Preface

Here are a set of problems for my Algebra notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.
Graphing and Functions

Introduction

Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.

Graphing
Lines
Circles
The Definition of a Function
Graphing Functions
Combining functions
Inverse Functions

Graphing

For problems 1 – 7 construct a table of at least 4 ordered pairs of points on the graph of the equation and use the ordered pairs from the table to sketch the graph of the equation.

1. \[ y = \frac{1}{2} x + \frac{3}{2} \]

2. \[ y = 4 - x \]

3. \[ y = 3x^2 \]
4. \( y = (x + 3)^2 \)
5. \( y = \sqrt{x} + 2 \)
6. \( y = |x| \)
7. \( y = x^3 \)

For problems 8 – 18 determine the \( x \)-intercepts and \( y \)-intercepts for the equation. Do not sketch the graph.

8. \( y = \frac{7}{3}x + 2 \)
9. \( 6y + 11x = -2 \)
10. \( y = 10x^2 \)
11. \( y = x^2 - 10x + 25 \)
12. \( y = 16x^2 - 8x + 17 \)
13. \( y = -x^2 - 25x - 24 \)
14. \( y = 2x^2 - 6x + 7 \)
15. \( y = -4x^2 - 3 \)
16. \( y = 6x^3 + 48 \)
17. \( y = |x + 4| - 7 \)
18. \( y = 4 - \sqrt{x - 2} \)

**Lines**

For problems 1 – 5 determine the slope of the line containing the two points and sketch the graph of the line.

1. \((2, 10), (2, 14)\)
2. \((-6,0), (-1,3)\)

3. \((2,12), (6,10)\)

4. \((-5,7), (1,-11)\)

5. \((-1,-6), (4,-6)\)

For problems 6 – 12 write down the equation of the line that passes through the two points. Give your answer in point-slope form and slope-intercept form.

6. \((2,10), (2,14)\)

7. \((-6,0), (-1,3)\)

8. \((2,12), (6,10)\)

9. \((-5,7), (1,-11)\)

10. \((-1,-6), (4,-6)\)

11. \((0,10), (4,2)\)

12. \((-9,2), (3,24)\)

For problems 13 – 17 determine the slope of the line and sketch the graph of the line.

13. \(6x - y = 8\)

14. \(y + 2x = -3\)

15. \(3x - y = 1\)

16. \(5y + 4x = 7\)

17. \(6y - 13x = -4\)

For problems 18 - 20 determine if the two given lines are parallel, perpendicular or neither.

18. The line containing the two points \((0,0), (3,18)\) and the line containing the two points \((-1,-5), (1,7)\).
19. \( y - 4x = 9 \) and \( 4y - x = -3 \)

20. \( y = \frac{2}{3}x - 4 \) and the line containing the two points \((-4, 7), (2, -2)\)

21. Find the equation of the line through \((6, -1)\) and is parallel to the line \(9x + 2y = 1\).

22. Find the equation of the line through \((6, -1)\) and is perpendicular to the line \(9x + 2y = 1\).

23. Find the equation of the line through \((-4, -9)\) and is parallel to the line \(-8y - x = 43\).

24. Find the equation of the line through \((-4, -9)\) and is perpendicular to the line \(-8y - x = 43\).

**Circles**

1. Write the equation of the circle with radius 1 and center \((11, 4)\).

2. Write the equation of the circle with radius 10 and center \((-6, 0)\).

3. Write the equation of the circle with radius \(\sqrt{19}\) and center \((7, -2)\).

4. Write the equation of the circle with radius \(\frac{7}{3}\) and center \((-\frac{1}{2}, \frac{3}{4})\).

For problems 5 – 10 determine the radius and center of the circle and sketch the graph of the circle.

5. \((x + 8)^2 + y^2 = 36\)

6. \((x - 1)^2 + (y - 7)^2 = 16\)

7. \((x + 10)^2 + (y - 6)^2 = 25\)

8. \(x^2 + (y + 4)^2 = \frac{49}{144}\)

9. \((x + 2)^2 + (y - 1)^2 = 3\)

10. \((x - 5)^2 + (y - 3)^2 = 11\)
For problems 11 – 17 determine the radius and center of the circle. If the equation is not the equation of a circle clearly explain why not.

