Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Solutions and Solution Sets

1. Is $x = 6$ a solution to $2x - 5 = 3(1 - x) + 22$?

Solution
There really isn’t all that much to do for these kinds of problems. All we need to do is plug the given number into both sides of the equation and check to see if the right and left side are the same value.

Here is that work for this particular problem.

\[
2(6) - 5 = 3(1 - 6) + 22 \\
7 = 7 \quad \text{OK}
\]

So, we can see that the right and left sides are the same and so we know that $x = 6$ is a solution to the equation.

2. Is $t = 7$ a solution to $t^2 + 3t - 10 = 4 + 8t$?

Solution
There really isn’t all that much to do for these kinds of problems. All we need to do is plug the given number into both sides of the equation and check to see if the right and left side are the same value.

Here is that work for this particular problem.

\[
(7)^2 + 3(7) - 10 = 4 + 8(7) \\
60 = 60 \quad \text{OK}
\]

So, we can see that the right and left sides are the same and so we know that $t = 7$ is a solution to the equation.

3. Is $t = -3$ a solution to $t^2 + 3t - 10 = 4 + 8t$?

Solution
There really isn’t all that much to do for these kinds of problems. All we need to do is plug the given number into both sides of the equation and check to see if the right and left side are the same value.
Here is that work for this particular problem.
\[
( -3 )^2 + 3(-3) - 10 = 4 + 8(-3) \\
-10 = -20 \quad \text{NOT OK}
\]

So, we can see that the right and left sides are the not the same and so we know that \( t = -3 \) is not a solution to the equation.

4. Is \( w = -2 \) a solution to \( \frac{w^2 + 8w + 12}{w + 2} = 0 \)?

Solution
There really isn’t all that much to do for these kinds of problems. All we need to do is plug the given number into both sides of the equation and check to see if the right and left side are the same value.

Note that for this problem we don’t even really need to plug the value into the equation. We can see by a quick inspection that if we were to plug \( w = -2 \) into this equation we would have division by zero and we know that is not allowed.

Therefore, \( w = -2 \) is not a solution to this equation.

Be very careful with this kind of problem. If we had plugged \( w = -2 \) into the equation we’d have gotten zero in the numerator as well and we might be tempted to say that it is a solution to the equation. We’d be wrong however. Regardless of the value of the numerator, we would still have division by zero and that is just not allowed and so \( w = -2 \) will not be a solution to this equation.

5. Is \( z = 4 \) a solution to \( 6z - z^2 \geq z^2 + 3 \)?

Solution
There really isn’t all that much to do for these kinds of problems. All we need to do is plug the given number into both sides of the inequality and check to see if the inequality is true. In this case that will mean checking to see if the left side is larger than or equal to the right side.

Here is that work for this particular problem.
\[
6(4) - (4)^2 \geq (4)^2 + 3 \\
8 \geq 19 \quad \text{NOT OK}
\]

So, we can see that the left side is neither larger than nor equal to the right side and so \( z = 4 \) is not a solution to this inequality.
6. Is \( y = 0 \) a solution to \( 2(y + 7) - 1 < 4(y + 1) + 3(4y + 10) \)?

Solution

There really isn’t all that much to do for these kinds of problems. All we need to do is plug the given number into both sides of the inequality and check to see if the inequality is true. In this case that will mean checking to see if the left side is less than the right side.

Here is that work for this particular problem.

\[
2(0 + 7) - 1 < 4(0 + 1) + 3(4(0) + 10) \\
13 < 34 \quad \text{OK}
\]

So, we can see that the left side is less than the right side and so \( y = 0 \) is a solution to this inequality.

7. Is \( x = 1 \) a solution to \( (x + 1)^2 > 3x + 1 \)?

Solution

There really isn’t all that much to do for these kinds of problems. All we need to do is plug the given number into both sides of the inequality and check to see if the inequality is true. In this case that will mean checking to see if the left side is greater than the right side.

Here is that work for this particular problem.

\[
(1+1)^2 > 3(1) + 1 \\
4 \neq 4 \quad \text{NOT OK}
\]

Be very careful with this type of problem! Four is not greater than 4 (it’s equal to 4 – big difference here) and so, we can see that \( x = 1 \) is not a solution to this inequality.

Contrast the inequality in this problem with,

\[
(x + 1)^2 \geq 3x + 1
\]

While \( x = 1 \) is not a solution to the inequality in the problem statement it is a solution to this inequality since 4 is in fact greater than or equal to 4. The presence of the equal sign in the inequality can make all the difference in the world and we really need to be on the lookout for it. It is easy to miss when it’s there and it is easy to sometimes assume it is there when in fact it isn’t.
**Linear Equations**

1. Solve the following equation and check your answer.

\[4x - 7(2 - x) = 3x + 2\]

Step 1
First we need to clear out the parenthesis on the left side and then simplify the left side.

\[
\begin{align*}
4x - 7(2 - x) &= 3x + 2 \\
4x - 14 + 7x &= 3x + 2 \\
11x - 14 &= 3x + 2
\end{align*}
\]

Step 2
Now we can subtract 3x and add 14 to both sides to get all the x’s on one side and the terms without an x on the other side.

\[
\begin{align*}
11x - 14 &= 3x + 2 \\
8x &= 16
\end{align*}
\]

Step 3
Finally, all we need to do is divide both sides by the coefficient of the x (i.e. the 8) to get the solution of \(x = 2\).

Step 4
Now all we need to do is check our answer from Step 3 and verify that it is a solution to the equation. It is important when doing this step to verify by plugging the solution from Step 3 into the equation given in the problem statement.

Here is the verification work.

\[
\begin{align*}
4(2) - 7(2 - 2) &= 3(2) + 2 \\
8 &= 8 \quad \text{OK}
\end{align*}
\]

So, we can see that our solution from Step 3 is in fact the solution to the equation.

2. Solve the following equation and check your answer.

\[2(w + 3) - 10 = 6(32 - 3w)\]
Step 1
First we need to clear out the parenthesis on each side and then simplify each side.

\[2(\,w+3\,)-10 = 6\, (32-3w)\]
\[2w+6-10 = 192-18w\]
\[2w-4 = 192-18w\]

Step 2
Now we can add \(18w\) and 4 to both sides to get all the \(w\)'s on one side and the terms without an \(w\) on the other side.

\[2w-4 = 192-18w\]
\[20w = 196\]

Step 3
Finally, all we need to do is divide both sides by the coefficient of the \(w\) (i.e. the 20) to get the solution of

\[w = \frac{196}{20} = \frac{49}{5}\]

Don’t get excited about solutions that are fractions. They happen more often than people tend to realize.

Step 4
Now all we need to do is check our answer from Step 3 and verify that it is a solution to the equation. It is important when doing this step to verify by plugging the solution from Step 3 into the equation given in the problem statement.

Here is the verification work.

\[2\left(\frac{49}{5}+3\right)-10 = 6\left(32-3\left(\frac{49}{5}\right)\right)\]
\[2\left(\frac{64}{5}\right)-10 = 6\left(\frac{13}{5}\right)\]
\[\frac{78}{5} = \frac{78}{5}\quad \text{OK}\]

So, we can see that our solution from Step 3 is in fact the solution to the equation.

3. Solve the following equation and check your answer.

\[\frac{4-2z}{3} = \frac{3}{4} - \frac{5z}{6}\]
Step 1
The first step here is to multiply both sides by the LCD, which happens to be 12 for this problem.

\[
12 \left( \frac{4 - 2z}{3} \right) = 12 \left( \frac{3 - 5z}{4} \right)
\]

\[
12 \left( \frac{4 - 2z}{3} \right) = 12 \left( \frac{3}{4} \right) - 12 \left( \frac{5z}{6} \right)
\]

\[
4(4 - 2z) = 3(3) - 2(5z)
\]

Step 2
Now we need to find the solution and so all we need to do is go through the same process that we used in the first two practice problems. Here is that work.

\[
4(4 - 2z) = 3(3) - 2(5z)
\]

\[
16 - 8z = 9 - 10z
\]

\[
2z = -7
\]

\[
z = \frac{-7}{2}
\]

Step 3
Now all we need to do is check our answer from Step 2 and verify that it is a solution to the equation. It is important when doing this step to verify by plugging the solution from Step 2 into the equation given in the problem statement.

Here is the verification work.

\[
4 - 2 \left( \frac{-7}{2} \right) = \frac{3}{4} - 5 \left( \frac{-7}{2} \right)
\]

\[
\frac{4 + 7}{4} = \frac{3}{4} - \frac{35}{6}
\]

\[
\frac{11}{4} = \frac{3}{4} + \frac{35}{12}
\]

\[
\frac{11}{3} = \frac{11}{3} \quad \text{OK}
\]

So, we can see that our solution from Step 2 is in fact the solution to the equation.

Note that the verification work can often be quite messy so don’t get excited about it when it does. Verification is an important step to always remember for these kinds of problems. You should always know if you got the answer correct before you check the answers and/or your instructor grades the problem!

4. Solve the following equation and check your answer.
\[ \frac{4t}{t^2 - 25} = \frac{1}{5 - t} \]

Hint: Do not forget to watch out for values of \( t \) that we’ll need to avoid!

Step 1
Let’s first factor the denominator on the left side so we can identify the LCD. While we are at it we will also factor a minus out of the denominator on the right side.

\[ \frac{4t}{(t-5)(t+5)} = \frac{1}{-(t-5)} \]

So, after factoring the left side and factoring the minus sign out of the denominator on the right side we can quickly see that the LCD for this equation is,

\[ (t-5)(t+5) \]

From this we can also see that we’ll need to avoid \( t = 5 \) and \( t = -5 \). Remember that we have to avoid division by zero and we will clearly get division by zero with each of these values of \( t \).

Step 2
Next we need to do find the solution. To get the solution we’ll need to multiply both sides by the LCD and the go through the same process we used in the first couple of practice problems. Here is that work.

\[ (t-5)(t+5) \left( \frac{4t}{(t-5)(t+5)} \right) = -\left( \frac{1}{t-5} \right)(t-5)(t+5) \]

\[ 4t = -(t+5) \]

\[ 4t = -t - 5 \]

\[ 5t = -5 \]

\[ t = -1 \]

Step 3
Finally we need to verify that our answer from Step 2 is in fact a solution.

The first thing to note is that it is not one of the values of \( t \) that we need to avoid. Having determined that we know that we do have a potential solution (i.e. it’s not a value of \( t \) we need to avoid) all we need to do is plug the solution into the equation given in the problem statement.
Here is the verification work.

\[
\frac{4(-1)}{(-1)^2 - 25} \quad ? \quad \frac{1}{5 - (-1)}
\]

\[
\frac{-4}{1 - 25} \quad ? \quad \frac{1}{5 + 1}
\]

\[
\frac{1}{6} = \frac{1}{6} \quad \text{OK}
\]

So, we can see that our solution from Step 2 is in fact the solution to the equation.

5. Solve the following equation and check your answer.

\[
\frac{3y + 4}{y - 1} = 2 + \frac{7}{y - 1}
\]

Hint : Do not forget to watch out for values of \( y \) that we’ll need to avoid!

Step 1
First we can see that the LCD for this equation is,

\[ y - 1 \]

From this we can also see that we’ll need to avoid \( y = 1 \). Remember that we have to avoid division by zero and we will clearly get division by zero with this value of \( y \).

Step 2
Next we need to do find the solution. To get the solution we’ll need to multiply both sides by the LCD and the go through the same process we used in the first couple of practice problems. Here is that work.

\[
(y - 1) \left( \frac{3y + 4}{y - 1} \right) = \left( 2 + \frac{7}{y - 1} \right) (y - 1)
\]

\[
3y + 4 = 2(y - 1) + 7
\]

\[
3y + 4 = 2y + 5
\]

\[
y = 1
\]

Step 3
Finally we need to verify that our answer from Step 2 is in fact a solution and in this case there isn’t a lot of work to that process. We can see that our potential solution from Step 2 is in fact the value of \( y \) that we need to avoid and so this equation has no solution.
We could also see this if we plugged the value of \( y \) from Step 2 into the equation given in the problem statement. Had we done that we would have gotten a division by zero in two of the terms! That, of course, is why we needed to avoid \( y = 1 \).

Note as well that we only have caught the division by zero if we verify by plugging into the equation in the problem statement. Had we checked in the equation we got by multiplying by the LCD it would have appeared to be a solution! This is the reason that we need to always check in the equation from the problem statement.

6. Solve the following equation and check your answer.

\[
\frac{5x}{3x-3} + \frac{6}{x+2} = \frac{5}{3}
\]

Hint : Do not forget to watch out for values of \( x \) that we’ll need to avoid!

Step 1
Let’s first factor a 3 out of the denominator of the first term on the left side so we can identify the LCD.

\[
\frac{5x}{3(x-1)} + \frac{6}{x+2} = \frac{5}{3}
\]

So, after factoring doing the factoring on the first term we can quickly see that the LCD for this equation is,

\[
3(x-1)(x+2)
\]

From this we can also see that we’ll need to avoid \( x = 1 \) and \( x = -2 \). Remember that we have to avoid division by zero and we will clearly get division by zero with each of these values of \( x \).

Step 2
Next we need to do find the solution. To get the solution we’ll need to multiply both sides by the LCD and the go through the same process we used in the first couple of practice problems. Here is that work.
\[
3(x - 1)(x + 2) \left( \frac{5x}{3(x - 1)} + \frac{6}{x + 2} \right) = \left( \frac{5}{3} \right) \left[ 3(x - 1)(x + 2) \right]
\]

\[
5x(x + 2) + 3(x - 1)(6) = 5(x - 1)(x + 2)
\]

\[
5x^2 + 10x + 18(x - 1) = 5(x^2 + x - 2)
\]

\[
5x^2 + 10x + 18x - 18 = 5x^2 + 5x - 10
\]

\[
28x - 18 = 5x - 10
\]

\[
23x = 8
\]

\[
x = \frac{8}{23}
\]

Step 3
Finally we need to verify that our answer from Step 2 is in fact a solution.

The first thing to note is that it is not one of the values of \(x\) that we need to avoid. Having determined that we know that we do have a potential solution (i.e. it’s not a value of \(x\) we need to avoid) all we need to do is plug the solution into the equation given in the problem statement.

Here is the verification work.

\[
\frac{5 \left( \frac{8}{23} \right)}{3 \left( \frac{8}{23} \right) - 3} + \frac{6}{\left( \frac{8}{23} \right) + 2} = \frac{5}{3}
\]

\[
\frac{\frac{40}{23}}{\frac{\frac{8}{23}}{23}} + \frac{\frac{6}{\frac{8}{23}}}{\frac{5}{3}} = \frac{5}{3}
\]

\[
\frac{\frac{40}{23}}{\frac{\frac{8}{23}}{23}} + \frac{\frac{6}{\frac{8}{23}}}{\frac{5}{3}} = \frac{5}{3}
\]

So, we can see that our solution from Step 2 is in fact the solution to the equation.

With this problem we have seen that both the solution and the verification step can be somewhat “messy”. That will happen on occasion and we shouldn’t get excited about it when it does. It is just the way these problems work on occasion.

