Step 1
Recall that by simplify we mean we want to put the expression in simplified radical form (which we defined in the notes for this section).

To do this for this expression we’ll need to write the radicand as,

\[
x^7 y^{20} z^{11} = x^4 y^{20} z^8 x^3 z^3 = x^4 \left( y^5 \right)^4 \left( z^2 \right)^4 x^3 z^3
\]

Step 2
Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.

\[
\sqrt[4]{x^7 y^{20} z^{11}} = \sqrt[4]{x^4 \left( y^5 \right)^4 \left( z^2 \right)^4 x^3 z^3} = x y^5 z^2 \sqrt[4]{x^3 z^3}
\]

11. Simplify the following expression. Assume that \(x, y\) and \(z\) are positive.

\[
\sqrt[3]{54x^6 y^7 z^2}
\]

Step 1
Recall that by simplify we mean we want to put the expression in simplified radical form (which we defined in the notes for this section).

To do this for this expression we’ll need to write the radicand as,

\[
54x^6 y^7 z^2 = \left( 27x^6 y^6 \right) \left( 2y^1 z^2 \right) = 3^3 \left( y^2 \right)^3 \left( 2yz^2 \right)
\]

Step 2
Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.
12. Simplify the following expression. Assume that \( x, y \) and \( z \) are positive.

\[
\sqrt[3]{54x^6y^7z^2} = \sqrt[3]{3^3(x^2)^3(y^2)^3(2yz)} = 3\sqrt[3]{x^2}(y^2)\sqrt[3]{2yz} = 3x^2y^2\sqrt[3]{2yz}
\]

Step 1
Remember that when we have a product of two radicals with the same index in an expression we first need to combine them into one root before we start the simplification process.

\[
\sqrt[3]{4x^3y} \sqrt[3]{8x^2y^3z^5} = \sqrt[3]{(4x^3y)(8x^2y^3z^5)} = \sqrt[3]{32x^5y^4z^5}
\]

Step 2
Now that the expression has been written as a single radical we can proceed as we did in the earlier problems.

The radicand can be written as,

\[
32x^5y^4z^5 = (2^4x^4y^4z^4)(2xz)
\]

Step 3
Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.

\[
\sqrt[3]{32x^5y^4z^5} = \sqrt[3]{2^4x^4y^4z^4} \sqrt[3]{2xz} = 2xz\sqrt[3]{2xz}
\]

13. Multiply the following expression. Assume that \( x \) is positive.

\[
\sqrt{x} \left( 4 - 3\sqrt{x} \right)
\]

Solution
All we need to do here is do the multiplication so here is that.

\[
\sqrt{x} \left( 4 - 3\sqrt{x} \right) = 4\sqrt{x} - 3\sqrt{x} \left( \sqrt{x} \right) = 4\sqrt{x} - 3x = 4\sqrt{x} - 3x
\]

Don’t forget to simplify any resulting roots that can be. That is an often missed part of these problems.
14. Multiply the following expression. Assume that \( x \) is positive.

\[
(2\sqrt{x} + 1)(3 - 4\sqrt{x})
\]

Solution

All we need to do here is do the multiplication so here is that.

\[
(2\sqrt{x} + 1)(3 - 4\sqrt{x}) = 6\sqrt{x} - 8\sqrt{x}(\sqrt{x}) + 3 - 4\sqrt{x} = 3 + 2\sqrt{x} - 8\sqrt{x} = 3 + 2\sqrt{x} - 8x
\]

Don’t forget to simplify any resulting roots that can be. That is an often missed part of these problems.

15. Multiply the following expression. Assume that \( x \) is positive.

\[
\left(\sqrt[3]{x} + 2\sqrt[3]{x^2}\right)\left(4 - \sqrt[3]{x^2}\right)
\]

Solution

All we need to do here is do the multiplication so here is that.

\[
\left(\sqrt[3]{x} + 2\sqrt[3]{x^2}\right)\left(4 - \sqrt[3]{x^2}\right) = 4\sqrt[3]{x} - \sqrt[3]{x}\sqrt[3]{x^2} + 8\sqrt[3]{x^2} - 2\sqrt[3]{x^2}\sqrt[3]{x^2} = 4\sqrt[3]{x} - \sqrt[3]{x^3} + 8\sqrt[3]{x^2} - 2\sqrt[3]{x^4} = 4\sqrt[3]{x} - \sqrt[3]{x^3} + 8\sqrt[3]{x^2} - 2\sqrt[3]{x^3}\sqrt[3]{x} = 4\sqrt[3]{x} - x + 8\sqrt[3]{x^2} - 2x\sqrt[3]{x}
\]

Don’t forget to simplify any resulting roots that can be. That is an often missed part of these problems and when dealing with roots other than square roots there can be quite a bit of work in the simplification process as we saw with this problem.