11. $x^2 + y^2 - 8y = 0$

12. $x^2 + y^2 - 6x - 4y - 12 = 0$

13. $x^2 + y^2 + 12x + 2y + 28 = 0$

14. $16x^2 + 16y^2 - 16x + 8y - 11 = 0$

15. $2x^2 + 2y^2 - 3x + 1 = 0$

16. $x^2 + y^2 + 2x - 2y + 11 = 0$

17. $x^2 + y^2 - 10x + 4y + 29 = 0$

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**The Definition of a Function**

For problems 1 – 6 determine if the given relation is a function.

1. \[\{(0,1),(2,6),(9,4),(7,2),(12,3)\}\]

2. \[\{(-4,1),(-2,1),(0,1),(3,1)\}\]

3. \[\{(0,4),(0,6),(0,8)\}\]

4. \[\{(1,6),(-3,4),(7,6),(2,-10)\}\]

5. \[\{(0,1),(2,3),(4,5),(6,7),(8,9),(10,11),(12,13)\}\]

6. \[\{(-7,0),(4,2),(4,1),(-2,3),(6,0)\}\]

For problems 7 – 13 determine if the given equation is a function.

7. \[y = \frac{2}{5}x + \frac{7}{5}\]

8. \[y = 3x^2 + 4x + 1\]
9. \( y = 2 - x^4 \)

10. \( y^2 = 10 - 3x \)

11. \( y^2 = x^2 + 1 \)

12. \( y^4 + x^3 = 1 \)

13. \( y^3 + x^4 = 1 \)

14. Given \( A(t) = 7t + 2 \) determine each of the following.
   (a) \( A(-9) \)  (b) \( A(0) \)  (c) \( A(2) \)  (d) \( A(6x) \)  (e) \( A(t^2 + 1) \)

15. Given \( f(x) = \frac{3}{x} \) determine each of the following.
   (a) \( f(-4) \)  (b) \( f\left(\frac{1}{3}\right) \)  (c) \( f\left(\frac{6}{7}\right) \)  (d) \( f(4t + 2) \)  (e) \( f\left(\frac{6}{x}\right) \)

16. Given \( h(w) = \sqrt{2w + 10} \) determine each of the following.
   (a) \( h(-1) \)  (b) \( h(0) \)  (c) \( h(3) \)  (d) \( h(-2t) \)  (e) \( h(w + 4) \)

17. Given \( P(x) = 3 - 2x - x^2 \) determine each of the following.
   (a) \( P(-6) \)  (b) \( P(0) \)  (c) \( P(3) \)  (d) \( P\left(z^2\right) \)  (e) \( P(4 - x) \)

18. Given \( f(z) = 2z^3 - z^2 \) determine each of the following.
   (a) \( f(-1) \)  (b) \( f(0) \)  (c) \( f(4) \)  (d) \( f\left(\frac{1}{2}t\right) \)  (e) \( f(z - 1) \)

19. Given \( g(t) = \begin{cases} 2 + t & \text{if } t \geq 10 \\ t - 7 & \text{if } t < 10 \end{cases} \) determine each of the following.
   (a) \( g(14) \)  (b) \( g(10) \)  (c) \( g(-1) \)

20. Given \( f(x) = \begin{cases} 4x^2 & \text{if } x < -4 \\ 6x & \text{if } x \geq -4 \end{cases} \) determine each of the following.
   (a) \( f(-6) \)  (b) \( f(-4) \)  (c) \( f(3) \)
21. Given \( g(x) = \begin{cases} \frac{1}{2}x & \text{if } x \leq 7 \\ x^2 + 1 & \text{if } 7 < x < 11 \\ 3 - x & \text{if } x \geq 11 \end{cases} \) determine each of the following.

(a) \( g(2) \)  
(b) \( g(7) \)  
(c) \( g(8) \)  
(d) \( g(11) \)  
(e) \( g(14) \)

22. Given \( A(w) = \begin{cases} 2 + 3w & \text{if } -10 \leq w \leq -8 \\ -1 & \text{if } w < -10 \end{cases} \) determine each of the following.

(a) \( A(-12) \)  
(b) \( A(-10) \)  
(c) \( A(-9) \)  
(d) \( A(-8) \)  
(e) \( A(0) \)

23. Given \( f(x) = \begin{cases} 2x & \text{if } x < 6 \\ 4 + x & \text{if } x = 6 \\ x^2 & \text{if } x > 6 \end{cases} \) determine each of the following.