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**Application of Linear Equations**
1. A widget is being sold in a store for $135.40 and has been marked up 7%. How much did the store pay for the widget?

Step 1
We’ll start by letting \( p \) be the price that the store paid for the widget. The widget has been marked up by 7% and so 0.07\( p \) has been added to the original price, \( p \).

The equation we get for this problem is then,

\[
p + 0.07p = 135.4
1.07p = 135.4
\]

Step 2
To finish all we need to do is divide both sides by 1.07 to get the price the store paid for the widget.

\[
p = 126.5421
\]

So, with rounding the store paid $126.54 for the widget.

2. A store is having a 30% off sale and one item is now being sold for $9.95. What was the original price of the item?

Step 1
We’ll start by letting \( p \) be the original price of the item. The price of the item has been reduced by 30% and so 0.30\( p \) has been subtracted from the original price, \( p \).

The equation we get for this problem is then,

\[
p - 0.3p = 9.95
0.7p = 9.95
\]

Step 2
To finish all we need to do is divide both sides by 0.7 to get the original price of the item.

\[
p = 14.2143
\]

So, with rounding the original price of the item was $14.21.

3. Two planes start out 2800 km apart and move towards each other meeting after 3.5 hours. One plane flies at 75 km/hour slower than the other plane. What was the speed of each plane?

Step 1
Let’s start with a diagram of what is going on in this situation.
Step 2
We can next set up a word equation for this situation.

\[
\text{Distance of Plane A} + \text{Distance of Plane B} = 2800
\]

We know that \(\text{Distance} = \text{Rate} \times \text{Time}\) so this gives to following word equation.

\[
\left( \frac{\text{Rate of Plane A}}{\text{Time of Plane A}} \right) + \left( \frac{\text{Rate of Plane B}}{\text{Time of Plane B}} \right) = 2800
\]

Step 3
Let’s let \(r\) be the speed of the faster plane. Therefore, the speed of the slower plane is \(r - 75\). We also know that each plane travels for 3.5 hours. Plugging all this information into the word equation above gives the following equation.

\[
(r)(3.5) + (r - 75)(3.5) = 2800
\]

\[
3.5r + 3.5(r - 75) = 2800
\]

Step 4
Now we can solve this equation for the speed of the faster plane.

\[
3.5r + 3.5(r - 75) = 2800
\]

\[
7r - 262.5 = 2800
\]

\[
7r = 3062.5
\]

\[
r = 437.5
\]

So, the faster plane is traveling at 437.5 km/hour while the slower plane is traveling at 362.5 km/hour (75 km/hour slower than faster plane).
Let’s start with a diagram of what is going on in this situation.

![Diagram showing distances and speeds of Kim and Mike.]  

Step 2
We can next set up a word equation for this situation.

\[
\frac{\text{Distance}}{\text{Kim Moved}} = 35 + \frac{\text{Distance}}{\text{Mike Moved}}
\]

We know that Distance = Rate X Time so this gives the following word equation.

\[
\left(\frac{\text{Rate of}}{\text{Kim}}\right)\left(\frac{\text{Time of}}{\text{Kim}}\right) = 35 + \left(\frac{\text{Rate of}}{\text{Mike}}\right)\left(\frac{\text{Time of}}{\text{Mike}}\right)
\]

Step 3
Both Kim and Mike move for the same amount of time so let’s call that \( t \). We also know that Kim moves at 3.4 ft/sec while Mike moves at 2 ft/sec. Plugging all this information into the word equation above gives the following equation.

\[
(3.4)(t) = 35 + (2)(t)
\]

\[
3.4t = 35 + 2t
\]

Step 4
Now we can solve this equation for the time they traveled.

\[
3.4t = 35 + 2t
\]

\[
1.4t = 35
\]

\[
t = 25
\]

So, Kim will catch up with Mike after she moves for 25 seconds.

5. A pump can empty a pool in 7 hours and a different pump can empty the same pool in 12 hours. How long does it take for both pumps working together to empty the pool?

Step 1
So, if we consider emptying the pool to be one job we have the following word equation describing both pumps working to empty the pool.

\[
\left( \text{Portion of job done by first pump} \right) + \left( \text{Portion of job done by second pump} \right) = 1 \text{ Job}
\]

We know that Portion of Job = Work Rate X Work Time so this gives the following word equation.

\[
\left( \text{Work Rate of first pump} \right) \left( \text{Work Time of first pump} \right) + \left( \text{Work Rate of second pump} \right) \left( \text{Work Time of second pump} \right) = 1
\]

Step 2
We’ll need the work rates of each pump and for that we can use the information we have in the problem statement on each pump working individually and the following word equation for each pump doing the job individually.

\[
\left( \text{Work Rate of pump} \right) \left( \text{Work Time of pump} \right) = 1
\]

For the first pump we have,

\[
\left( \text{Work Rate of first pump} \right)(7) = 1 \quad \Rightarrow \quad \text{Work Rate of first pump} = \frac{1}{7}
\]

and for the second pump we have,

\[
\left( \text{Work Rate of second pump} \right)(12) = 1 \quad \Rightarrow \quad \text{Work Rate of second pump} = \frac{1}{12}
\]

Step 3
Now let \( t \) be the amount of time it takes both pumps working together to empty the pool. Using this and the work rates we found in the second step our word equation from the first step becomes,

\[
\left( \frac{1}{7} \right)(t) + \left( \frac{1}{12} \right)t = 1
\]

\[
\frac{19}{84}t = 1
\]

Step 4
Now we can solve this for \( t \).

\[
\frac{19}{84}t = 1 \quad \Rightarrow \quad t = \frac{84}{19} = 4.4211
\]
So, it will take both pumps approximately 4.4211 hours to empty the pool if they both work together.

6. John can paint a house in 28 hours. John and Dave can paint the house in 17 hours working together. How long would it take Dave to paint the house by himself?

Step 1
So, if we consider painting the house to be a single job we have the following word equation if both John and Dave work together to paint the house.

\[
\left( \text{Portion of job done by John} \right) + \left( \text{Portion of job done by Dave} \right) = 1 \text{ Job}
\]

We know that Portion of Job = Work Rate X Work Time so this gives the following word equation.

\[
\left( \text{Work Rate of John} \right) \times \left( \text{Work Time of John} \right) + \left( \text{Work Rate of Dave} \right) \times \left( \text{Work Time of Dave} \right) = 1
\]

Step 2
We know that John can paint the house in 28 hours so we can use the following equation to determine the John’s work rate.

\[
\left( \text{Work Rate of John} \right) \times \left( \text{Work Time of John} \right) = \left( \text{Work Rate of John} \right) \times \left( 28 \right) = 1 \quad \Rightarrow \quad \text{Work Rate of John} = \frac{1}{28}
\]

Similarly, if we let \( t \) be the amount of time it takes Dave to paint the house by himself we have the following relationship between the time and work rate of Dave.

\[
\left( \text{Work Rate of Dave} \right) \times \left( \text{Work Time of Dave} \right) = \left( \text{Work Rate of Dave} \right) \times \left( t \right) = 1 \quad \Rightarrow \quad \text{Work Rate of Dave} = \frac{1}{t}
\]

Step 3
We can now plug in the information from the second step as well as the fact that it takes John and Dave 17 hours to paint the house by themselves into the word equation from the first step to get,

\[
\left( \frac{1}{28} \right) \left( 17 \right) + \left( \frac{1}{t} \right) \left( 17 \right) = 1
\]

\[
\frac{17}{28} + \frac{17}{t} = 1
\]

Step 4
Now we can solve this for \( t \).
\[
\frac{17}{t} = 1 - \frac{17}{28} \\
\frac{17}{t} = \frac{11}{28} \\
\frac{476}{11} = t \implies t = 43.2727
\]

So, it will take Dave approximately 43.2727 hours to paint the house by himself.

7. How much of a 20% acid solution should we add to 20 gallons of a 42% acid solution to get a 35% acid solution?

Step 1
We’ll start by letting \( x \) be the amount of the 20% solution we’ll need. This in turn means that we’ll have \( x + 20 \) gallons of the 35% solution once we’re done mixing.

The basic word equation is then,

\[
\left( \text{Amount of acid in 20\% solution} \right) + \left( \text{Amount of acid in 42\% solution} \right) = \left( \text{Amount of acid in 35\% solution} \right)
\]

We know that \( \text{Amount of Acid in Solution} = \text{Percentage of Solution} \times \text{Volume of Solution} \). This gives the following word equation.

\[
(0.20)\left( \text{Volume of 20\% solution} \right) + (0.42)\left( \text{Volume of 42\% solution} \right) = (0.35)\left( \text{Volume of 35\% solution} \right)
\]

Step 2
So, plugging all the known information in gives the following equation that we can solve for \( x \).  

\[
0.2x + (0.42)(20) = 0.35(x + 20) \\
0.2x + 8.4 = 0.35x + 7 \\
0.15x = 1.4 \\
x = 9.33
\]

So, we’ll need 9.33 gallons of the 20% acid solution.

8. We need 100 liters of a 25% saline solution and we only have a 14% solution and a 60% solution. How much of each should we mix together to get the 100 liters of the 25% solution?
Step 1
We’ll start by letting $x$ be the amount of the 14\% solution we’ll need. This in turn means that we’ll need $100 - x$ gallons of the 60\% solution.

The basic word equation is then,

\[
\left( \text{Amount of salt in 14\% solution} \right) + \left( \text{Amount of salt in 60\% solution} \right) = \left( \text{Amount of salt in 25\% solution} \right)
\]

We know that Amount of Salt in Solution = Percentage of Solution X Volume of Solution. This gives the following word equation.

\[
(0.14) \left( \text{Volume of 14\% solution} \right) + (0.6) \left( \text{Volume of 60\% solution} \right) = (0.25) \left( \text{Volume of 25\% solution} \right)
\]

Step 2
So, plugging all the known information in gives the following equation that we can solve for $x$.

\[
0.14x + 0.6(100 - x) = 0.25(100)
\]
\[
0.14x + 60 - 0.6x = 25
\]
\[
-0.46x = -35
\]
\[
x = 76.09
\]

So, we’ll need 76.09 liters of the 14\% saline solution and 23.91 liters of the 60\% saline solution.

9. We want to fence in a field whose length is twice the width and we have 80 feet of fencing material. If we use all the fencing material what would the dimensions of the field be?

Step 1
We’ll start by letting $x$ be width of the field and so $2x$ will be the length of the field.

Next, we have the following word equation for the length of the fencing material.

\[
2(\text{Length of Fence}) + 2(\text{Width of Fence}) = 80
\]

Step 2
So, plugging all the known information in gives the following equation that we can solve for $x$.

\[
2(x) + 2(2x) = 80
\]
\[
6x = 80
\]
\[
x = 13.33
\]

So, the width of the fence will be 13.33 feet while the length will be 26.66 feet.
Equations With More Than One Variable

1. Solve \( E = 3v \left( 4 - \frac{2}{r} \right) \) for \( r \).

Step 1
Note that there quite a few solution “paths” that you can take to get the solution to this problem. For this solution let’s first distribute the \( 3v \) though the parenthesis.

\[
E = 12v - \frac{6v}{r}
\]

Step 2
Next, let’s clear the denominator out by multiplying both sides by \( r \).

\[
Er = 12vr - 6v
\]

Step 3
Now let’s get all the terms with \( r \) on one side and the terms without \( r \) on the other side. We’ll also factor the \( r \) out when we’re done as well. Doing this gives,

\[
Er - 12vr = -6v
\]

\[
(E - 12v)r = -6v
\]

Step 4
Finally, all we need to do is divide by both sides by the coefficient of the \( r \) to get,

\[
r = \frac{-6v}{E - 12v}
\]

Note that depending upon the path you chose for your solution you may have something slightly different for your answer. However, you could do some manipulation of your answer to make it look like mine (or you could manipulate mine to make it look like yours).

2. Solve \( Q = \frac{6h}{7s} + 4(1 - h) \) for \( s \).

Step 1
Note that there quite a few solution “paths” that you can take to get the solution to this problem. For this solution let’s first clear the denominator out by multiplying both sides by $7s$.

\[
(Q)(7s) = 7s\left(\frac{6h}{7s} + 4(1-h)\right)
\]

\[
7sQ = 6h + 28s(1-h)
\]

Step 2
Now let’s get all the terms with $s$ on one side and the terms without $s$ on the other side. We’ll also factor the $s$ out when we’re done as well. Doing this gives,

\[
7sQ - 28s(1-h) = 6h
\]

\[
\left[7Q - 28(1-h)\right]s = 6h
\]

Step 3
Finally, all we need to do is divide by both sides by the coefficient of the $s$ to get,

\[
s = \frac{6h}{7Q - 28(1-h)}
\]

Note that depending upon the path you chose for your solution you may have something slightly different for your answer. However, you could do some manipulation of your answer to make it look like mine (or you could manipulate mine to make it look like yours).

3. Solve $Q = \frac{6h}{7s} + 4(1-h)$ for $h$.

Step 1
Note that there quite a few solution “paths” that you can take to get the solution to this problem. For this solution let’s first clear the denominator out by multiplying both sides by $7s$.

\[
(Q)(7s) = 7s\left(\frac{6h}{7s} + 4(1-h)\right)
\]

\[
7sQ = 6h + 28s(1-h)
\]

\[
7sQ = 6h + 28s - 28sh
\]

We also distributed the $28s$ through the parenthesis in anticipation of the next step.

Step 2
Now let’s get all the terms with $h$ on one side and the terms without $h$ on the other side. We’ll also factor the $h$ out when we’re done as well. Doing this gives,
7sQ − 28s = 6h − 28sh
7sQ − 28s = (6 − 28s)h

Step 3
Finally, all we need to do is divide by both sides by the coefficient of the h to get,

\[ h = \frac{7sQ − 28s}{6 − 28s} \]

Note that depending upon the path you chose for your solution you may have something slightly different for your answer. However, you could do some manipulation of your answer to make it look like mine (or you could manipulate mine to make it look like yours).

4. Solve \( A - \frac{1 - 2t}{4p} = \frac{4 + 3t}{5p} \) for t.

Step 1
Note that there quite a few solution “paths” that you can take to get the solution to this problem. For this solution let’s first clear the denominator out by multiplying both sides by \( 20p \).

\[
A - \frac{1 - 2t}{4p} = \frac{4 + 3t}{5p} \\
20p \left( A - \frac{1 - 2t}{4p} \right) = 20p \left( \frac{4 + 3t}{5p} \right) \\
20Ap - 5(1 - 2t) = 4(4 + 3t) \\
20Ap - 5 + 10t = 16 + 12t
\]

We also distributed the constants through the parenthesis in anticipation of the next step.

Step 2
Now let’s get all the terms with t on one side and the terms without t on the other side. Doing this gives,

\[ 20Ap - 21 = 2t \]

Step 3
Finally, all we need to do is divide by both sides by the coefficient of the t to get,

\[ t = \frac{20Ap - 21}{2} \]
Note that depending upon the path you chose for your solution you may have something slightly different for your answer. However, you could do some manipulation of your answer to make it look like mine (or you could manipulate mine to make it look like yours).