16. Rationalize the denominator. Assume that \( x \) is positive.

\[
\frac{6}{\sqrt{x}}
\]

Solution

For this problem we need to multiply the numerator and denominator by \( \sqrt{x} \) in order to rationalize the denominator.
17. Rationalize the denominator. Assume that $x$ is positive.

\[
\frac{9}{\sqrt[3]{2x}}
\]

Solution

For this problem we need to multiply the numerator and denominator by $\sqrt[3]{(2x)^2}$ in order to rationalize the denominator.

\[
\frac{9}{\sqrt[3]{2x}} = \frac{9 \sqrt[3]{(2x)^2}}{\sqrt[3]{2x} \sqrt[3]{(2x)^2}} = \frac{9 \sqrt[3]{(2x)^2}}{\sqrt[3]{(2x)^3}} = \frac{9 \sqrt[3]{(2x)^2}}{2x} = \frac{9 \sqrt[3]{4x^2}}{2x}
\]

18. Rationalize the denominator. Assume that $x$ and $y$ are positive.

\[
\frac{4}{\sqrt{x} + 2\sqrt{y}}
\]

Solution

For this problem we need to multiply the numerator and denominator by $\sqrt{x} - 2\sqrt{y}$ in order to rationalize the denominator.

\[
\frac{4}{\sqrt{x} + 2\sqrt{y}} = \frac{4 \sqrt{x} - 2\sqrt{y}}{\sqrt{x} + 2\sqrt{y} \sqrt{x} - 2\sqrt{y}} = \frac{4 (\sqrt{x} - 2\sqrt{y})}{(\sqrt{x} + 2\sqrt{y})(\sqrt{x} - 2\sqrt{y})} = \frac{4\sqrt{x - 8\sqrt{y}}}{x - 4y}
\]

19. Rationalize the denominator. Assume that $x$ is positive.

\[
\frac{10}{3 - 5\sqrt{x}}
\]

Solution

For this problem we need to multiply the numerator and denominator by $3 + 5\sqrt{x}$ in order to rationalize the denominator.
Polynomials

1. Perform the indicated operation and identify the degree of the result.

Add $4x^3 - 2x^2 + 1$ to $7x^2 + 12x$

Step 1
Here is the operation we’re being asked to perform.

$$\left(4x^3 - 2x^2 + 1\right) + \left(7x^2 + 12x\right)$$

Note that the parenthesis are only there to illustrate how each polynomial is being used in the indicated operation and are not needed (or used) in general.

Here’s the result of the operation.

$$\left(4x^3 - 2x^2 + 1\right) + \left(7x^2 + 12x\right) = 4x^3 + 5x^2 + 12x + 1$$

Step 2
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is **three**.

2. Perform the indicated operation and identify the degree of the result.

Subtract $4z^6 - 3z^2 + 2z$ from $-10z^6 + 7z^2 - 8$

Step 1
Here is the operation we’re being asked to perform.

$$-10z^6 + 7z^2 - 8 - \left(4z^6 - 3z^2 + 2z\right)$$

Be careful with the order here! We are subtracting the first polynomial from the second and that implies the order we’ve got here. Also be careful with the parenthesis on the second polynomial. We are subtracting the whole polynomial and so we need to have the parenthesis to do that.
Here’s the result of the operation.

\[-10z^6 + 7z^2 - 8 - (4z^6 - 3z^2 + 2z) = -10z^6 + 7z^2 - 8 - 4z^6 + 3z^2 - 2z\]

\[= -14z^6 + 10z^2 - 2z - 8\]

Step 2
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is six.

3. Perform the indicated operation and identify the degree of the result.

Subtract \(-3x^2 + 7x + 8\) from \(x^4 + 7x^3 - 12x - 1\)

Step 1
Here is the operation we’re being asked to perform.

\[x^4 + 7x^3 - 12x - 1 - (-3x^2 + 7x + 8)\]

Be careful with the order here! We are subtracting the first polynomial from the second and that implies the order we’ve got here. Also be careful with the parenthesis on the second polynomial. We are subtracting the whole polynomial and so we need to have the parenthesis to do that.

Here’s the result of the operation.

\[x^4 + 7x^3 - 12x - 1 - (-3x^2 + 7x + 8) = x^4 + 7x^3 - 12x - 1 + 3x^2 - 7x - 8\]

\[= x^4 + 7x^3 + 3x^2 - 19x - 9\]

Step 2
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is four.

4. Perform the indicated operation and identify the degree of the result.

\[12y(3y^4 - 7y^2 + 1)\]

Step 1
All we need to do is multiply the 12y through the second polynomial. Here is the result of doing that.

\[12y(3y^4 - 7y^2 + 1) = 36y^5 - 84y^3 + 12y\]
Step 2
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is five.

5. Perform the indicated operation and identify the degree of the result.

\[(3x + 1)(2 - 9x^2)\]

Step 1
All we need to do is foil out the two polynomials. Here is the result of doing that.

\[(3x + 1)(2 - 9x^2) = 2 + 6x - 9x^2 - 27x^3\]

Step 2
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is three.

6. Perform the indicated operation and identify the degree of the result.

\[(w^2 + 2)(3w^2 + w)\]

Step 1
All we need to do is foil out the two polynomials. Here is the result of doing that.

\[(w^2 + 2)(3w^2 + w) = 3w^4 + w^3 + 6w^2 + 2w\]

Step 2
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is four.

7. Perform the indicated operation and identify the degree of the result.

\[(4x^6 - 3x)(4x^6 + 3)\]

Step 1
All we need to do is foil out the two polynomials. Here is the result of doing that.

\[(4x^6 - 3x)(4x^6 + 3) = 16x^{12} - 9x^2\]
Step 2
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is **twelve**.

---

8. Perform the indicated operation and identify the degree of the result.

\[ 3(10 - 4y^3)^2 \]

Step 1
Remember that this is just another way of writing,

\[ 3(10 - 4y^3)^2 = 3(10 - 4y^3)(10 - 4y^3) \]

Now all we need to do is foil out the two polynomials. Here is the result of doing that.

\[ 3(10 - 4y^3)^2 = 3(10 - 4y^3)(10 - 4y^3) = 3(100 - 80y^3 + 16y^6) = 300 - 240y^3 + 48y^6 \]

Be careful with dealing with the three! Make sure you take care of the exponent first (i.e. make sure you multiply out the product first) before you multiply the three through the result.