5. Solve \( y = \frac{10}{3-7x} \) for \( x \).

Step 1
Note that there quite a few solution “paths” that you can take to get the solution to this problem. For this solution let’s first clear the denominator out.

\[
y(3-7x) = 10
\]

\[
3y - 7xy = 10
\]

We also distributed the \( y \) through the parenthesis in anticipation of the next step.

Step 2
Now let’s get all the terms with \( x \) on one side and the terms without \( x \) on the other side. Doing this gives,

\[
-7xy = 10 - 3y
\]

Step 3
Finally, all we need to do is divide by both sides by the coefficient of the \( x \) to get,

\[
x = \frac{10 - 3y}{-7y} = \frac{3y - 10}{7y}
\]

We distributed the minus sign in the denominator into the numerator to reduce the number of minus signs in the answer but doesn’t need to be done if you don’t want to.

Note that depending upon the path you chose for your solution you may have something slightly different for your answer. However, you could do some manipulation of your answer to make it look like mine (or you could manipulate mine to make it look like yours).

6. Solve \( y = \frac{3+x}{12-9x} \) for \( x \).

Step 1
Note that there quite a few solution “paths” that you can take to get the solution to this problem. For this solution let’s first clear the denominator out.
\[ y(12 - 9x) = 3 + x \]
\[ 12y - 9xy = 3 + x \]

We also distributed the \( y \) through the parenthesis in anticipation of the next step.

Step 2
Now let’s get all the terms with \( x \) on one side and the terms without \( x \) on the other side. Doing this gives,
\[ 12y - 3 = x + 9xy \]
\[ 12y - 3 = x(1 + 9y) \]

Step 3
Finally, all we need to do is divide by both sides by the coefficient of the \( x \) to get,
\[
\begin{align*}
  x &= \frac{12y - 3}{1 + 9y} \\
\end{align*}
\]

Note that depending upon the path you chose for your solution you may have something slightly different for your answer. However, you could do some manipulation of your answer to make it look like mine (or you could manipulate mine to make it look like yours).

**Quadratic Equations – Part I**

1. Solve the following quadratic equation by factoring.
\[ u^2 - 5u - 14 = 0 \]

Step 1
Not much to this problem. We already have zero on one side of the equation, which we need to proceed with this problem. Therefore, all we need to do is actually factor the quadratic.
\[ (u + 2)(u - 7) = 0 \]

Step 2
Now all we need to do is use the zero factor property to get,
\[
\begin{align*}
  u + 2 &= 0 \quad & \text{OR} \quad & u - 7 &= 0 \\
  u &= -2 \quad & u &= 7 \\
\end{align*}
\]

Therefore the two solutions are: \[ u = -2 \text{ and } u = 7 \]
We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.

2. Solve the following quadratic equation by factoring.

\[ x^2 + 15x = -50 \]

**Step 1**
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.

\[ x^2 + 15x + 50 = 0 \]

\[ (x + 5)(x + 10) = 0 \]

**Step 2**
Now all we need to do is use the zero factor property to get,

\[ x + 5 = 0 \quad \text{OR} \quad x + 10 = 0 \]

\[ x = -5 \quad \text{OR} \quad x = -10 \]

Therefore the two solutions are: \[ x = -5 \text{ and } x = -10 \]

We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.

3. Solve the following quadratic equation by factoring.

\[ y^2 = 11y - 28 \]

**Step 1**
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.

\[ y^2 - 11y + 28 = 0 \]

\[ (y - 4)(y - 7) = 0 \]

**Step 2**
Now all we need to do is use the zero factor property to get,

\[ y - 4 = 0 \quad \text{OR} \quad y - 7 = 0 \]

\[ y = 4 \quad \text{OR} \quad y = 7 \]

Therefore the two solutions are: \[ y = 4 \text{ and } y = 7 \]

We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.
4. Solve the following quadratic equation by factoring.
\[ 19x = 7 - 6x^2 \]

Step 1
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.
\[ 6x^2 + 19x - 7 = 0 \]
\[ (3x - 1)(2x + 7) = 0 \]

Step 2
Now all we need to do is use the zero factor property to get,
\[ 3x - 1 = 0 \quad \text{OR} \quad 2x + 7 = 0 \]
\[ x = \frac{1}{3} \quad \quad \quad \quad \quad x = -\frac{7}{2} \]

Therefore the two solutions are : \[ x = \frac{1}{3} \quad \text{and} \quad x = -\frac{7}{2} \]

We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.

5. Solve the following quadratic equation by factoring.
\[ 6w^2 - w = 5 \]

Step 1
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.
\[ 6w^2 - w - 5 = 0 \]
\[ (6w + 5)(w - 1) = 0 \]

Step 2
Now all we need to do is use the zero factor property to get,
\[ 6w + 5 = 0 \quad \text{OR} \quad w - 1 = 0 \]
\[ w = -\frac{5}{6} \quad \quad \quad \quad \quad w = 1 \]

Therefore the two solutions are : \[ w = -\frac{5}{6} \quad \text{and} \quad w = 1 \]

We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.
6. Solve the following quadratic equation by factoring.

\[ z^2 - 16z + 61 = 2z - 20 \]

Step 1
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.

\[ z^2 - 18z + 81 = 0 \]

\[ (z - 9)^2 = 0 \]

Step 2
From the factored form we can quickly see that the solution is \[ z = 9 \]

7. Solve the following quadratic equation by factoring.

\[ 12x^2 = 25x \]

Step 1
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.

\[ 12x^2 - 25x = 0 \]

\[ x(12x - 25) = 0 \]

Make sure that you do not just cancel an \( x \) from both sides of the equation!

Step 2
Now all we need to do is use the zero factor property to get,

\[ x = 0 \quad \text{OR} \quad 12x - 25 = 0 \]

\[ x = \frac{25}{12} \]

Therefore the two solutions are \[ x = 0 \text{ and } x = \frac{25}{12} \]

Note that if we’d canceled an \( x \) from both sides of the equation in the first step we would have missed the solution \( x = 0 \! \)

8. Use factoring to solve the following equation.

\[ x^4 - 2x^3 - 3x^2 = 0 \]

Step 1
Do not let the fact that this equation is not a quadratic equation convince you that you can’t do it! Note that we can factor an \( x^2 \) out of the equation. Doing that gives,
9. Use factoring to solve the following equation.

\[ t^5 = 9t^3 \]

**Step 1**

Do not let the fact that this equation is not a quadratic equation convince you that you can’t do it! Note that we move both terms to one side we can factor a \( t^3 \) out of the equation. Doing that gives,

\[
\begin{align*}
  t^5 - 9t^3 &= 0 \\
  t^3(t^2 - 9) &= 0
\end{align*}
\]

The quantity in the parenthesis is a quadratic and we can factor it. The full factoring of the equation is then,

\[
\begin{align*}
  t^3(t - 3)(t + 3) &= 0
\end{align*}
\]

**Step 2**

Now all we need to do is use the zero factor property to get,

\[
\begin{align*}
  t^3 &= 0 & \text{OR} & & t - 3 &= 0 & \text{OR} & & t + 3 &= 0 \\
  t &= 0 & & & t &= 3 & & & t &= -3
\end{align*}
\]

Therefore the three solutions are: \( t = 0, t = 3 \) and \( t = -3 \)
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\[
\frac{w^2 - 10}{w + 2} + w - 4 = w - 3
\]

Step 1
This is an equation containing rational expressions so we know that the first step is to clear out the denominator by multiplying by the LCD, which is \(w + 2\) in this case. Also, note that we now know that we must avoid \(w = -2\) so we do not get division by zero.

Multiplying by the LCD and doing some basic simplification gives,

\[
(w + 2)\left(\frac{w^2 - 10}{w + 2} + w - 4\right) = (w - 3)(w + 2)
\]

\[
w^2 - 10 + (w - 4)(w + 2) = (w - 3)(w + 2)
\]

\[
w^2 - 10 + w^2 - 2w - 8 = w^2 - w - 6
\]

\[
w^2 - w - 12 = 0
\]

Step 2
We can now factor the quadratic to get,

\[
(w - 4)(w + 3) = 0
\]

The zero factor property now tells us,

\[
w - 4 = 0 \quad \text{OR} \quad w + 3 = 0
\]

\[
w = 4 \quad \text{OR} \quad w = -3
\]

Therefore the two solutions are: \(w = 4\) and \(w = -3\).

Note as well that because neither of these are \(w = -2\) we know that we won’t get division by zero. Do not forget this important part of the solution process for equations involving rational expressions!

11. Use factoring to solve the following equation.

\[
\frac{4z}{z + 1} + \frac{5}{z} = \frac{6z + 5}{z^2 + z}
\]

Step 1
This is an equation containing rational expressions so we know that the first step is to clear out the denominator by multiplying by the LCD, which is \(z(z + 1)\) in this case. Also, note that we now know that we must avoid \(z = 0\) and \(z = -1\) so we do not get division by zero.

Multiplying by the LCD and doing some basic simplification gives,
Step 2
We can now factor the quadratic to get,

\[ z(4z - 1) = 0 \]

The zero factor property now tells us,

\[ z = 0 \quad \text{OR} \quad 4z - 1 = 0 \]

\[ z = \frac{1}{4} \]

Note that we cannot use the first potential solution since that would give us division by zero! Therefore the only solution is: \( z = \frac{1}{4} \).

When dealing with equations that have rational expressions do not forget to verify that you do not get division by zero with any of the potential solutions! As we saw in this case if we had not checked we would have gotten a value of \( z \) that seemed to be a solution but in fact was not because of the division by zero issue.

12. Use factoring to solve the following equation.

\[ x + 1 = \frac{2x - 7}{x + 5} - \frac{5x + 8}{x + 5} \]

Step 1
This is an equation containing rational expressions so we know that the first step is to clear out the denominator by multiplying by the LCD, which is \( x + 5 \) in this case. Also, note that we now know that we must avoid \( x = -5 \) so we do not get division by zero.

Multiplying by the LCD and doing some basic simplification gives,
\[(x + 5)(x + 1) = \left(\frac{2x - 7}{x + 5} - \frac{5x + 8}{x + 5}\right)(x + 5)\]
\[(x + 5)(x + 1) = \left(\frac{2x - 7}{x + 5}\right)(x + 5) - \left(\frac{5x + 8}{x + 5}\right)(x + 5)\]
\[(x + 5)(x + 1) = 2x - 7 - (5x + 8)\]
\[x^2 + 6x + 5 = 2x - 7 - 5x - 8\]
\[x^2 + 9x + 20 = 0\]

Step 2
We can now factor the quadratic to get,
\[(x + 4)(x + 5) = 0\]

The zero factor property now tells us,
\[x + 4 = 0 \quad \text{OR} \quad x + 5 = 0\]
\[x = -4 \quad \text{OR} \quad x = -5\]

Note that we cannot use the second potential solution since that would give us division by zero!

Therefore the only solution is: \(x = -4\).

When dealing with equations that have rational expressions do not forget to verify that you do not get division by zero with any of the potential solutions! As we saw in this case if we had not checked we would have gotten a value of \(x\) that seemed to be a solution but in fact was not because of the division by zero issue.

13. Use the Square Root Property to solve the equation.
\[9u^2 - 16 = 0\]

Step 1
There really isn’t too much to this problem. Just recall that we need to get the variable on one side of the equation by itself with a coefficient of one. For this problem that gives,
\[9u^2 = 16\]
\[u^2 = \frac{16}{9}\]

Step 2
Now all we need to do is use the Square Root Property to get,
\[u = \pm \sqrt{\frac{16}{9}} = \pm \frac{\sqrt{16}}{\sqrt{9}} = \pm \frac{4}{3}\]
So we have the following two solutions: $u = -\frac{4}{3}$ and $u = \frac{4}{3}$.

14. Use the Square Root Property to solve the equation.

$$x^2 + 15 = 0$$

Step 1
There really isn’t too much to this problem. Just recall that we need to get the variable on one side of the equation by itself with a coefficient of one. For this problem that gives,

$$x^2 = -15$$

Step 2
Now all we need to do is use the Square Root Property to get,

$$x = \pm \sqrt{-15} = \pm \sqrt{15}i$$

So we have the following two solutions: $x = -\sqrt{15}i$ and $x = \sqrt{15}i$.

Do not get excited about complex solutions. They will happen fairly regularly when solving quadratic equations so we need to be able to deal with them.

15. Use the Square Root Property to solve the equation.

$$\left(z - 2\right)^2 - 36 = 0$$

Step 1
There really isn’t too much to this problem. Just recall that we need to get the squared term on one side of the equation by itself with a coefficient of one. For this problem that gives,

$$\left(z - 2\right)^2 = 36$$

Step 2
Using the Square Root Property gives,

$$z - 2 = \pm \sqrt{36} = \pm 6$$

To finish this off all we need to do then is solve for $z$ by adding 2 to both sides. This gives,

$$z = 2 \pm 6 \quad \Rightarrow \quad z = 2 - 6 = -4, \quad z = 2 + 6 = 8$$
So, after we did a little arithmetic, have the following two solutions: $z = -4$ and $z = 8$.

16. Use the Square Root Property to solve the equation. 

$$ (6t + 1)^2 + 3 = 0 $$

Step 1
There really isn’t too much to this problem. Just recall that we need to get the squared term on one side of the equation by itself with a coefficient of one. For this problem that gives,

$$ (6t + 1)^2 = -3 $$

Step 2
Using the Square Root Property gives,

$$ 6t + 1 = \pm \sqrt{-3} = \pm \sqrt{3}i $$

To finish this off all we need to do then is solve for $t$ by subtracting 1 from both sides and then dividing by the 6. This gives,

$$ 6t = -1 \pm \sqrt{3}i $$

$$ t = \frac{-1 \pm \sqrt{3}i}{6} = -\frac{1}{6} \pm \frac{\sqrt{3}}{6}i $$

Note that we did a little rewrite after dividing by the 6 to put the answer in a more standard form for complex numbers.

We then have the following two solutions: $t = -\frac{1}{6} - \frac{\sqrt{3}}{6}i$ and $t = -\frac{1}{6} + \frac{\sqrt{3}}{6}i$.

---

**Quadratic Equations – Part II**

1. Complete the square on the following expression.

$$ x^2 + 8x $$

**Step 1**
First we need to identify the number we need to add to this. Recall that we will need the coefficient of the $x$ to do this. The number we need is,
\[
\left(\frac{8}{2}\right)^2 = (4)^2 = 16
\]

**Step 2**
To complete the square all we need to do then is add this to the expression and factor the result. Doing this gives,

\[
x^2 + 8x + 16 = (x + 4)^2
\]

2. Complete the square on the following expression.

\[u^2 - 11u\]

**Step 1**
First we need to identify the number we need to add to this. Recall that we will need the coefficient of the \(u\) to do this. The number we need is,

\[
\left(\frac{-11}{2}\right)^2 = \frac{(-11)^2}{(2)^2} = \frac{121}{4}
\]

**Step 2**
To complete the square all we need to do then is add this to the expression and factor the result. Doing this gives,

\[
u^2 - 11u + \frac{121}{4} = \left(u - \frac{11}{2}\right)^2
\]

Recall that this will always factor as \(u\) plus the number inside the parenthesis in the first step, \(-\frac{11}{2}\) in this case.

Do not get too excited about the fractions that can show up in these problems. They will be there occasionally and so we need to be able to deal with them. Luckily, if you can recall the “trick” to the factoring they aren’t all that bad.

3. Complete the square on the following expression.

\[2z^2 - 12z\]

**Step 1**
Remember that prior to completing the square we need a coefficient of one on the squared variable. However, we can’t just “cancel” it since that requires an equation which we don’t have.
Therefore, we need to first factor a 2 out of the expression as follows,

\[2z^2 - 12z = 2\left(z^2 - 6z\right)\]

We can now proceed with completing the square on the expression inside the parenthesis.

Step 2
Next we’ll need the number we need to add onto the expression inside the parenthesis. We’ll need the coefficient of the \(z\) to do this. The number we need is,

\[
\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9
\]

Step 3
To complete the square all we need to do then is add this to the expression inside the parenthesis and factor the result. Doing this gives,

\[2z^2 - 12z = 2\left(z^2 - 6z + 9\right) = 2(z - 3)^2\]

Be careful when the coefficient of the squared term is not a one! In order to get the correct answer to completing the square we must have a coefficient of one on the squared term!

4. Solve the following quadratic equation by completing the square.

\[t^2 - 10t + 34 = 0\]

Step 1
First, let’s get the equation put into the form where all the variables are on one side and the number is on the other side. Doing this gives,

\[t^2 - 10t = -34\]

Step 2
We can now complete the square on the expression on the left side of the equation.

The number that we’ll need to do this is,

\[
\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25
\]

Step 3
At this point we need to recall that we have an equation here and what we do to one side of the equation we also need to do the other. In other words, don’t forget to add the number from the previous step to both sides of the equation from Step 1.
\[ t^2 - 10t + 25 = -34 + 25 \]
\[ (t - 5)^2 = -9 \]

Step 4
Now all we need to do to finish solving the equation is to use the Square Root Property on the equation from the previous step. Doing this gives,

\[ t - 5 = \pm \sqrt{-9} = \pm 3i \]
\[ t = 5 \pm 3i \]

The two solutions to this equation are then: \[ t = 5 - 3i \text{ and } t = 5 + 3i \].

5. Solve the following quadratic equation by completing the square.

\[ v^2 + 8v - 9 = 0 \]

Step 1
First, let’s get the equation put into the form where all the variables are on one side and the number is on the other side. Doing this gives,

\[ v^2 + 8v = 9 \]

Step 2
We can now complete the square on the expression on the left side of the equation.  The number that we’ll need to do this is,

\[ \left( \frac{8}{2} \right)^2 = (4)^2 = 16 \]

Step 3
At this point we need to recall that we have an equation here and what we do to one side of the equation we also need to do the other.  In other words, don’t forget to add the number from the previous step to both sides of the equation from Step 1.

\[ v^2 + 8v + 16 = 9 + 16 \]
\[ (v + 4)^2 = 25 \]

Step 4
Now all we need to do to finish solving the equation is to use the Square Root Property on the equation from the previous step. Doing this gives,
\[ v + 4 = \pm \sqrt{25} = \pm 5 \]
\[ v = -4 \pm 5 \quad \Rightarrow \quad v = -4 - 5 = -9, \quad v = -4 + 5 = 1 \]

The two solutions to this equation are then: \( v = -9 \) and \( v = 1 \).

6. Solve the following quadratic equation by completing the square.

\[ x^2 + 9x + 16 = 0 \]

Step 1
First, let’s get the equation put into the form where all the variables are on one side and the number is on the other side. Doing this gives,

\[ x^2 + 9x = -16 \]

Step 2
We can now complete the square on the expression on the left side of the equation.

The number that we’ll need to do this is,

\[ \left( \frac{9}{2} \right)^2 = \left( \frac{9}{2} \right)^2 = \frac{81}{4} \]

Step 3
At this point we need to recall that we have an equation here and what we do to one side of the equation we also need to do the other. In other words, don’t forget to add the number from the previous step to both sides of the equation from Step 1.

\[ x^2 + 9x + \frac{81}{4} = -16 + \frac{81}{4} \]

\[ \left( x + \frac{9}{2} \right)^2 = \frac{17}{4} \]

Step 4
Now all we need to do to finish solving the equation is to use the Square Root Property on the equation from the previous step. Doing this gives,

\[ x + \frac{9}{2} = \pm \sqrt{\frac{17}{4}} = \pm \frac{\sqrt{17}}{2} \]

\[ x = -\frac{9}{2} \pm \frac{\sqrt{17}}{2} \]
The two solutions to this equation are then: 
\[ x = -\frac{9}{2} - \frac{\sqrt{17}}{2} \quad \text{and} \quad x = -\frac{9}{2} + \frac{\sqrt{17}}{2}. \]

Often we will get “messy” answers when using completing the square to solve equations. This is not something to get too excited about as many applications that involve solving quadratic equations have this kind of solution and so it is something that we just need to be able to deal with.

7. Solve the following quadratic equation by completing the square.

\[ 4u^2 - 8u + 5 = 0 \]

Step 1
First, let’s get the equation put into the form where all the variables are on one side, with a coefficient of one on the \( u^2 \), and the number is on the other side. Doing this gives,

\[ u^2 - 2u + \frac{5}{4} = 0 \]

\[ u^2 - 2u = -\frac{5}{4} \]

Step 2
We can now complete the square on the expression on the left side of the equation.

The number that we’ll need to do this is,

\[ \left(\frac{-2}{2}\right)^2 = \left(-1\right)^2 = 1 \]

Step 3
At this point we need to recall that we have an equation here and what we do to one side of the equation we also need to do the other. In other words, don’t forget to add the number from the previous step to both sides of the equation from Step 1.

\[ u^2 - 2u + 1 = -\frac{5}{4} + 1 \]

\[ (u-1)^2 = -\frac{1}{4} \]

Step 4
Now all we need to do to finish solving the equation is to use the Square Root Property on the equation from the previous step. Doing this gives,
\[ u - 1 = \pm \sqrt{-\frac{1}{4}} = \pm \frac{\sqrt{1}}{\sqrt{4}}i = \pm \frac{1}{2}i \]

\[ u = 1 \pm \frac{1}{2}i \]

The two solutions to this equation are then: \[ u = 1 - \frac{1}{2}i \text{ and } u = 1 + \frac{1}{2}i \].

8. Solve the following quadratic equation by completing the square.

\[ 2x^2 + 5x + 3 = 0 \]

Step 1
First, let’s get the equation put into the form where all the variables are on one side, with a coefficient of one on the \( x^2 \), and the number is on the other side. Doing this gives,

\[ x^2 + \frac{5}{2}x + \frac{3}{2} = 0 \]

\[ x^2 + \frac{5}{2}x = -\frac{3}{2} \]

Step 2
We can now complete the square on the expression on the left side of the equation.

The number that we’ll need to do this is,

\[ \left( \frac{5}{2} \right)^2 = \left( \frac{5}{4} \right)^2 = \frac{25}{16} \]

Step 3
At this point we need to recall that we have an equation here and what we do to one side of the equation we also need to do the other. In other words, don’t forget to add the number from the previous step to both sides of the equation from Step 1.

\[ x^2 + \frac{5}{2}x + \frac{25}{16} = -\frac{3}{2} + \frac{25}{16} \]

\[ \left( x + \frac{5}{4} \right)^2 = \frac{1}{16} \]

Step 4
Now all we need to do to finish solving the equation is to use the Square Root Property on the equation from the previous step. Doing this gives,

\[ x + \frac{5}{4} = \pm \sqrt{\frac{1}{16}} = \pm \frac{1}{4} \]

\[ x = -\frac{5}{4} \pm \frac{1}{4} \Rightarrow x = -\frac{5}{4} - \frac{1}{4}, \quad x = -\frac{5}{4} + \frac{1}{4} = -1 \]

The two solutions to this equation are then: \( x = -\frac{3}{2} \) and \( x = -1 \).

9. Use the quadratic formula to solve the following quadratic equation.

\[ x^2 - 6x + 4 = 0 \]

**Step 1**
There really isn’t too much to this problem. First we need to identify the values for the quadratic formula.

\[ a = 1, \quad b = -6, \quad c = 4 \]

**Step 2**
Plugging these into the quadratic formula gives,

\[ x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)} = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5} \]

The two solutions to this equation are then: \( x = 3 - \sqrt{5} \) and \( x = 3 + \sqrt{5} \).

10. Use the quadratic formula to solve the following quadratic equation.

\[ 9w^2 - 6w = 101 \]

**Step 1**
First we need to get the quadratic equation in standard form. This is,

\[ 9w^2 - 6w - 101 = 0 \]

**Step 2**
Now we need to identify the values for the quadratic formula.

\[ a = 9, \quad b = -6, \quad c = -101 \]
Step 3
Plugging these into the quadratic formula gives,

\[ w = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(-101)}}{2(9)} = \frac{6 \pm \sqrt{3672}}{18} = \frac{6 \pm \sqrt{(36)(102)}}{18} \]

\[= \frac{6 \pm 6\sqrt{102}}{18} = \frac{1 \pm \sqrt{102}}{3} \]

The two solutions to this equation are then:

\[ w = \frac{1}{3} - \frac{\sqrt{102}}{3} \text{ and } w = \frac{1}{3} + \frac{\sqrt{102}}{3} \]

11. Use the quadratic formula to solve the following quadratic equation.

\[ 8u^2 + 5u + 70 = 5 - 7u \]

Step 1
First we need to get the quadratic equation in standard form. This is,

\[ 8u^2 + 12u + 65 = 0 \]

Step 2
Now we need to identify the values for the quadratic formula.

\[ a = 8 \quad b = 12 \quad c = 65 \]

Step 3
Plugging these into the quadratic formula gives,

\[ u = \frac{-12 \pm \sqrt{(12)^2 - 4(8)(65)}}{2(8)} = \frac{-12 \pm \sqrt{-1936}}{16} = \frac{-12 \pm 44i}{16} = -\frac{3 \pm 11i}{4} \]

The two solutions to this equation are then:

\[ u = -\frac{3}{4} - \frac{11}{4}i \text{ and } u = -\frac{3}{4} + \frac{11}{4}i \]

12. Use the quadratic formula to solve the following quadratic equation.

\[ 169 - 20t + 4t^2 = 0 \]

Step 1
First we need to get the quadratic equation in standard form. This is,

$$4t^2 - 20t + 169 = 0$$

Note that in this case we just rearranged the terms to have decreasing exponents.

Step 2
Now we need to identify the values for the quadratic formula.

$$a = 4 \quad b = -20 \quad c = 169$$

Step 3
Plugging these into the quadratic formula gives,

$$t = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(4)(169)}}{2(4)} = \frac{20 \pm \sqrt{-2304}}{8} = \frac{20 \pm 48i}{8} = \frac{5 \pm 6i}{2}$$

The two solutions to this equation are then:

$$t = \frac{5}{2} - 6i \quad \text{and} \quad t = \frac{5}{2} + 6i$$

Problem 13.

Use the quadratic formula to solve the following quadratic equation.

$$2z^2 + z - 72 = z^2 - 2z + 58$$

Step 1
First we need to get the quadratic equation in standard form. This is,

$$z^2 + 3z - 130 = 0$$

Note that in this case we just rearranged the terms to have decreasing exponents.

Step 2
Now we need to identify the values for the quadratic formula.

$$a = 1 \quad b = 3 \quad c = -130$$

Step 3
Plugging these into the quadratic formula gives,

$$z = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-130)}}{2(1)} = \frac{-3 \pm \sqrt{529}}{2} = \frac{-3 \pm 23}{2}$$

In this case because there are no roots or complex numbers we can go further to reduce the solutions to “nicer” values.
The two solutions to this equation are then: \( z = -13 \) and \( z = 10 \).

As a final comment we can also note that because the solutions were integers we could have also gotten the answer by factoring! The quadratic (once written in standard form) factors as,

\[(z + 13)(z - 10) = 0\]

and this clearly would arrive at the same solutions.

The point of all this is to note that if more than one technique can be used it won’t matter which we use. Regardless of the solution technique used you will arrive at the same solutions.

---

**Solving Quadratic Equations : A Summary**

1. Use the discriminant to determine the type of roots for the following equation. Do not find any roots.

\[169x^2 - 182x + 49 = 0\]

**Step 1**

There really isn’t too much to this problem. First we need to identify the values for computing the discriminant.

\[a = 169 \quad b = -182 \quad c = 49\]

**Step 2**

Plugging these into the formula for the discriminant gives,

\[b^2 - 4ac = (-182)^2 - 4(169)(49) = 0\]

**Step 3**

The discriminant is zero and so we know that this equation will have a **double root**.

---

2. Use the discriminant to determine the type of roots for the following equation. Do not find any roots.

\[x^2 + 28x + 61 = 0\]

**Step 1**
There really isn’t too much to this problem. First we need to identify the values for computing the discriminant.

\[ a = 1 \quad b = 28 \quad c = 61 \]

Step 2
Plugging these into the formula for the discriminant gives,

\[ b^2 - 4ac = (28)^2 - 4(1)(61) = 540 \]

Step 3
The discriminant is positive and so we know that this equation will have two real roots.

---

3. Use the discriminant to determine the type of roots for the following equation. Do not find any roots.

\[ 49x^2 - 126x + 102 = 0 \]

Step 1
There really isn’t too much to this problem. First we need to identify the values for computing the discriminant.

\[ a = 49 \quad b = -126 \quad c = 102 \]

Step 2
Plugging these into the formula for the discriminant gives,

\[ b^2 - 4ac = (-126)^2 - 4(49)(102) = -4116 \]

Step 3
The discriminant is negative and so we know that this equation will have two complex roots.

---

4. Use the discriminant to determine the type of roots for the following equation. Do not find any roots.

\[ 9x^2 + 151 = 0 \]

Step 1
There really isn’t too much to this problem. First we need to identify the values for computing the discriminant.

\[ a = 9 \quad b = 0 \quad c = 151 \]

Step 2
Plugging these into the formula for the discriminant gives,
Step 3
The discriminant is negative and so we know that this equation will have two complex roots.

Application of Quadratic Equations

1. The width of a rectangle is 1 m less than twice the length. If the area of the rectangle is 100 m² what are the dimensions of the rectangle?

Step 1
We'll start by letting $L$ be the length of the rectangle. From the problem statement we now know that the width of the rectangle is 1 m less than twice the length and so must be $2L - 1$.

Step 2
We also know that the area of any rectangle is length times width and we are given that the area of this particular rectangle is 100. Therefore the equation for this problem is,

$$A = (\text{length})(\text{width})$$

$$100 = (L)(2L - 1)$$

$$100 = 2L^2 - L$$

Step 3
This is a quadratic equation and we know how to solve that so let's do that. First, we need to get the quadratic equation in standard form.

$$2L^2 - L - 100 = 0$$

We can now use the quadratic formula on this to get,

$$L = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-100)}}{2(2)} = \frac{1 \pm \sqrt{801}}{4}$$

Step 4
Reducing the two values we got in the previous steps to decimals we arrive at the following two solutions to the quadratic equation from Step 2.

$$L = \frac{1 - \sqrt{801}}{4} = -6.8255$$

$$L = \frac{1 + \sqrt{801}}{4} = 7.3255$$
We are dealing with a rectangle and so having a negative length doesn’t make much sense. Therefore the first solution to the quadratic equation can’t be the length of the rectangle.