Step 2
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is **six**.

---

9. Perform the indicated operation and identify the degree of the result.

\[ (x^2 + x - 2)(3x^2 - 8x - 7) \]

Step 1
Remember that the foil method only works for binomials and these are both trinomials (i.e. they each have three terms).

So, all we need to do is multiply each term in the second polynomial by each term in the first polynomial. Here is the result of doing that.

\[ (x^2 + x - 2)(3x^2 - 8x - 7) = 3x^4 - 8x^3 - 7x^2 + 3x^3 - 8x^2 - 7x - 6x^2 + 16x + 14 \]

\[ = 3x^4 - 5x^3 - 21x^2 + 9x + 14 \]

Step 2
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is **four**.

10. Perform the indicated operation and identify the degree of the result.

Subtract \(3\left(x^2 + 1\right)^2\) from \(6x^3 - 9x^2 - 13x - 4\)

**Step 1**  
Here is the operation we’re being asked to perform.

\[
6x^3 - 9x^2 - 13x - 4 - 3\left(x^2 + 1\right)^2
\]

Now, before we actually do the subtraction we need to actually multiply out the second term before we do the subtraction. Here are the results of all these operations.

\[
6x^3 - 9x^2 - 13x - 4 - 3\left(x^2 + 1\right)^2 = 6x^3 - 9x^2 - 13x - 4 - 3\left(x^4 + 2x^2 + 1\right)
\]

\[
= 6x^3 - 9x^2 - 13x - 4 - 3x^4 - 6x^2 - 3
\]

\[
= -3x^4 + 6x^3 - 15x^2 - 13x - 7
\]

**Step 2**  
Remember the degree of a polynomial is just the largest exponent in the polynomial and so the degree of the result of this operation is **four**.

---

**Factoring Polynomials**

1. Factor out the greatest common factor from the following polynomial.

\[
6x^7 + 3x^4 - 9x^3
\]

**Step 1**  
The first step is to identify the greatest common factor. In this case it looks like we can factor a 3 and an \(x^3\) out of each term and so the greatest common factor is \(3x^3\).

**Step 2**  
Okay, now let’s do the factoring.

\[
6x^7 + 3x^4 - 9x^3 = 3x^3\left(2x^4 + x - 3\right)
\]
Don’t forget to also identify any numbers in the greatest common factor as well. That can often greatly simplify the problem for later work (when we have later work for the problem anyway…).

2. Factor out the greatest common factor from the following polynomial.

\[ a^3b^8 - 7a^{10}b^4 + 2a^5b^2 \]

Step 1
The first step is to identify the greatest common factor. In this case it looks like we can factor an \( a^3 \) and a \( b^2 \) out of each term and so the greatest common factor is \( a^3b^2 \).

Step 2
Okay, now let’s do the factoring.

\[ a^3b^8 - 7a^{10}b^4 + 2a^5b^2 = a^3b^2 \left( b^{6} - 7a^7b^2 + 2a^2 \right) \]

3. Factor out the greatest common factor from the following polynomial.

\[ 2x(x^2 + 1)^3 - 16(x^2 + 1)^5 \]

Step 1
The first step is to identify the greatest common factor. In this case it looks like we can factor a 2 and an \((x^2 + 1)^3\) out of each term and so the greatest common factor is \( 2(x^2 + 1)^3 \).

Step 2
Okay, now let’s do the factoring.

\[ 2x(x^2 + 1)^3 - 16(x^2 + 1)^5 = 2(x^2 + 1)^3 \left( x - 8(x^2 + 1) \right) \]

Don’t get excited if the greatest common factor has more “complicated” terms in it as this one did. The greatest common factor won’t always be just variables to powers.

4. Factor out the greatest common factor from the following polynomial.

\[ x^2(2 - 6x) + 4x(4 - 12x) \]

Step 1
The first step is to identify the greatest common factor and in this case we’ll need to be a little careful. If we just do a quick glance we might be tempted to just say the greatest common factor is just $x$ since there is clearly an $x$ in both terms.

However, notice that we can factor a 2 out of the $4-12x$ in the second term to get,

$$x^2(2-6x) + 4x(4-12x) = x^2(2-6x) + 8x(2-6x)$$

Upon doing this we see that not only do we have an $x$ in both terms we also have a $2-6x$ in both terms and so the greatest common factor in this case is $x(2-6x)$.

Step 2
Okay, now let’s do the factoring.

$$x^2(2-6x) + 4x(4-12x) = x^2(2-6x) + 8x(2-6x) = x(2-6x)(x+8)$$

Sometimes we need to do a little “pre factoring” work on a polynomial in order to determine just what the greatest common factor is. It won’t happen often, but it does need to be done often enough that we can’t forget about it.

5. Factor the following polynomial by grouping.

$$7x + 7x^3 + x^4 + x^6$$

Step 1
The first step here is to group the first two term and the last two terms as follows.

$$(7x + 7x^3) + (x^4 + x^6)$$

Step 2
We can now see that we can factor a $7x$ out of the first grouping and an $x^4$ out of the second grouping. Doing this gives,

$$7x + 7x^3 + x^4 + x^6 = 7x(1 + x^2) + x^4(1 + x^2)$$

Step 3
Finally we see that we can factor an $x(1 + x^2)$ out of both of the new terms to get,

$$7x + 7x^3 + x^4 + x^6 = x(1 + x^2)(7 + x^3)$$
6. Factor the following polynomial by grouping.

\[ 18x + 33 - 6x^4 - 11x^3 \]

**Step 1**
The first step here is to group the first two terms and the last two terms as follows.

\[
(18x + 33) - (6x^4 + 11x^3)
\]

Be careful with the last grouping. Because both of the terms were negative we needed to factor out an “-” as we did the grouping.