This means that the length of the rectangle must be 7.3255 m and the width of the rectangle is then \(2(7.3255) - 1 = 13.651\text{ m}\).

2. Two cars start out at the same spot. One car starts to drive north at 40 mph and 3 hours later the second car starts driving to the east at 60 mph. How long after the first car starts driving does it take for the two cars to be 500 miles apart?

Step 1
Let’s start out this problem by defining Car A to be the car that drives 40 mph and Car B to be the car that drives 60 mph. Let’s also let \(t\) be the time that Car A is driving. From the problem statement we know that Car B starts 3 hours after Car A and so drives for 3 hours less than Car A. This means that \(t - 3\) is the time that Car B is driving.

Step 2
Next let’s set up a sketch for this situation.

Step 3
Okay. Now we need to get an equation for this situation. The first thing to notice about our sketch is that we have a right triangle! This means we can relate all three lengths using the Pythagorean Theorem (this is one of the reasons to have a sketch – to see these kinds of things).

The Pythagorean Theorem tells us that,

\[
\left(\text{Distance Car A drives}\right)^2 + \left(\text{Distance Car B drives}\right)^2 = (500)^2 = 250,000.
\]
Step 4
Next, we know that we can find the distance of each car using the formula,

\[ \text{Distance} = \text{(Speed of Car)} \times \text{(Time driving)} \]

So, for each car we have,

Distance of Car A = (40)(t) = 40t
Distance of Car B = (60)(t - 3) = 60(t - 3)

Putting all of this into the “word equation” we wrote down in Step 3 we get the following equation.

\[ (40t)^2 + (60(t - 3))^2 = 250,000 \]
\[ 40^2t^2 + 60^2(t - 3)^2 = 250,000 \]
\[ 1600t^2 + 3600(t^2 - 6t + 9) = 250,000 \]
\[ 1600t^2 + 3600t^2 - 21,600t + 32,400 = 250,000 \]
\[ 5200t^2 - 21,600t - 217,600 = 0 \]

Note as well that we did quite a bit of simplification to get the equation into a standard form. Also, do not get excited about the “large” numbers here! They happen on occasion so they are nothing to worry about. This is still just a quadratic and we know how to solve quadratic equations. It doesn’t matter if the numbers are single digit numbers or significantly larger numbers as they are here.

Step 5
As noted in the previous step this is just a quadratic equation and we know how to solve those! Using the quadratic formula gives,

\[ t = \frac{-(-21,600) \pm \sqrt{(-21,600)^2 - 4(5200)(-217,600)}}{2(5200)} = \frac{21,600 \pm \sqrt{4,992,640,000}}{10,400} \]

Step 6
Reducing the two values we got in the previous steps to decimals we arrive at the following two solutions to the quadratic equation from Step 4.

\[ t = \frac{21,600 - \sqrt{4,992,640,000}}{10,400} = -4.7172 \quad t = \frac{21,600 + \sqrt{4,992,640,000}}{10,400} = 8.8710 \]

The first solution to the equation doesn’t make any sense since it is negative (we are working with time and so it’s safe to assume we are starting at \( t = 0 \) after all!) so that means the second is the answer we need.
This means that Car A (i.e. the one traveling at 40 mph) travels for 8.871 hours while Car B (i.e. the one traveling at 60 mph) travels for 5.871 hours (three hours less than Car A time!).

3. Two people can paint a house in 14 hours. Working individually one of the people takes 2 hours more than it takes the other person to paint the house. How long would it take each person working individually to paint the house?

Step 1
First, let Person A be the faster of the two painters and let \( t \) be the amount of time it takes to paint the house by himself. Next, let Person B be the slower of the two painters and so it will take this person \( t + 2 \) hours to paint the house by himself.

Step 2
Working together they can paint the house in 14 hours so we have the following word equation for them working together to paint the house.

\[
\left( \text{Portion of job done by Person A} \right) + \left( \text{Portion of job done by Person B} \right) = 1 \text{ Job}
\]

We know that Portion of Job = Work Rate X Work Time so this gives the following word equation.

\[
\left( \frac{\text{Work Rate of Person A}}{\text{Work Time of Person A}} \right) + \left( \frac{\text{Work Rate of Person B}}{\text{Work Time of Person B}} \right) = 1
\]

\[
\left( \frac{\text{Work Rate of Person A}}{\text{Work Time of Person A}} \right) (14) + \left( \frac{\text{Work Rate of Person B}}{\text{Work Time of Person B}} \right) (14) = 1
\]

Step 3
Now we need the work rate of each person which we can get from their individual painting times as follows,

\[
\left( \frac{\text{Work Rate of Person A}}{\text{Work Time of Person A}} \right) = \left( \frac{\text{Work Rate of Person A}}{t} \right) = 1
\]

\[
\Rightarrow \text{ Work Rate of Person A} = \frac{1}{t}
\]

\[
\left( \frac{\text{Work Rate of Person B}}{\text{Work Time of Person B}} \right) = \left( \frac{\text{Work Rate of Person B}}{t + 2} \right) = 1
\]

\[
\Rightarrow \text{ Work Rate of Person B} = \frac{1}{t + 2}
\]

Step 4
Plugging these into the word equation from Step 2 we arrive at the following equation.
To solve this we know that we’ll need to multiply by the LCD, \( t(t + 2) \) in this case, to clear the denominators. Doing this gives,

\[
14(t + 2) + 14t = t(t + 2)
\]

\[
28t + 28 = t^2 + 2t
\]

\[
t^2 - 26t - 28 = 0
\]

After some simplification we arrive a fairly simple quadratic equation to solve. Using the quadratic formula gives,

\[
L = \frac{-(-26) \pm \sqrt{(-26)^2 - 4(1)(-28)}}{2(1)} = \frac{26 \pm \sqrt{788}}{2}
\]

Reducing the two values we got in the previous steps to decimals we arrive at the following two solutions to the quadratic equation from Step 2.

\[
t = \frac{26 - \sqrt{788}}{2} = -1.0357 \quad t = \frac{26 + \sqrt{788}}{2} = 27.0357
\]

The first solution to the equation doesn’t make any sense since it is negative (we are working with time and so it’s safe to assume we are starting at \( t = 0 \) after all!) so that means the second is the answer we need.

This means that Person A can paint the house in 27.0357 hours while Person B can paint the house in 29.0357 hours (two hours more than Person A).

---

**Equations Reducible to Quadratic Form**

1. Solve the following equation.

\[x^6 - 9x^3 + 8 = 0\]
Hint: Remember to look at the exponents of the first two terms and try to find a substitution that will turn this into a “normal” quadratic equation.

Step 1
First let’s notice that \( 6 = 2 \cdot 3 \) and so we can use the following substitution to reduce the equation to a quadratic equation.

\[ u = x^3 \quad u^2 = (x^3)^2 = x^6 \]

Step 2
Using this substitution the equation becomes,

\[ u^2 - 9u + 8 = 0 \]
\[ (u - 1)(u - 8) = 0 \]

We can easily see that the solution to this equation is: \( u = 1 \) and \( u = 8 \).

Step 3
Now all we need to do is use our substitution from the first step to determine the solution to the original equation.

\[ u = 1: \quad x^3 = 1 \quad \Rightarrow \quad x = (1)^{\frac{1}{3}} = 1 \]
\[ u = 8: \quad x^3 = 8 \quad \Rightarrow \quad x = (8)^{\frac{1}{3}} = 2 \]

Therefore the two solutions to the original equation are: \( x = 1 \) and \( x = 2 \).

2. Solve the following equation.

\[ x^4 - 7x^2 - 18 = 0 \]

Hint: Remember to look at the exponents of the first two terms and try to find a substitution that will turn this into a “normal” quadratic equation.

Step 1
First let’s notice that \( -4 = 2 \cdot (-2) \) and so we can use the following substitution to reduce the equation to a quadratic equation.

\[ u = x^{-2} \quad u^2 = (x^{-2})^2 = x^{-4} \]

Step 2
Using this substitution the equation becomes,
\( \begin{align*}
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u^2 - 7u - 18 = 0 \\
(u - 9)(u + 2) = 0
\end{align*} \)

We can easily see that the solution to this equation is \( u = -2 \) and \( u = 9 \).

Step 3
Now all we need to do is use our substitution from the first step to determine the solution to the original equation.

\[
\begin{align*}
\text{If } u = -2: \quad & x^2 = \frac{1}{x^2} = -2 \quad \Rightarrow \quad x^2 = \frac{1}{2} \quad \Rightarrow \quad x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}i \\
\text{If } u = 9: \quad & x^2 = \frac{1}{x^2} = 9 \quad \Rightarrow \quad x^2 = \frac{1}{9} \quad \Rightarrow \quad x = \pm \sqrt{\frac{1}{9}} = \pm \frac{1}{3}
\end{align*}
\]

Therefore the four solutions to the original equation are: \( x = \pm \frac{1}{\sqrt{2}}i \) and \( x = \pm \frac{1}{3} \).

3. Solve the following equation.

\[
4x^\frac{2}{3} + 21x^\frac{1}{3} + 27 = 0
\]

Hint: Remember to look at the exponents of the first two terms and try to find a substitution that will turn this into a “normal” quadratic equation.

Step 1
First let’s notice that \( \frac{2}{3} = 2 \left( \frac{1}{3} \right) \) and so we can use the following substitution to reduce the equation to a quadratic equation.

\[
\begin{align*}
u = x^\frac{1}{3} \quad \Rightarrow \quad u^2 = \left( x^\frac{1}{3} \right)^2 = x^\frac{2}{3}
\end{align*}
\]

Step 2
Using this substitution the equation becomes,

\[
\begin{align*}
4u^2 + 21u + 27 = 0 \\
(4u + 9)(u + 3) = 0
\end{align*}
\]

We can easily see that the solution to this equation is: \( u = -\frac{9}{4} \) and \( u = -3 \).

Step 3
Now all we need to do is use our substitution from the first step to determine the solution to the original equation.
Therefore the two solutions to the original equation are: $x = \frac{-729}{64}$ and $x = -27$.

4. Solve the following equation.

$$x^8 - 6x^4 + 7 = 0$$

Hint: Remember to look at the exponents of the first two terms and try to find a substitution that will turn this into a “normal” quadratic equation.

Step 1
First let’s notice that $8 = 2(4)$ and so we can use the following substitution to reduce the equation to a quadratic equation.

$$u = x^4 \quad u^2 = (x^4)^2 = x^8$$

Step 2
Using this substitution the equation becomes,

$$u^2 - 6u + 7 = 0$$

Now, this equation does not factor. That happens on occasion but luckily enough we know how to solve it anyway. All we need to do is use the quadratic formula to find the solutions to this equation.

$$u = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)} = \frac{6 \pm \sqrt{36 - 28}}{2} = \frac{6 \pm 2\sqrt{2}}{2} = 3 \pm \sqrt{2}$$

Do not get excited about the “messy” numbers here! These kinds of solutions happen on occasion and we just need to be able to deal with them. Just keep in mind that they are just numbers even if they are not the integers we are used to seeing!

Step 3
Now all we need to do is use our substitution from the first step to determine the solution to the original equation.

$$u = 3 + \sqrt{2} : \quad x^4 = 3 + \sqrt{2} \quad \Rightarrow \quad x = (3 + \sqrt{2})^{\frac{1}{4}} = (4.4142)^{\frac{1}{4}} = 1.4495$$
\[ u = 3 - \sqrt{2} : \quad x^4 = 3 - \sqrt{2} \Rightarrow x = \left(3 - \sqrt{2}\right)^{\frac{1}{4}} = (1.5858)^{\frac{1}{4}} = 1.1222 \]

Therefore the two solutions to the original equation are: \( x = 1.1222 \) and \( x = 1.4495 \).

5. Solve the following equation.

\[
\frac{2}{x^2} + \frac{17}{x} + 21 = 0
\]

Hint: This works exactly the same as the first four problems even though the \( x \)'s are in the denominator. The only difference here is that the \( x \)'s will be in the denominator of our substitution.

Step 1
First let’s notice that \( 2 = 2(1) \) and so we can use the following substitution to reduce the equation to a quadratic equation.

\[
u = \frac{1}{x} \quad u^2 = \left(\frac{1}{x}\right)^2 = \frac{1}{x^2} = \frac{1}{x^2}
\]

Step 2
Using this substitution the equation becomes,

\[
2u^2 + 17u + 21 = 0
\]

\[
(2u + 3)(u + 7) = 0
\]

We can easily see that the solution to this equation is: \( u = -\frac{3}{2} \) and \( u = -7 \).

Step 3
Now all we need to do is use our substitution from the first step to determine the solution to the original equation.

\[
u = -\frac{3}{2} : \quad \frac{1}{x} = -\frac{3}{2} \Rightarrow x = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}
\]

\[
u = -7 : \quad \frac{1}{x} = -7 \Rightarrow x = \frac{1}{-7} = -\frac{1}{7}
\]

Therefore the two solutions to the original equation are: \( x = -\frac{2}{3} \) and \( x = -\frac{1}{7} \).

6. Solve the following equation.
$$\frac{1}{x} - \frac{11}{\sqrt{x}} + 18 = 0$$

Hint: This works exactly the same as the first four problems even though the x’s are in the denominator. The only difference here is that the x’s will be in the denominator of our substitution.

Step 1
First let’s notice that \(1 = 2\left(\frac{1}{x}\right)\) and recall that \(\sqrt{x} = x^{\frac{1}{2}}\). So we can use the following substitution to reduce the equation to a quadratic equation.

\[
\frac{1}{\sqrt{x}} \quad u = \frac{1}{\sqrt{x}} \quad u^2 = \left(\frac{1}{\sqrt{x}}\right)^2 = \left(\frac{1}{x^{\frac{1}{2}}}\right)^2 = \frac{1}{x}
\]

Step 2
Using this substitution the equation becomes,

\[
u^2 - 11u + 18 = 0 \quad (u - 2)(u - 9) = 0
\]

We can easily see that the solution to this equation is: \(u = 2\) and \(u = 9\).

Step 3
Now all we need to do is use our substitution from the first step to determine the solution to the original equation.

\[
u = 2: \quad \frac{1}{\sqrt{x}} = 2 \quad \Rightarrow \quad \sqrt{x} = \frac{1}{2} \quad \Rightarrow \quad x = \left(\frac{1}{2}\right)^2 = \frac{1}{4}
\]

\[
u = 9: \quad \frac{1}{\sqrt{x}} = 9 \quad \Rightarrow \quad \sqrt{x} = \frac{1}{9} \quad \Rightarrow \quad x = \left(\frac{1}{9}\right)^2 = \frac{1}{81}
\]

Therefore the two solutions to the original equation are: \(x = \frac{1}{4}\) and \(x = \frac{1}{81}\).

---

**Equations with Radicals**

1. Solve the following equation.

\[2x = \sqrt{x} + 3\]
Step 1
The first step here is to square both sides to get,

\[ (2x)^2 = \left(\sqrt{x + 3}\right)^2 \]

\[ 4x^2 = x + 3 \]

\[ 4x^2 - x - 3 = 0 \]

Step 2
This is just a quadratic equation and we know how to solve it so let’s do that.

\[ (4x + 3)(x - 1) = 0 \quad \Rightarrow \quad x = -\frac{3}{4}, \quad x = 1 \]

As shown we have two solutions to the quadratic we got from the first step.

Hint: Recall that the solution process used here can, and often does, introduce values that are not in fact solutions to the original equation!

Step 3
We’re not done with this problem. Recall from the notes that the solution process we used here has the unfortunate side effect of sometimes introducing values that are not solutions to the original equation.

So, to finish this out we need to check both of the potential solutions from the previous step in the original equation (recall it’s important to check the potential solutions in the original equation).

\[ x = -\frac{3}{4} : \quad 2\left(-\frac{3}{4}\right) = \sqrt{-\frac{3}{4} + 3} \quad \Rightarrow \quad -\frac{3}{2} = \sqrt{\frac{9}{4}} \quad \Rightarrow \quad -\frac{3}{2} \neq \frac{3}{2} \quad \text{NOT OK} \]

\[ x = 1 : \quad 2(1) = \sqrt{1 + 3} \quad \Rightarrow \quad 2 = \sqrt{4} \quad \Rightarrow \quad 2 = 2 \quad \text{OK} \]

Only one of the potential solutions work out and so the original equation has a single solution: \[ x = 1 \].

2. Solve the following equation.

\[ \sqrt{33 - 2x} = x + 1 \]

Step 1
The first step here is to square both sides to get,

\[ \left(\sqrt{33 - 2x}\right)^2 = (x + 1)^2 \]

\[ 33 - 2x = x^2 + 2x + 1 \]

\[ x^2 + 4x - 32 = 0 \]

Step 2
This is just a quadratic equation and we know how to solve it so let’s do that.

\[(x + 8)(x - 4) = 0 \Rightarrow x = -8, \ x = 4\]

As shown we have two solutions to the quadratic we got from the first step.

Hint : Recall that the solution process used here can, and often does, introduce values that are not in fact solutions to the original equation!

Step 3
We’re not done with this problem. Recall from the notes that the solution process we used here has the unfortunate side effect of sometimes introducing values that are not solutions to the original equation.

So, to finish this out we need to check both of the potential solutions from the previous step in the original equation (recall it’s important to check the potential solutions in the original equation).

\[x = -8:\  \sqrt{33 - 2(-8)} = -8 + 1 \rightarrow \sqrt{49} = 7 \rightarrow 7 \neq -7 \quad \text{NOT OK}\]

\[x = 4:\  \sqrt{33 - 2(4)} = 4 + 1 \rightarrow \sqrt{25} = 5 \rightarrow 5 = 5 \quad \text{OK}\]

Only one of the potential solutions work out and so the original equation has a single solution : \[x = 4\].

3. Solve the following equation.

\[7 = \sqrt{39 + 3x} - x\]

Step 1
The first step here is to square both sides. However, before we do that we need to get the root on one side by itself.

\[7 + x = \sqrt{39 + 3x}\]

Now we can square both sides to get,

\[(7 + x)^2 = (\sqrt{39 + 3x})^2\]

\[x^2 + 14x + 49 = 39 + 3x\]

\[x^2 + 11x + 10 = 0\]

Step 2
This is just a quadratic equation and we know how to solve it so let’s do that.

\[(x + 10)(x + 1) = 0 \Rightarrow x = -10, \ x = -1\]

As shown we have two solutions to the quadratic we got from the first step.
Hint: Recall that the solution process used here can, and often does, introduce values that are not in fact solutions to the original equation!

Step 3
We’re not done with this problem. Recall from the notes that the solution process we used here has the unfortunate side effect of sometimes introducing values that are not solutions to the original equation.

So, to finish this out we need to check both of the potential solutions from the previous step in the original equation (recall it’s important to check the potential solutions in the original equation).

\[
x = -10: \quad 7 = \sqrt{39 + 3(-10) - (-10)} \quad \rightarrow \quad 7 = \sqrt{9 + 10} \quad \rightarrow \quad 7 \neq 13 \quad \text{NOT OK}
\]

\[
x = -1: \quad 7 = \sqrt{39 + 3(-1) - (-1)} \quad \rightarrow \quad 7 = \sqrt{36 + 1} \quad \rightarrow \quad 7 = 7 \quad \text{OK}
\]

Only one of the potential solutions work out and so the original equation has a single solution: \( x = -1 \).

---

4. Solve the following equation.

\[x = 1 + \sqrt{2x - 2}\]

Step 1
The first step here is to square both sides. However, before we do that we need to get the root on one side by itself.

\[x - 1 = \sqrt{2x - 2}\]

Now we can square both sides to get,

\[(x - 1)^2 = \left(\sqrt{2x - 2}\right)^2\]

\[x^2 - 2x + 1 = 2x - 2\]

\[x^2 - 4x + 3 = 0\]

Step 2
This is just a quadratic equation and we know how to solve it so let’s do that.

\[(x - 1)(x - 3) = 0 \quad \Rightarrow \quad x = 1, \quad x = 3\]

As shown we have two solutions to the quadratic we got from the first step.

Hint: Recall that the solution process used here can, and often does, introduce values that are not in fact solutions to the original equation!
We’re not done with this problem. Recall from the notes that the solution process we used here has the unfortunate side effect of sometimes introducing values that are not solutions to the original equation.

So, to finish this out we need to check both of the potential solutions from the previous step in the original equation (recall it’s important to check the potential solutions in the original equation).

\[ x = 1: \quad 1 = 1 + \sqrt{2(1) - 2} \quad \Rightarrow \quad 1 = 1 + 0 \quad \Rightarrow \quad 1 = 1 \quad \text{OK} \]

\[ x = 3: \quad 3 = 1 + \sqrt{2(3) - 2} \quad \Rightarrow \quad 3 = 1 + 4 \quad \Rightarrow \quad 3 = 3 \quad \text{OK} \]

Both of the potential solutions work out and so the original equation has a two solutions: \( x = 1 \) and \( x = 3 \).

---

5. Solve the following equation.

\[ 1 + \sqrt{1 - x} = \sqrt{2x + 4} \]

Step 1

The first step here is to square both sides to get,

\[ (1 + \sqrt{1 - x})^2 = (\sqrt{2x + 4})^2 \]

\[ 1 + 2\sqrt{1 - x} + (\sqrt{1 - x})^2 = 2x + 4 \]

\[ 1 + 2\sqrt{1 - x} + 1 - x = 2x + 4 \]

\[ 2\sqrt{1 - x} = 3x + 2 \]

Step 2

Unlike the previous problems squaring both sides once isn’t sufficient to eliminate the square roots from the problem. So, once we get the remaining root on one side by itself, as we did in the previous step, we need to square both sides once again.

Doing that gives,

\[ (2\sqrt{1 - x})^2 = (3x + 2)^2 \]

\[ 4(1 - x) = 9x^2 + 12x + 4 \]

\[ 9x^2 + 16x = 0 \]

Step 3

This is just a quadratic equation and we know how to solve it so let’s do that.

\[ x(9x + 16) = 0 \quad \Rightarrow \quad x = 0, \quad x = -\frac{16}{9} \]
As shown we have two solutions to the quadratic we got from the first step.

Hint : Recall that the solution process used here can, and often does, introduce values that are not in fact solutions to the original equation!

Step 4
We’re not done with this problem. Recall from the notes that the solution process we used here has the unfortunate side effect of sometimes introducing values that are not solutions to the original equation.

So, to finish this out we need to check both of the potential solutions from the previous step in the original equation (recall it’s important to check the potential solutions in the original equation).

\[
\begin{align*}
\text{x = 0 : } & \quad 1 + \sqrt{1-0} = \sqrt{2(0)+4} \quad \rightarrow \quad 1 + \sqrt{1} = \sqrt{4} \quad \rightarrow \quad 2 = 2 \quad \text{OK} \\
\text{x = } & -\frac{16}{9} : \quad 1 + \sqrt{1-\left(-\frac{16}{9}\right)} = \sqrt{2\left(-\frac{16}{9}\right)+4} \quad \rightarrow \quad 1 + \frac{25}{9} = \frac{\sqrt{1}}{\sqrt{9}} \quad \rightarrow \quad \frac{8}{3} \neq \frac{2}{3} \quad \text{NOT OK}
\end{align*}
\]

Only one of the potential solutions work out and so the original equation has a single solution : \( x = 0 \).

---

**Linear Inequalities**

1. Solve the following inequality and give the solution in both inequality and interval notation.

\[
4(z + 2) - 1 > 5 - 7(4 - z)
\]

Hint : Remember that solving linear inequalities is pretty much the same as solving a linear equation. Just remember to be careful when multiplying/dividing by a negative number.

Step 1
We know that the process of solving a linear inequality is pretty much the same process as solving a linear equation. We do basic algebraic manipulations to get all the \( z \)’s on one side of the inequality and the numbers on the other side. Just remember that what you do to one side of the inequality you have to do to the other side as well. So, let’s go through the solution process for this linear inequality.

First, we should clear out the parenthesis on both sides and do any simplification that we can. Doing this gives,

\[
4z + 8 - 1 > 5 - 28 + 7z
\]

\[
4z + 7 > -23 + 7z
\]

Step 2
We can now subtract \( 7z \) from both sides and subtract 7 to both sides to get,
Note that we could just have easily subtracted 4\(z\) from both sides and added 23 to both sides. Each will get the same result in the end.

Step 3
For the final step we need to divide both sides by -3. Recall however that because we are dividing by a negative number we need to switch the direction of the inequality to get,

\[ z < 10 \]

So, the inequality form of the solution is \( z < 10 \) and the interval notation form of the solution is \((-\infty, 10]\).

Remember that we use a parenthesis, \( i.e. \) \(^{\text{“)}}\), for the right side of the interval notation because we are not including 10 in the solution. Also recall that infinities always get parenthesis!

2. Solve the following inequality and give the solution in both inequality and interval notation.

\[
\frac{1}{2}(3 + 4t) \leq 6\left(\frac{1}{3} - \frac{1}{2}t\right) - \frac{1}{4}(2 + 10t)
\]

Hint: Remember that solving linear inequalities is pretty much the same as solving a linear equation. Just remember to be careful when multiplying/dividing by a negative number.

Step 1
We know that the process of solving a linear inequality is pretty much the same process as solving a linear equation. We do basic algebraic manipulations to get all the \(t\)'s on one side of the inequality and the numbers on the other side. Just remember that what you do to one side of the inequality you have to do to the other side as well. So, let's go through the solution process for this linear inequality.

First, we should clear out the parenthesis on both sides and do any simplification that we can. Doing this gives,

\[
\frac{3}{2} + 2t \leq 2 - 3t - \frac{5}{2}t
\]

\[
\frac{3}{2} + 2t \leq \frac{3}{2} - \frac{11}{2}t
\]

Step 2
We can now add \(\frac{1}{2}t\) to both sides and subtract \(\frac{3}{2}\) from both sides to get,

\[
\frac{15}{2}t \leq 0
\]
Step 3
For the final step we need to multiply both sides by \( \frac{2}{15} \) to get,

\[
t \leq 0
\]

So, the inequality form of the solution is \([t \leq 0]\) and the interval notation form of the solution is \((\infty, 0]\).

Remember that we use a square bracket, i.e. “[“, for the left portion of the interval because we are including zero in the solution. Also recall that infinities never get square brackets!

3. Solve the following inequality and give the solution in both inequality and interval notation.

\[-1 < 4x + 2 < 10\]

Hint: Solving double inequalities uses the same basic process as solving single inequalities. Just remember that what you do to one part you have to do to all parts of the inequality.

Step 1
Just like with single inequalities solving these follow pretty much the same process as solving a linear equation. The only difference between this and a single inequality is that we now have three parts of the inequality and so we just need to remember that what we do to one part we need to do to all parts.

Also, recall that the main goal is to get the variable all by itself in the middle and all the numbers on the two outer parts of the inequality.

So, let’s start by subtracting 2 from all the parts. This gives,

\[-3 < 4x < 8\]

Step 2
Finally, all we need to do is divide all three parts by 4 to get,

\[-\frac{3}{4} < x < 2\]

So, the inequality form of the solution is \([ -\frac{3}{4}, 2]\) and the interval notation form of the solution is \(( -\frac{3}{4}, 2]\).

4. Solve the following inequality and give the solution in both inequality and interval notation.

\[8 \leq 3 - 5z < 12\]
Hint: Solving double inequalities uses the same basic process as solving single inequalities. Just remember that what you do to one part you have to do to all parts of the inequality.

Step 1
Just like with single inequalities solving these follow pretty much the same process as solving a linear equation. The only difference between this and a single inequality is that we now have three parts of the inequality and so we just need to remember that what we do to one part we need to do to all parts.

Also, recall that the main goal is to get the variable all by itself in the middle and all the numbers on the two outer parts of the inequality.

So, let’s start by subtracting 3 from all the parts. This gives,

\[ 5 \leq -5z < 9 \]

Step 2
Finally, all we need to do is divide all three parts by -5 to get,

\[ -1 \geq z > -\frac{9}{5} \]

Don’t forget that because we were dividing everything by a negative number we needed to switch the direction of the inequalities.

So, the inequality form of the solution is \[ -\frac{9}{5} < z \leq -1 \] (we flipped the inequality around to get the smaller number on the left as that is a more “standard” form). The interval notation form of the solution is \( (-\frac{9}{5}, -1] \).

For the interval notation form remember that the smaller number is always on the left (hence the reason for flipping the inequality form above!) and be careful with parenthesis and square brackets. We use parenthesis if we don’t include the number and square brackets if we do include the number.

5. Solve the following inequality and give the solution in both inequality and interval notation.

\[ 0 \leq 10w - 15 \leq 23 \]

Hint: Solving double inequalities uses the same basic process as solving single inequalities. Just remember that what you do to one part you have to do to all parts of the inequality.

Step 1
Just like with single inequalities solving these follow pretty much the same process as solving a linear equation. The only difference between this and a single inequality is that we now have three parts of the inequality and so we just need to remember that what we do to one part we need to do to all parts.

Also, recall that the main goal is to get the variable all by itself in the middle and all the numbers on the two outer parts of the inequality.
So, let’s start by add 15 to all the parts. This gives, 

\[ 15 \leq 10w \leq 38 \]

Step 2  
Finally, all we need to do is divide all three parts by 10 to get, 

\[ \frac{3}{2} \leq w \leq \frac{19}{5} \]

So, the inequality form of the solution is \( \frac{3}{2} \leq w \leq \frac{19}{5} \) and the interval notation form of the solution is \( \left[ \frac{3}{2}, \frac{19}{5} \right] \).

6. Solve the following inequality and give the solution in both inequality and interval notation.  
\[ 2 < \frac{1}{6} - \frac{1}{2} x \leq 4 \]

Hint: Solving double inequalities uses the same basic process as solving single inequalities. Just remember that what you do to one part you have to do to all parts of the inequality.