**Step 2**
We can now see that we can factor a 3 out of the first grouping and an \( x^3 \) out of the second grouping.

Doing this gives,

\[ 18x + 33 - 6x^4 - 11x^3 = 3(6x + 11) - x^3(6x + 11) \]

**Step 3**
Finally we see that we can factor a \( 6x + 11 \) out of both of the new terms to get,

\[ 18x + 33 - 6x^4 - 11x^3 = (6x + 11)(3 - x^3) \]

7. Factor the following polynomial.

\[ x^2 - 2x - 8 \]

**Step 1**
The initial form for the factoring will be,

\[
(x + \_)(x + \_)
\]

and the factors of -8 are,

\[
(-1)(8) \quad (1)(-8) \quad (-2)(4) \quad (2)(-4)
\]

**Step 2**
Now, recalling that we need the pair of factors from the above list that will add to get -2. So, we can see that the correct factoring will then be,

\[ x^2 - 2x - 8 = (x - 4)(x + 2) \]
8. Factor the following polynomial. 

\[ z^2 - 10z + 21 \]

**Step 1**
The initial form for the factoring will be,

\[
(z + \_)(z + \_)
\]

and the factors of 21 are,

\[
(-1)(-21) \quad (1)(21) \quad (-3)(-7) \quad (3)(7)
\]

**Step 2**
Now, recalling that we need the pair of factors from the above list that will add to get -10. So, we can see that the correct factoring will then be,

\[
z^2 - 10z + 21 = (z - 3)(z - 7)
\]

9. Factor the following polynomial. 

\[ y^2 + 16y + 60 \]

**Step 1**
The initial form for the factoring will be,

\[
(y + \_)(y + \_)
\]

and the factors of 60 are,

\[
(-1)(-60) \quad (-2)(-30) \quad (-3)(-20) \quad (-4)(-15) \quad (-5)(-12) \quad (-6)(-10) \\
(1)(60) \quad (2)(30) \quad (3)(20) \quad (4)(15) \quad (5)(12) \quad (6)(10)
\]

Sometimes there are a lot of factors that we need to deal with. As you get more practice you will start to be able to do most of this in your head and won’t need to actually write all of the factors down.

**Step 2**
Now, recalling that we need the pair of factors from the above list that will add to get 16. So, we can see that the correct factoring will then be,

\[
y^2 + 16y + 60 = (y + 6)(y + 10)
\]
10. Factor the following polynomial.

\[ 5x^2 + 14x - 3 \]

Step 1
There are only two positive factors of 5 so the initial form for the factoring will be,

\[(5x + \underline{\phantom{0}})(x + \underline{\phantom{0}})\]

and the factors of -3 are,

\[(-1)(3) \quad (1)(-3)\]

Step 2
After some trial and error we see that the correct factoring will then be,

\[ 5x^2 + 14x - 3 = (5x - 1)(x + 3) \]

11. Factor the following polynomial.

\[ 6t^2 - 19t - 7 \]

Step 1
There are two sets of positive factors of 6 and so we will have one of the two following possible initial forms for the factoring.

\[(2t + \underline{\phantom{0}})(3t + \underline{\phantom{0}}) \quad (6t + \underline{\phantom{0}})(t + \underline{\phantom{0}})\]

and the factors of -7 are,

\[(-1)(7) \quad (1)(-7)\]

Step 2
After some trial and error we see that the correct factoring will then be,

\[ 6t^2 - 19t - 7 = (2t - 7)(3t + 1) \]

12. Factor the following polynomial.

\[ 4z^2 + 19z + 12 \]

Step 1
There are two sets of positive factors of 4 and so we will have one of the two following possible initial forms for the factoring.
\[(2z + \_)(2z + \_)
\]
\[(4z + \_)(z + \_)
\]
and the factors of 12 are,

\((-1)(-12) \quad (-2)(-6) \quad (-3)(-4) \quad (1)(12) \quad (2)(6) \quad (3)(4)\)

**Step 2**
After some trial and error we see that the correct factoring will then be,

\[4z^2 + 19z + 12 = (4z + 3)(z + 4)\]

13. Factor the following polynomial.

\[x^2 + 14x + 49\]

**Solution**
We can do this in the manner of the previous problems if we wanted to. On the other hand we can notice that the constant is a perfect square and that \[2(7) = 14\] and so we can see that this is one of the special forms.

Therefore the factoring of this polynomial is,

\[x^2 + 14x + 49 = (x + 7)^2\]

Note that while you don’t need necessarily need to know the special forms if you do and can easily recognize them it will make the factoring easier.

14. Factor the following polynomial.

\[4w^2 - 25\]

**Solution**
We can do this in the manner of the previous problems if we wanted to. On the other hand we can notice that we have a difference of perfect squares and so this is one of the special forms.

Therefore the factoring of this polynomial is,

\[4w^2 - 25 = (2w - 5)(2w + 5)\]

Note that while you don’t need necessarily need to know the special forms if you do and can easily recognize them it will make the factoring easier.
15. Factor the following polynomial.

\[ 81x^2 - 36x + 4 \]

Solution
We can do this in the manner of the previous problems if we wanted to. On the other hand we can notice that the constant is a perfect square and the coefficient of the \( x^2 \) is also a perfect square. We can also notice that that \( 2(9)(2) = 36 \) and so we can see that this is one of the special forms.