Step 1  
Just like with single inequalities solving these follow pretty much the same process as solving a linear equation. The only difference between this and a single inequality is that we now have three parts of the inequality and so we just need to remember that what we do to one part we need to do to all parts.

Also, recall that the main goal is to get the variable all by itself in the middle and all the numbers on the two outer parts of the inequality.

So, let’s start by subtracting \( \frac{1}{6} \) from all the parts. This gives, 

\[ \frac{11}{6} < -\frac{1}{2} x \leq \frac{23}{6} \]

Step 2  
Finally, all we need to do is multiply all three parts by -2 to get, 

\[ -\frac{11}{3} > x \geq -\frac{23}{3} \]

Don’t forget that because we were multiplying everything by a negative number we needed to switch the direction of the inequalities.
So, the inequality form of the solution is \(- \frac{23}{3} \leq x < - \frac{11}{3}\) (we flipped the inequality around to get the smaller number on the left as that is a more “standard” form). The interval notation form of the solution is \([- \frac{23}{3}, - \frac{11}{3})\).

For the interval notation form remember that the smaller number is always on the left (hence the reason for flipping the inequality form above!) and be careful with parenthesis and square brackets. We use parenthesis if we don’t include the number and square brackets if we do include the number.

7. If \(0 \leq x < 3\) determine \(a\) and \(b\) for the inequality: \(a \leq 4x + 1 < b\)

Hint: Can you make the middle part of the first inequality look like the middle part of the second inequality?

Step 1
This problem is really the reverse of the previous problems in this section. In the previous problems we started with something like the second inequality (of course we also had numbers in the two outer portions instead of \(a\) and \(b\)) and we had to manipulate it to get the \(x\) by itself in the middle.

The process here is basically the same just in reverse. We need to do algebraic manipulations to make the middle part of the first inequality look like the middle part of the second manipulation. The only real difference is that with the solving problems we added/subtracted the number before we dealt with the coefficient of the \(x\). Here we need to get the coefficient on the \(x\) before we get the number.

So, the first thing we’ll do is multiply all three parts of the first inequality by 4. This gives,

\[0(4) \leq 4x < 3(4) \Rightarrow 0 \leq 4x < 12\]

Step 2
Now all we need to do is add one to all three parts.

\[1 \leq 4x + 1 < 13\]

Step 3
Comparing this inequality in the second step to the second inequality in the problem statement we can see that we must have \(a = 1\) and \(b = 13\).

Polynomial Inequalities
1. Solve the following inequality.

\[ u^2 + 4u \geq 21 \]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the polynomial.

\[
\begin{align*}
(u + 7)(u - 3) & \geq 0 \\
\end{align*}
\]

Hint: Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored form we can quickly see that the polynomial will be zero at,

\[ u = -7 \quad \text{and} \quad u = 3 \]

Remember that these points are important because they are the only places where the polynomial on the left side of the inequality might change sign. Given that we want to know where the polynomial is zero (which we now know) or positive knowing where the polynomial might change sign will help considerably with determining the answer we’re looking for.

Hint: Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[
\begin{align*}
\text{at} & \quad u = -8 \quad | \quad u = 0 \quad | \quad u = 4 \\
\text{compute} & \quad (-1)(-11) > 0 \quad | \quad (7)(-3) < 0 \quad | \quad (11)(1) > 0 \\
\text{at} & \quad u = -8 \quad | \quad u = 0 \quad | \quad u = 4 \\
\end{align*}
\]

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.
2. Solve the following inequality.

\[ x^2 + 8x + 12 < 0 \]

Step 1
The first thing we need to do is get a zero on one side of the inequality (which is already done for this problem) and then, if possible, factor the polynomial.

\[ (x + 6)(x + 2) < 0 \]

Hint: Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored form we can quickly see that the polynomial will be zero at,

\[ x = -6 \quad \text{ and } \quad x = -2 \]

Remember that these points are important because they are the only places where the polynomial on the left side of the inequality might change sign. Given that we want to know where the polynomial is negative knowing where the polynomial might change sign will help considerably with determining the answer we’re looking for.

Hint: Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.
Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[-6 < x < -2\]

\[(-6, -2)\]

3. Solve the following inequality.

\[4t^2 \leq 15 - 17t\]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the polynomial.

\[4t^2 + 17t - 15 \leq 0\]

\[(t + 5)(4t - 3) \leq 0\]

Hint : Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored form we can quickly see that the polynomial will be zero at,

\[t = -5\] \[t = \frac{3}{4}\]

Remember that these points are important because they are the only places where the polynomial on the left side of the inequality might change sign. Given that we want to know where the polynomial is zero (which we now know) or negative knowing where the polynomial might change sign will help considerably with determining the answer we’re looking for.

Hint : Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.
Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the
inequality and interval notation from of the answer.

\[
\begin{align*}
-5 & \leq t \leq \frac{3}{4} \\
\left[-5, \frac{3}{4}\right] &
\end{align*}
\]

4. Solve the following inequality.

\[z^2 + 34 > 12z\]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the
polynomial.

\[z^2 - 12z + 34 > 0\]

In this case the polynomial doesn’t factor.

Hint : Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. Because the
polynomial didn’t factor we’ll need to use the quadratic formula to determine where it’s zero.

\[
z = \frac{12 \pm \sqrt{144 - 4(1)(34)}}{2(1)} = \frac{12 \pm \sqrt{8}}{2} \implies z = 4.5858, 7.4142
\]

We’ll need these points in decimal form to make the rest of the problem easier.

Remember that these points are important because they are the only places where the polynomial on the
left side of the inequality might change sign. Given that we want to know where the polynomial is
positive knowing where the polynomial might change sign will help considerably with determining the
answer we’re looking for.
Hint: Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[
\begin{align*}
\quad &\quad 4^2 - 12(4) + 34 > 0 \\
\quad &\quad 6^2 - 12(6) + 34 < 0 \\
\quad &\quad 8^2 - 12(8) + 34 > 0
\end{align*}
\]

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[
\begin{align*}
z < 4.5858 & \quad \text{and} \quad z > 7.4142 \\
(\infty, 4.5858) & \quad \text{and} \quad (7.4142, \infty)
\end{align*}
\]

5. Solve the following inequality.

\[y^2 - 2y + 1 \leq 0\]

Step 1
The first thing we need to do is get a zero on one side of the inequality (which is already done for this problem) and then, if possible, factor the polynomial.

\[(y - 1)^2 \leq 0\]

Hint: Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored form we can quickly see that the polynomial will be zero at,

\[y = 1\]
Hint: Is it possible for the polynomial to ever be negative?

Step 3
This problem works a little differently than the others in this section. Because the polynomial is a perfect square we know that it can never be negative! It is only possible for it to be zero or positive.

We are being asked to determine where the polynomial is negative or zero. As noted however it isn’t possible for it to be negative. Therefore the only solution we can get for this inequality is where it is zero and we found that in the previous step.

The answer is then,

\[ y = 1 \]

In this case the answer is a single number and not an inequality. This happens on occasion and we shouldn’t worry about these kinds of “unusual” answers.

6. Solve the following inequality.

\[ t^4 + t^3 - 12t^2 < 0 \]

Step 1
The first thing we need to do is get a zero on one side of the inequality (which is already done for this problem) and then, if possible, factor the polynomial.

\[ t^2 (t^2 + t - 12) < 0 \]
\[ t^2 (t + 4)(t - 3) < 0 \]

Hint: Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored from we can quickly see that the polynomial will be zero at,

\[ t = -4 \quad t = 0 \quad t = 3 \]

Remember that these points are important because they are the only places where the polynomial on the left side of the inequality might change sign. Given that we want to know where the polynomial is negative knowing where the polynomial might change sign will help considerably with determining the answer we’re looking for.

Hint: Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[
\begin{align*}
\text{Be careful with the first term in the factored form when plugging in the test points! It is squared and so will always be positive regardless of the sign of the test points. One of the bigger mistakes that students make with this kind of problem is to miss the square and treat that term as negative when plugging in a negative test point.}
\end{align*}
\]

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[
\begin{align*}
\quad & -4 < t < 0 \quad \text{and} \quad 0 < t < 3 \\
\quad & (-4, 0) \quad \text{and} \quad (0, 3)
\end{align*}
\]

Be careful with your answer here and don’t include \( t = 0 \)! It might be tempting to do that to “simplify” the answer into a single inequality/interval but the polynomial is zero at \( t = 0 \) and we only want to know where the polynomial is negative! Therefore we cannot include \( t = 0 \) in our answer and we’ll need to write it as two inequalities/intervals.

**Rational Inequalities**

1. Solve the following inequality.

\[
\frac{4 - x}{x + 3} > 0
\]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the numerator and denominator as much as possible.

For this problem we already have zero on one side of the inequality and there is no factoring to do with the problem.

Hint : Where are the only places where the rational expression might change signs?

Step 2
Recall from the discussion in the notes for this section that the rational expression can only change sign where the numerator is zero and/or where the denominator is zero.

We can see that the numerator will be zero at,

$$x = 4$$

and the denominator will be zero at,

$$x = -3$$

Hint : Knowing that the rational expression can only change sign at the points above how can we quickly determine if the rational expression is positive or negative in the ranges between those points?

Step 3
Just as we did with polynomial inequalities all we need to do is check the rational expression at test points in each region between the points from the previous step. The rational expression will have the same sign as the sign at the test point since it can only change sign at those points.

Here is a sketch of a number line with the points from the previous step graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

<table>
<thead>
<tr>
<th>x = -4</th>
<th>x = 0</th>
<th>x = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{8}{-1} &lt; 0$</td>
<td>$\frac{4}{3} &gt; 0$</td>
<td>$\frac{-1}{8} &lt; 0$</td>
</tr>
</tbody>
</table>

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

$$-3 < x < 4$$

$$(-3, 4)$$
2. Solve the following inequality.

\[ \frac{2z - 5}{z - 7} \leq 0 \]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the numerator and denominator as much as possible.

For this problem we already have zero on one side of the inequality and there is no factoring to do with the problem.

Hint : Where are the only places where the rational expression might change signs?

Step 2
Recall from the discussion in the notes for this section that the rational expression can only change sign where the numerator is zero and/or where the denominator is zero.

We can see that the numerator will be zero at,

\[ z = \frac{5}{2} \]

and the denominator will be zero at,

\[ z = 7 \]

Hint : Knowing that the rational expression can only change sign at the points above how can we quickly determine if the rational expression is positive or negative in the ranges between those points?

Step 3
Just as we did with polynomial inequalities all we need to do is check the rational expression at test points in each region between the points from the previous step. The rational expression will have the same sign as the sign at the test point since it can only change sign at those points.

Here is a sketch of a number line with the points from the previous step graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[ \frac{-1}{-5} > 0 \quad \frac{5}{-2} < 0 \quad \frac{11}{1} > 0 \]

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[
\frac{5}{2} \leq z < 7
\]

\[
\left[ \frac{5}{2}, 7 \right)
\]

Be careful with the endpoints for this problem. Because we have an equal sign in the original inequality we need to include \( z = \frac{5}{2} \) because the numerator and hence the rational expression will be zero there. However, we can’t include \( z = 7 \) because the denominator is zero there and so the rational expression has division by zero at that point!

3. Solve the following inequality.

\[
\frac{w^2 + 5w - 6}{w - 3} \geq 0
\]

Step 1

The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the numerator and denominator as much as possible.

For this problem we already have zero on one side of the inequality but we do need to factor the numerator.

\[
\frac{(w + 6)(w - 1)}{w - 3} \geq 0
\]

Hint : Where are the only places where the rational expression might change signs?

Step 2

Recall from the discussion in the notes for this section that the rational expression can only change sign where the numerator is zero and/or where the denominator is zero.

We can see that the numerator will be zero at,

\[
w = -6 \quad w = 1
\]

and the denominator will be zero at,

\[
w = 3
\]

Hint : Knowing that the rational expression can only change sign at the points above how can we quickly determine if the rational expression is positive or negative in the ranges between those points?

Step 3
Just as we did with polynomial inequalities all we need to do is check the rational expression at test points in each region between the points from the previous step. The rational expression will have the same sign as the sign at the test point since it can only change sign at those points.

Here is a sketch of a number line with the points from the previous step graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[
-6 \leq w \leq 1 \quad \text{and} \quad w > 3
\]

\[
[-6, 1] \quad \text{and} \quad (3, \infty)
\]

Be careful with the endpoints for this problem. Because we have an equal sign in the original inequality we need to include \( w = -6 \) and \( w = 1 \) because the numerator and hence the rational expression will be zero there. However, we can’t include \( w = 3 \) because the denominator is zero there and so the rational expression has division by zero at that point!

4. Solve the following inequality.

\[
\frac{3x + 8}{x - 1} < -2
\]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the numerator and denominator as much as possible.

So, we first need to add 2 to both sides to get,

\[
\frac{3x + 8}{x - 1} + 2 < 0
\]

We now need to combine the two terms in to a single rational expression.
\[
\frac{3x + 8}{x - 1} + \frac{2(x - 1)}{x - 1} < 0
\]
\[
\frac{3x + 8 + 2x - 2}{x - 1} < 0
\]
\[
\frac{5x + 6}{x - 1} < 0
\]

At this point we can also see that factoring will not be needed for this problem.

Hint : Where are the only places where the rational expression might change signs?

Step 2
Recall from the discussion in the notes for this section that the rational expression can only change sign where the numerator is zero and/or where the denominator is zero.

We can see that the numerator will be zero at,
\[
x = -\frac{6}{5}
\]
and the denominator will be zero at,
\[
x = 1
\]

Hint : Knowing that the rational expression can only change sign at the points above how can we quickly determine if the rational expression is positive or negative in the ranges between those points?

Step 3
Just as we did with polynomial inequalities all we need to do is check the rational expression at test points in each region between the points from the previous step. The rational expression will have the same sign as the sign at the test point since it can only change sign at those points.

Here is a sketch of a number line with the points from the previous step graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[
\frac{-4}{-3} > 0 \quad \left| \quad \frac{6}{-1} < 0 \quad \left| \quad \frac{16}{1} > 0
\]

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.
5. Solve the following inequality.

\[ u \leq \frac{4}{u-3} \]

**Step 1**
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the numerator and denominator as much as possible.

So, we first need to get zero on one side of the inequality.

\[ u - \frac{4}{u-3} \leq 0 \]

We now need to combine the two terms into a single rational expression.

\[ \frac{u(u-3) - 4}{u-3} \leq 0 \]

\[ \frac{u^2 - 3u - 4}{u-3} \leq 0 \]

Finally, we need to factor the numerator.

\[ \frac{(u-4)(u+1)}{u-3} \leq 0 \]

**Hint:** Where are the only places where the rational expression might change signs?

**Step 2**
Recall from the discussion in the notes for this section that the rational expression can only change sign where the numerator is zero and/or where the denominator is zero.

We can see that the numerator will be zero at,

\[ u = -1 \quad u = 4 \]

and the denominator will be zero at,

\[ u = 3 \]
Hint: Knowing that the rational expression can only change sign at the points above how can we quickly determine if the rational expression is positive or negative in the ranges between those points?

Step 3
Just as we did with polynomial inequalities all we need to do is check the rational expression at test points in each region between the points from the previous step. The rational expression will have the same sign as the sign at the test point since it can only change sign at those points.

Here is a sketch of a number line with the points from the previous step graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[
\begin{array}{cccccccc}
\quad & u = -2 & \quad & u = 0 & \quad & u = 3.5 & \quad & u = 5 \\
\frac{(-5)(-1)}{-5} < 0 & \quad & \frac{(-4)(1)}{-3} > 0 & \quad & \frac{(-0.5)(4.5)}{0.5} < 0 & \quad & \frac{(1)(6)}{2} > 0 \\
\end{array}
\]

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation form of the answer.

\[
\frac{u}{-1} \leq -1 \quad \text{and} \quad 3 < u \leq 4
\]

Be careful with the endpoints for this problem. Because we have an equal sign in the original inequality we need to include \( u = -1 \) and \( u = 4 \) because the numerator and hence the rational expression will be zero there. However, we can’t include \( u = 3 \) because the denominator is zero there and so the rational expression has division by zero at that point!

6. Solve the following inequality.

\[
\frac{t^3 - 6t^2}{t - 2} > 0
\]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the numerator and denominator as much as possible.

We already have zero on one side of the inequality but we still need to factor the numerator.

\[
\frac{t^2(t - 6)}{t - 2} > 0
\]
Hint: Where are the only places where the rational expression might change signs?

Step 2
Recall from the discussion in the notes for this section that the rational expression can only change sign where the numerator is zero and/or where the denominator is zero.

We can see that the numerator will be zero at,

\[ t = 0 \quad t = 6 \]

and the denominator will be zero at,

\[ t = 2 \]

Hint: Knowing that the rational expression can only change sign at the points above how can we quickly determine if the rational expression is positive or negative in the ranges between those points?

Step 3
Just as we did with polynomial inequalities all we need to do is check the rational expression at test points in each region between the points from the previous step. The rational expression will have the same sign as the sign at the test point since it can only change sign at those points.

Here is a sketch of a number line with the points from the previous step graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[
\begin{array}{c|c|c|c|c|c}
 t = -1 & t = 1 & t = 3 & t = 7 \\
\frac{(-1)^2(-7)}{-3} > 0 & \frac{(1)^2(-5)}{-1} > 0 & \frac{(3)^2(-3)}{1} < 0 & \frac{(7)^2(1)}{5} > 0 \\
\end{array}
\]

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[
\begin{align*}
 t < 0 & \quad 0 < t < 2 & \quad \text{and} & \quad t > 6 \\
(-\infty, 0) & \quad (0, 2) & \quad \text{and} & \quad (6, \infty)
\end{align*}
\]

Be careful to not include \( t = 0 \) in the answer! It might be tempting to “simplify” the first two inequalities in our answer into a single inequality. However, we’re looking for where the rational expression is positive only and at \( t = 0 \) the rational expression is zero and so we need to exclude \( t = 0 \) from our answer.
**Absolute Value Equations**

1. Solve the following equation.

\[ |4p - 7| = 3 \]

Step 1
There really isn’t all that much to this problem. All we need to do is use the formula we discussed in the notes for this section. Doing that gives,

\[ 4p - 7 = -3 \quad \text{or} \quad 4p - 7 = 3 \]

Do not make the common mistake of just turning every minus sign inside the absolute value bars into a plus sign. That is just not how these work. The only way for the value of the absolute value to be 3 is for the quantity inside to be either -3 or 3. In other words, we get rid of the absolute value bars by using the formula from the notes.

Step 2
At this point all we need to do is solve each of the linear equations we got in the previous step. Doing that gives,

\[ 4p = 4 \quad \text{or} \quad 4p = 10 \]

\[ p = 1 \quad \text{or} \quad p = \frac{10}{4} = \frac{5}{2} \]

The two solutions are then: \( p = 1 \quad \text{and} \quad p = \frac{5}{2} \).

2. Solve the following equation.

\[ |2 - 4x| = 1 \]

Step 1
There really isn’t all that much to this problem. All we need to do is use the formula we discussed in the notes for this section. Doing that gives,

\[ 2 - 4x = -1 \quad \text{or} \quad 2 - 4x = 1 \]

Do not make the common mistake of just turning every minus sign inside the absolute value bars into a plus sign. That is just not how these work. The only way for the value of the absolute value to be 1 is for the quantity inside to be either -1 or 1. In other words, we get rid of the absolute value bars by using the formula from the notes.

Step 2
At this point all we need to do is solve each of the linear equations we got in the previous step. Doing that gives,

\[-4x = -3 \quad \text{or} \quad -4x = -1\]
\[x = \frac{3}{4} \quad \text{or} \quad x = \frac{1}{4}\]

The two solutions are then: \[x = \frac{1}{4} \text{ and } x = \frac{3}{4}\].

3. Solve the following equation.

\[6u = |1 + 3u|\]

Hint: Just because the quantity outside of the absolute value bars in not a number does not mean this problem works any differently. Just remember to be careful with your answers!

Step 1
Despite the fact that the quantity outside of the absolute value bars is not a positive number doesn’t mean that we can’t use the same process that we used in the first two problems.

Using the formula from the notes gives,

\[1 + 3u = -6u \quad \text{or} \quad 1 + 3u = 6u\]

Step 2
Now solving each these linear equations gives,

\[1 + 3u = -6u \quad \text{or} \quad 1 + 3u = 6u\]
\[1 = -9u \quad \text{or} \quad 1 = 3u\]
\[u = -\frac{1}{9} \quad \text{or} \quad u = \frac{1}{3}\]

Step 3
Now, because the quantity outside of the absolute value bars was not a positive constant we need to be careful with the answers we got in the previous step. It is possible that one or both are not in fact solutions to the original equation. So, we need to verify each of the possible solutions from the previous step by checking them in the original equation.

\[u = -\frac{1}{9} : \quad 6\left(-\frac{1}{9}\right) = \left|1 + 3\left(-\frac{1}{9}\right)\right| \rightarrow -\frac{2}{3} = \frac{2}{3} \rightarrow -\frac{2}{3} \neq \frac{2}{3} \quad \text{NOT OK}\]

\[u = \frac{1}{3} : \quad 6\left(\frac{1}{3}\right) = \left|1 + 3\left(\frac{1}{3}\right)\right| \rightarrow \frac{2}{3} = 2 \rightarrow 2 \neq 2 \quad \text{OK}\]
Therefore, the only solution to the original equation is then: $u = \frac{1}{3}$.

4. Solve the following equation.

$$|2x - 3| = 4 - x$$

Hint: Just because the quantity outside of the absolute value bars in not a number does not mean this problem works any differently. Just remember to be careful with your answers!

Step 1
Despite the fact that the quantity outside of the absolute value bars is not a positive number doesn’t mean that we can’t use the same process that we used in the first two problems.

Using the formula from the notes gives,

$$2x - 3 = -(4 - x) = x - 4$$ or $$2x - 3 = 4 - x$$

Step 2
Now solving each these linear equations gives,

$$2x - 3 = x - 4$$ or $$2x - 3 = 4 - x$$

$x = -1$ or $3x = 7$

$x = -1$ or $x = \frac{7}{3}$

Step 3
Now, because the quantity outside of the absolute value bars was not a positive constant we need to be careful with the answers we got in the previous step. It is possible that one or both are not in fact solutions to the original equation. So, we need to verify each of the possible solutions from the previous step by checking them in the original equation.

$$x = -1: \quad |2(-1) - 3| = 4 - (-1) \quad \rightarrow \quad |-5| = 5 \quad \rightarrow \quad 5 \neq 5$$ OK

$$x = \frac{7}{3}: \quad |2\left(\frac{7}{3}\right) - 3| = 4 - \left(\frac{7}{3}\right) \quad \rightarrow \quad \frac{5}{3} = \frac{5}{3} \quad \rightarrow \quad \frac{5}{3} = \frac{5}{3}$$ OK

Therefore, the two solutions to the original equation are then: $x = -1$ and $x = \frac{7}{3}$.

5. Solve the following equation.

$$\left|\frac{1}{2}z + 4\right| = |4z - 6|$$
Hint: This problem works the same as all the others in this section do.

Step 1
This problem works identically to all the problems in this section. The only way the two absolute values can be equal is if the quantities inside them are the same value or the same value except for opposite signs. Doing this gives,

\[
\frac{1}{2}z + 4 = -(4z - 6) = 6 - 4z \quad \text{or} \quad \frac{1}{2}z + 4 = 4z - 6
\]

In other words, we can use the formula discussed in this section to do this problem!

Step 2
Now solving each these linear equations gives,

\[
\frac{1}{2}z + 4 = 6 - 4z \quad \text{or} \quad \frac{1}{2}z + 4 = 4z - 6
\]

\[
\frac{9}{2}z = 2 \quad \text{or} \quad -\frac{7}{2}z = -10
\]

\[
z = \frac{4}{9} \quad \text{or} \quad z = \frac{20}{7}
\]

Now, because both sides of the equation have absolute values, we know that regardless of the value of \(x\) we plug into the original equation the absolute value will guarantee that the result will be positive and so we don’t need to verify either of these solutions.

Therefore, the two solutions to the original equation are then: \(z = \frac{4}{9}\) and \(z = \frac{20}{7}\).

6. Find all the real valued solutions to the equation.

\[
\left|x^2 + 2x\right| = 15
\]

Hint: Don’t let the fact that there is a quadratic term in the absolute value throw you off. This problem works exactly the same as the previous problems!

Step 1
To this point we’ve only worked problems that have linear terms in the absolute value bars. In this case we have a quadratic in the absolute value bars. That doesn’t change how the problem works however. We work this exactly like the previous problems.

Applying the formula from this section gives,

\[
x^2 + 2x = -15 \quad \text{or} \quad x^2 + 2x = 15
\]

Step 2
7. Find all the real valued solutions to the equation.

\[ |x^2 + 4| = 1 \]

Hint: Don’t let the fact that there is a quadratic term in the absolute value throw you off. This problem works exactly the same as the previous problems!

Step 1
To this point we’ve only worked problems that have linear terms in the absolute value bars. In this case we have a quadratic in the absolute value bars. That doesn’t change how the problem works however. We work this exactly like the previous problems.

Applying the formula from this section gives,

\[ x^2 + 4 = -1 \quad \text{or} \quad x^2 + 4 = 1 \]

Step 2
To finish this problem all we need to do is solve each of the quadratic equations we got in the previous step. Here is the solution to each of them.
Note that the instructions asked for “real valued solutions”. This basically means that we don’t want complex solutions and the solutions to both of the quadratic equations from the first step are complex and so, for this equation, there are no solutions.

**Absolute Value Inequalities**

1. Solve the following equation.

$$|4t + 9| < 3$$

Step 1
There really isn’t all that much to this problem. All we need to do is use the formula for “less than” inequalities we discussed in the notes for this section. Doing that gives,

$$-3 < 4t + 9 < 3$$

Step 2
To get the solution all we need to do then is solve the double inequality from the previous step. Here is that work.

$$-3 < 4t + 9 < 3$$

$$-12 < 4t < -6$$

$$-3 < t < -\frac{3}{2}$$

2. Solve the following equation.

$$|6 - 5x| \leq 10$$

Step 1
There really isn’t all that much to this problem. All we need to do is use the formula for “less than” inequalities we discussed in the notes for this section. Doing that gives,

$$-10 \leq 6 - 5x \leq 10$$

Step 2
To get the solution all we need to do then is solve the double inequality from the previous step. Here is that work.
\[ -10 \leq 6 - 5x \leq 10 \\
-16 \leq -5x \leq 4 \\
\frac{16}{5} \geq x \geq -\frac{4}{5} \]

Remember that when dividing all parts of an inequality by a negative number (as we did here) we need to also switch the direction of the inequalities!

3. Solve the following equation.

\[ |12x + 1| \leq -9 \]

Solution

There is no solution to this inequality.

We know that absolute value will only give positive or zero answers and so this inequality is asking what values of \( x \) will give a value on the left side (after taking the absolute value of course) that is less than a -9. In other words, any solution requires that the absolute value give a negative number and we know that can’t happen. Therefore, there are no solutions to this inequality. This kinds of thing happens occasionally so don’t get too excited about it when it does.

4. Solve the following equation.

\[ |2w - 1| < 1 \]

Step 1

There really isn’t all that much to this problem. All we need to do is use the formula for “less than” inequalities we discussed in the notes for this section. Doing that gives,

\[ -1 < 2w - 1 < 1 \]

Step 2

To get the solution all we need to do then is solve the double inequality from the previous step. Here is that work.

\[ -1 < 2w - 1 < 1 \\
0 < 2w < 2 \\
0 < w < 1 \]

5. Solve the following equation.
\[ |2z - 7| > 1 \]

**Step 1**
There really isn’t all that much to this problem. All we need to do is use the formula for “greater than” inequalities we discussed in the notes for this section. Doing that gives,

\[ 2z - 7 < -1 \quad \text{or} \quad 2z - 7 > 1 \]

**Step 2**
To get the solution all we need to do then is solve the two inequalities from the previous step. Here is that work.

\[
\begin{align*}
2z - 7 &< -1 \quad \text{or} \quad 2z - 7 > 1 \\
2z &< 6 \quad \text{or} \quad 2z > 8 \\
2z &< 6 \quad \text{or} \quad 2z > 8 \\
z &< 3 \quad \text{or} \quad z > 4 \\
\end{align*}
\]

---

6. Solve the following equation.

\[ |10 - 3w| \geq 4 \]

**Step 1**
There really isn’t all that much to this problem. All we need to do is use the formula for “greater than” inequalities we discussed in the notes for this section. Doing that gives,

\[ 10 - 3w \leq -4 \quad \text{or} \quad 10 - 3w \geq 4 \]

**Step 2**
To get the solution all we need to do then is solve the two inequalities from the previous step. Here is that work.

\[
\begin{align*}
10 - 3w &\leq -4 \quad \text{or} \quad 10 - 3w \geq 4 \\
-3w &\leq -14 \quad \text{or} \quad -3w \geq -6 \\
-3w &\leq -14 \quad \text{or} \quad -3w \geq -6 \\
w &\geq \frac{14}{3} \quad \text{or} \quad w \leq 2 \\
\end{align*}
\]

Remember that when dividing all parts of an inequality by a negative number (as we did here) we need to also switch the direction of the inequalities!

---

7. Solve the following equation.

\[ |4 - 3z| > 7 \]

**Step 1**
There really isn’t all that much to this problem. All we need to do is use the formula for “greater than” inequalities we discussed in the notes for this section. Doing that gives,

\[ 4 - 3z < -7 \quad \text{or} \quad 4 - 3z > 7 \]

Step 2
To get the solution all we need to do then is solve the two inequalities from the previous step. Here is that work.

\[
\begin{align*}
4 - 3z &< -7 & \text{or} & & 4 - 3z &> 7 \\
-3z &< -11 & \text{or} & & -3z &> 3 \\
\frac{z}{3} &> \frac{11}{3} & \text{or} & & z &< -1 \\
\end{align*}
\]

Remember that when dividing all parts of an inequality by a negative number (as we did here) we need to also switch the direction of the inequalities!