Therefore the factoring of this polynomial is,

\[ 81x^2 - 36x + 4 = (9x - 2)^2 \]

Note that while you don’t need necessarily need to know the special forms if you do and can easily recognize them it will make the factoring easier.

16. Factor the following polynomial.

\[ x^6 + 3x^3 - 4 \]

Step 1
Don’t let the fact that this polynomial is not a quadratic. That doesn’t mean that we can’t factor the polynomial.

For this polynomial we can see that \( (x^3)^2 = x^6 \) and so it looks like we can factor this into the form,

\[ (x^3 + \_)(x^3 + \_) \]

At this point all we need to do is proceed as we did with the quadratics we were factoring above.

Step 2
After writing down the factors of -4 we can see that we need to have the following factoring.

\[ x^6 + 3x^3 - 4 = (x^3 + 4)(x^3 - 1) \]

Step 3
Now, we need to be careful here. Sometimes these will have further factoring we can do. In this case we can see that the second factor is a difference of perfect cubes and we have a formula for factoring a difference of perfect cubes.

Therefore the factoring of this polynomial is,

\[ x^6 + 3x^3 - 4 = (x^3 + 4)(x^3 - 1) = (x^3 + 4)(x - 1)(x^2 + x + 1) \]
17. Factor the following polynomial.
\[ 3z^3 - 17z^4 - 28z^3 \]

Step 1
Don’t let the fact that this polynomial is not a quadratic. That doesn’t mean that we can’t factor the polynomial.

For this polynomial note that we can factor a \( z^3 \) out of each term to get,
\[ 3z^5 - 17z^4 - 28z^3 = z^3 \left( 3z^2 - 17z - 28 \right) \]

Step 2
Now, notice that the second factor is a quadratic and we know how to factor these. So, it looks like the form of the factoring should be,
\[ 3z^5 - 17z^4 - 28z^3 = z^3 \left( 3z + \_ight)\left(z + \_ \right) \]

Step 3
Finally once we write down the factors of the -28 we can see that the factoring of this polynomial is,
\[ 3z^5 - 17z^4 - 28z^3 = z^3 \left(3z + 4\right) \left(z - 7\right) \]

18. Factor the following polynomial.
\[ 2x^{14} - 512x^6 \]

Step 1
Don’t let the fact that this polynomial is not a quadratic. That doesn’t mean that we can’t factor the polynomial.

For this polynomial note that we can factor a \( 2x^6 \) out of each term to get,
\[ 2x^{14} - 512x^6 = 2x^6 \left( x^8 - 256 \right) \]

Step 2
Now, notice that the second factor is a difference of perfect squares and so we can further factor this as,
\[ 2x^{14} - 512x^6 = 2x^6 \left( x^4 + 16 \right) \left( x^4 - 16 \right) \]

Step 3
Next, we can see that the third term is once again a difference of perfect squares and so can also be factored. After doing that the factoring of this polynomial is,
\[ 2x^{14} - 512x^6 = 2x^6 \left( x^4 + 16 \right) \left( x^2 + 4 \right) \left( x^2 - 4 \right) \]
Step 4
Finally we can see that we can do one more factoring on the last factor.

$$2x^{14} - 512x^6 = 2x^6 (x^4 + 16)(x^2 + 4)(x+2)(x-2)$$

Do not get too excited about polynomials that have lots of factoring in them. They will happen on occasion so don’t worry about it when they do.

---

**Rational Expressions**

1. Reduce the following rational expression to lowest terms.

$$\frac{x^2 - 6x - 7}{x^2 - 10x + 21}$$

**Step 1**
First, we need to factor the numerator and denominator as much as we can. Doing that gives,

$$\frac{x^2 - 6x - 7}{x^2 - 10x + 21} = \frac{(x-7)(x+1)}{(x-7)(x-3)}$$

**Step 2**
Now all we need to do is cancel all the factors that we can in order to reduce the rational expression to lowest terms.

$$\frac{x^2 - 6x - 7}{x^2 - 10x + 21} = \frac{x+1}{x-3}$$

2. Reduce the following rational expression to lowest terms.

$$\frac{x^2 + 6x + 9}{x^2 - 9}$$

**Step 1**
First, we need to factor the numerator and denominator as much as we can. Doing that gives,

$$\frac{x^2 + 6x + 9}{x^2 - 9} = \frac{(x+3)^2}{(x+3)(x-3)}$$
Step 2  
Now all we need to do is cancel all the factors that we can in order to reduce the rational expression to lowest terms.

\[
\frac{x^2 + 6x + 9}{x^2 - 9} = \frac{x + 3}{x - 3}
\]

3. Reduce the following rational expression to lowest terms.

\[
\frac{2x^2 - x - 28}{20 - x - x^2}
\]

Step 1  
First, we need to factor the numerator and denominator as much as we can. Doing that gives,

\[
\frac{2x^2 - x - 28}{20 - x - x^2} = \frac{(2x + 7)(x - 4)}{-(x^2 + x - 20)} = \frac{(2x + 7)(x - 4)}{-(x + 5)(x - 4)}
\]

Notice that in order to make factoring the denominator somewhat easier we first factored a minus sign out of the denominator.

Step 2  
Now all we need to do is cancel all the factors that we can in order to reduce the rational expression to lowest terms.

\[
\frac{2x^2 - x - 28}{20 - x - x^2} = \frac{2x + 7}{x + 5}
\]

Recall that the minus sign in the denominator can be put out in front of the rational expression if we choose to put it there (as we did here).

4. Perform the indicated operation in the following expression and reduce the answer to lowest terms.

\[
\frac{x^2 + 5x - 24}{x^2 + 6x + 8} \cdot \frac{x^2 + 4x + 4}{x^2 - 3x}
\]

Step 1  
So, we first need to factor each of the polynomials as much as possible.

\[
\frac{(x + 8)(x - 3)}{(x + 4)(x + 2)} \cdot \frac{(x + 2)^2}{x(x - 3)} = \frac{(x + 8)}{(x + 4)} \cdot \frac{(x + 2)}{x}
\]
5. Perform the indicated operation in the following expression and reduce the answer to lowest terms.

\[ \frac{x^2 - 49}{2x^2 - 3x - 5} \div \frac{x^2 - x - 42}{x^2 + 7x + 6} \]

Step 1
So, we first need to do is convert this into a product.

\[ \frac{x^2 - 49}{2x^2 - 3x - 5} \div \frac{x^2 - x - 42}{x^2 + 7x + 6} = \frac{x^2 - 49}{2x^2 - 3x - 5} \cdot \frac{x^2 + 7x + 6}{x^2 - x - 42} \]

Make sure that you don’t do the factoring and canceling until you’ve converted the division to a product.

Step 2
Now we can factor each of the terms as much as possible to get,

\[ \frac{x^2 - 49}{2x^2 - 3x - 5} \div \frac{x^2 - x - 42}{x^2 + 7x + 6} = \frac{(x - 7)(x + 7)}{(2x - 5)(x + 1)} \cdot \frac{(x + 1)(x + 6)}{(x - 7)(x + 6)} \]

Step 3
Finally cancel as much as possible to reduce to lowest terms and do the product.

\[ \frac{x^2 - 49}{2x^2 - 3x - 5} \div \frac{x^2 - x - 42}{x^2 + 7x + 6} = \frac{x + 7}{2x - 5} \]

6. Perform the indicated operation in the following expression and reduce the answer to lowest terms.

\[ \frac{x^2 - 2x - 8}{2x^2 - 8x - 24} \div \frac{x^2 - 9x + 20}{x^2 - 11x + 30} \]

Step 1
So, we first need to do is convert this into a product.

\[ \frac{x^2 - 2x - 8}{2x^2 - 8x - 24} \div \frac{x^2 - 9x + 20}{x^2 - 11x + 30} = \frac{x^2 - 2x - 8}{2x^2 - 8x - 24} \cdot \frac{x^2 - 11x + 30}{x^2 - 9x + 20} \]
Make sure that you don’t do the factoring and canceling until you’ve converted the division to a product.

Step 2
Now we can factor each of the terms as much as possible to get,

\[
\frac{x^2 - 2x - 8}{2x^2 - 8x - 24} \div \frac{x^2 - 9x + 20}{x^2 - 11x + 30} = \frac{(x - 4)(x + 2)}{2(x + 2)(x - 6)} \cdot \frac{(x - 5)(x - 6)}{(x - 5)(x - 4)}
\]

Step 3
Finally cancel as much as possible to reduce to lowest terms and do the product.

\[
\frac{x^2 - 2x - 8}{2x^2 - 8x - 24} \div \frac{x^2 - 9x + 20}{x^2 - 11x + 30} = \frac{1}{2}
\]

Don’t worry if all the variables end up cancelling out after you are done reducing to lowest terms. It will happen on occasion so don’t worry about it when it does.

7. Perform the indicated operation in the following expression and reduce the answer to lowest terms.

\[
\frac{3}{x + 1} \div \frac{x + 4}{x^2 + 11x + 10}
\]

Step 1
This is just a division and so let’s first convert it to a product.

\[
\frac{3}{x + 1} \div \frac{x + 4}{x^2 + 11x + 10} = \frac{3}{x + 1} \cdot \frac{x^2 + 11x + 10}{x + 4}
\]

Step 2
Now we can factor each of the second term as much as possible to get,

\[
\frac{3}{x + 1} \div \frac{x + 4}{x^2 + 11x + 10} = \frac{3}{x + 1} \cdot \frac{(x + 1)(x + 10)}{x + 4}
\]

Step 3
Now cancel as much as possible to reduce to lowest terms and do the product.
8. Perform the indicated operation in the following expression.

\[
\frac{3}{x+1} + \frac{x}{x+4} = \frac{3(x+10)}{x+4} \quad \frac{x^2 + 11x + 10}{x+4}
\]

Step 1
We first need the least common denominator for this rational expression.

\[
\text{lcd } (x-4)(2x+7)
\]

Step 2
Now multiply each term by an appropriate quantity to get the least common denominator into the denominator of each term.

\[
\frac{3}{x-4} + \frac{x}{2x+7} = \frac{3(2x+7)}{(x-4)(2x+7)} + \frac{x(x-4)}{(2x+7)(x-4)}
\]

Step 3
All we need to do now is do the addition and simplify the numerator of the result.

\[
\frac{3}{x-4} + \frac{x}{2x+7} = \frac{3(2x+7) + x(x-4)}{(x-4)(2x+7)} = \frac{6x + 21 + x^2 - 4x}{(x-4)(2x+7)} = \frac{x^2 + 2x + 21}{(x-4)(2x+7)}
\]

9. Perform the indicated operation in the following expression.

\[
\frac{2}{3x^2} - \frac{1}{9x^4} + \frac{2}{x+4}
\]

Step 1
We first need the least common denominator for this rational expression.

\[
\text{lcd } 9x^4(x+4)
\]

Step 2
Now multiply each term by an appropriate quantity to get the least common denominator into the denominator of each term.
\[
\frac{2}{3x^2} - \frac{1}{9x^4} + \frac{2}{x + 4} = \frac{2(3x^2)(x + 4)}{3x^2(3x^2)(x + 4)} - \frac{1(3x^2)(x + 4)}{9x^4(3x^2)(x + 4)} + \frac{2(9x^4)(x + 4)}{9x^4(x + 4)(3x^2)(x + 4)}
\]

**Step 3**

All we need to do now is do the subtraction and addition then simplify the numerator of the result.

\[
\frac{2}{3x^2} - \frac{1}{9x^4} + \frac{2}{x + 4} = \frac{6x^3 + 24x^2 - (x + 4) + 18x^4}{9x^4(x + 4)} = \frac{18x^4 + 6x^3 + 24x^2 - x - 4}{9x^4(x + 4)}
\]

10. Perform the indicated operation in the following expression.

\[
\frac{x}{x^2 + 12x + 36} - \frac{x - 8}{x + 6}
\]

**Step 1**

We first need the least common denominator for this rational expression. However, before we get that we’ll need to factor the denominator of the first term. Doing this gives,

\[
\frac{x}{x^2 + 12x + 36} - \frac{x - 8}{x + 6} = \frac{x}{(x + 6)^2} - \frac{x - 8}{x + 6}
\]

**Step 2**

The least common denominator is then,

\[
\text{lcd} : (x + 6)^2
\]

Remember that we only take the highest power on each term in the denominator when setting up the least common denominator.

**Step 3**

Next, multiply each term by an appropriate quantity to get the least common denominator into the denominator of each term.

\[
\frac{x}{x^2 + 12x + 36} - \frac{x - 8}{x + 6} = \frac{x}{(x + 6)^2} = \frac{(x - 8)(x + 6)}{(x + 6)(x + 6)}
\]

**Step 4**

Finally all we need to do is the subtraction and simplify the numerator of the result.

\[
\frac{x}{x^2 + 12x + 36} - \frac{x - 8}{x + 6} = \frac{x - (x - 8)(x + 6)}{(x + 6)^2} = \frac{x - (x^2 - 2x - 48)}{(x + 6)^2} = \frac{48 + 3x - x^2}{(x + 6)^2}
\]
11. Perform the indicated operation in the following expression.

\[
\frac{1}{x^2-13x+42} + \frac{x+1}{x-6} - \frac{x^2}{x-7}
\]

Step 1
We first need the least common denominator for this rational expression. However, before we get that we’ll need to factor the denominator of the first term. Doing this gives,

\[
\frac{1}{(x-6)(x-7)} + \frac{(x+1)(x-7)}{(x-6)(x-7)} - \frac{x^2}{(x-7)(x-6)}
\]

Step 2
The least common denominator is then,

\[
\text{lcd} : (x-6)(x-7)
\]

Remember that we only take the highest power on each term in the denominator when setting up the least common denominator.

Step 3
Next, multiply each term by an appropriate quantity to get the least common denominator into the denominator of each term.

\[
\frac{1}{x^2-13x+42} + \frac{x+1}{x-6} - \frac{x^2}{x-7} = \frac{1}{(x-6)(x-7)} + \frac{(x+1)(x-7)}{(x-6)(x-7)} - \frac{x^2}{(x-7)(x-6)}
\]

Step 4
Finally all we need to do is the addition and subtraction then simplify the numerator of the result.

\[
\frac{1}{x^2-13x+42} + \frac{x+1}{x-6} - \frac{x^2}{x-7} = \frac{1+(x+1)(x-7)-x^2(x-6)}{(x-6)(x-7)} = \frac{-x^3+7x^2-6x-6}{(x-6)(x-7)}
\]

12. Perform the indicated operation in the following expression.

\[
\frac{x+10}{(3x+8)^3} + \frac{x}{(3x+8)^2}
\]
Step 1
We first need the least common denominator for this rational expression.

\[
\text{lcd} : (3x + 8)^3
\]

Remember that we only take the highest power on each term in the denominator when setting up the least common denominator.

Step 2
Now multiply each term by an appropriate quantity to get the least common denominator into the denominator of each term.

\[
\frac{x + 10}{(3x + 8)^3} + \frac{x}{(3x + 8)^2} = \frac{x + 10}{(3x + 8)^3} + \frac{x(3x + 8)}{(3x + 8)^3 (3x + 8)}
\]

Step 3
All we need to do now is do the addition and simplify the numerator of the result.

\[
\frac{x + 10}{(3x + 8)^3} + \frac{x}{(3x + 8)^2} = \frac{x + 10 + 3x^2 + 8x}{(3x + 8)^3} = \frac{3x^2 + 9x + 10}{(3x + 8)^3}
\]

**Complex Numbers**

1. Perform the indicated operation and write your answer in standard form.

\[(4 - 5i)(12 + 11i)\]

Hint: You know how to do the operation with polynomials so you can do the operation here! Just recall that you need to be careful to deal with any \(i^2\) that might happen to show up in the process.

Solution
We know how to multiply two polynomials and so we also know how to multiply two complex numbers. All we need to do is “foil” the two complex numbers to get,

\[(4 - 5i)(12 + 11i) = 48 + 44i - 60i - 55i^2 = 48 - 16i - 55i^2\]

All we need to do to finish the problem is to recall that \(i^2 = -1\). Upon using this fact we can finish the problem.
(4 − 5i)(12 + 11i) = 48 − 16i − 55(−1) = 103 − 16i

2. Perform the indicated operation and write your answer in standard form.

(−3 − i) − (6 − 7i)

Hint: You know how to do the operation with polynomials so you can do the operation here!

Solution
We know how to subtract two polynomials and so we also know how to subtract two complex numbers.

(−3 − i) − (6 − 7i) = −3 − i − 6 + 7i = −9 + 6i

3. Perform the indicated operation and write your answer in standard form.

(1 + 4i) − (−16 + 9i)

Hint: You know how to do the operation with polynomials so you can do the operation here!

Solution
We know how to subtract two polynomials and so we also know how to subtract two complex numbers.

(1 + 4i) − (−16 + 9i) = 1 + 4i + 16 − 9i = 17 − 5i

4. Perform the indicated operation and write your answer in standard form.

8i(10 + 2i)

Hint: You know how to do the operation with polynomials so you can do the operation here! Just recall that you need to be careful to deal with any $i^2$ that might happen to show up in the process.

Solution
We know how to multiply two polynomials and so we also know how to multiply two complex numbers. All we need to do is distribute the $8i$ to get,

\[8i(10 + 2i) = 80i + 16i^2\]

All we need to do to finish the problem is to recall that $i^2 = −1$. Upon using this fact we can finish the problem.

\[8i(10 + 2i) = 80i + 16(−1) = −16 + 80i\]
5. Perform the indicated operation and write your answer in standard form.

\[ (-3 - 9i)(1 + 10i) \]

Hint: You know how to do the operation with polynomials so you can do the operation here! Just recall that you need to be careful to deal with any \(i^2\) that might happen to show up in the process.

Solution
We know how to multiply two polynomials and so we also know how to multiply two complex numbers. All we need to do is “foil” the two complex numbers to get,

\[ (-3 - 9i)(1 + 10i) = -30i - 9i - 90i^2 \]

All we need to do to finish the problem is to recall that \(i^2 = -1\). Upon using this fact we can finish the problem.

\[ (-3 - 9i)(1 + 10i) = -30i - 9i - 90(-1) = 87 - 39i \]

6. Perform the indicated operation and write your answer in standard form.

\[ (2 + 7i)(8 + 3i) \]

Hint: You know how to do the operation with polynomials so you can do the operation here! Just recall that you need to be careful to deal with any \(i^2\) that might happen to show up in the process.

Solution
We know how to multiply two polynomials and so we also know how to multiply two complex numbers. All we need to do is “foil” the two complex numbers to get,

\[ (2 + 7i)(8 + 3i) = 16 + 6i + 56i + 21i^2 \]

All we need to do to finish the problem is to recall that \(i^2 = -1\). Upon using this fact we can finish the problem.

\[ (2 + 7i)(8 + 3i) = 16 + 6i + 56i + 21(-1) = -5 + 62i \]

7. Perform the indicated operation and write your answer in standard form.

\[ \frac{7 - i}{2 + 10i} \]
Hint: Recall that standard form does not allow any $i$'s in the denominator.

Step 1
Because standard form does not allow for $i$'s to be in the denominator we’ll need to multiply the numerator and denominator by the conjugate of the denominator, which is $2 - 10i$.

Step 2
Multiplying by the conjugate gives,

$$\frac{7-i}{2+10i} \cdot \frac{2-10i}{2-10i} = \frac{(7-i)(2-10i)}{(2+10i)(2-10i)}$$

Step 3
Now all we need to do is do the multiplication in the numerator and denominator and put the result in standard form.

$$\frac{7-i}{2+10i} = \frac{14-72i+10i^2}{4-100i^2} = \frac{4-72i}{104} = \frac{4}{104} - \frac{72}{104}i = \frac{1}{26} - \frac{9}{13}i$$

8. Perform the indicated operation and write your answer in standard form.

$$\frac{1+5i}{-3i}$$

Hint: Recall that standard form does not allow any $i$'s in the denominator.

Step 1
Because standard form does not allow for $i$'s to be in the denominator we’ll need to multiply the numerator and denominator by the conjugate of the denominator, which is $3i$.

Step 2
Multiplying by the conjugate gives,

$$\frac{1+5i}{-3i} \cdot \frac{3i}{3i} = \frac{(1+5i)(3i)}{(-3i)(3i)}$$

Step 3
Now all we need to do is do the multiplication in the numerator and denominator and put the result in standard form.

$$\frac{1+5i}{-3i} = \frac{3i+15i^2}{-9i^2} = \frac{-15+3i}{9} = \frac{-15}{9} + \frac{3}{9}i = \frac{-5+1}{3} + \frac{1}{3}i$$
9. Perform the indicated operation and write your answer in standard form.

\[ \frac{6 + 7i}{8 - i} \]

Hint: Recall that standard form does not allow any \( i \)'s in the denominator.

Step 1
Because standard form does not allow for \( i \)'s to be in the denominator we’ll need to multiply the numerator and denominator by the conjugate of the denominator, which is \( 8 + i \).

Step 2
Multiplying by the conjugate gives,

\[ \frac{6 + 7i}{8 - i} \frac{8 + i}{8 + i} = \frac{(6 + 7i)(8 + i)}{(8 - i)(8 + i)} \]

Step 3
Now all we need to do is do the multiplication in the numerator and denominator and put the result in standard form.

\[ \frac{6 + 7i}{8 - i} \frac{8 + i}{8 + i} = \frac{48 + 62i + 7i^2}{64 - i^2} = \frac{41 + 62i}{65} = \frac{41}{65} + \frac{62}{65}i \]