(a) \( f(0) \)  
(b) \( f(2) \)  
(c) \( f(6) \)  
(d) \( f(8) \)  
(e) \( f(10) \)

For problems 24 – 28 compute the difference quotient for the given function. The difference quotient for the function \( f(x) \) is defined to be,

\[
\frac{f(x+h) - f(x)}{h}
\]

24. \( f(x) = 8x - 1 \)

25. \( f(x) = 3x^2 \)

26. \( f(x) = 7 - x^2 \)

27. \( f(x) = 3x^2 + 7x - 4 \)

28. \( f(x) = \frac{2}{x} \)

For problems 29 – 39 determine the domain of the function.

29. \( f(x) = 9 - x \)

30. \( P(z) = z^2 - 4 \)
31. \( h(x) = \frac{2+x}{8x-1} \)

32. \( A(t) = \frac{t^2 - 4}{t^2 + 6t - 7} \)

33. \( h(w) = \frac{w^2 + 3w + 2}{w^2 + 12w + 36} \)

34. \( g(x) = \sqrt{10x - 15} \)

35. \( f(t) = \frac{10t}{\sqrt{6 - 4t}} \)

36. \( f(w) = \frac{\sqrt{w + 7}}{\sqrt{2 - w}} \)

37. \( A(z) = \sqrt{z^2 - 9z} \)

38. \( h(z) = \sqrt{z^2 - z - 20} \)

39. \( g(t) = \sqrt{\frac{6 + t}{5t - 10}} \)

**Graphing Functions**

For problems 1 – 13 construct a table of at least 4 ordered pairs of points on the graph of the function and use the ordered pairs from the table to sketch the graph of the function.

1. \( f(x) = 6x - 1 \)

2. \( f(x) = 3 - 5x \)

3. \( f(x) = 2x^2 \)

4. \( f(x) = x^2 + 7 \)

5. \( f(x) = \sqrt{x + 3} \)
6. \( f(x) = \sqrt{6-x} \)

7. \( f(x) = \frac{1}{x} \), use only positive \( x \)'s

8. \( f(x) = \frac{1}{x} \), use only negative \( x \)'s

9. \( f(x) = \begin{cases} 3 & \text{if } x \geq 0 \\ 4-x & \text{if } x < 0 \end{cases} \)

10. \( f(x) = \begin{cases} 4x & \text{if } x \leq -2 \\ 3-2x & \text{if } x > -2 \end{cases} \)

11. \( f(x) = \begin{cases} 2-x^2 & \text{if } x < 1 \\ (x-2)^2 & \text{if } x \geq 1 \end{cases} \)

12. \( f(x) = \begin{cases} x^2 & \text{if } x > 3 \\ 4 & \text{if } -2 \leq x \leq 3 \\ 1-x & \text{if } x < -2 \end{cases} \)

13. \( f(x) = \begin{cases} x^2-1 & \text{if } -1 < x < 1 \\ -1-x & \text{if } x \leq -1 \end{cases} \)

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**Combining Functions**

1. Given \( f(x) = x+12 \) and \( g(x) = 9+4x \) compute each of the following.
   (a) \( f + g \)  (b) \( (f - g)(1) \)  (c) \( (f g)(x) \)  (d) \( \frac{f}{g} \)

2. Given \( h(w) = w^2 - 4w \) and \( f(w) = 2 + w^2 \) compute each of the following.
   (a) \( (h - f)(w) \)  (b) \( (f + h)(-4) \)  (c) \( f h \)  (d) \( \left( \frac{h}{f} \right)(w) \)

3. Given \( A(x) = 6x - 1 \) and \( P(x) = \frac{1}{4-x} \) compute each of the following.
(a) \((A + P)(0)\)  \hspace{1cm} (b) \((P - A)(-2)\)  \hspace{1cm} (c) \(AP\)  \hspace{1cm} (d) \(\left(\frac{A}{P}\right)(x)\)

4. Given \(f(t) = 2t + 9\) and \(g(t) = 2t - 1\) compute each of the following.
(a) \((f \cdot g)(t)\)  \hspace{1cm} (b) \((f \circ g)(t)\)  \hspace{1cm} (c) \((g \circ f)(t)\)  \hspace{1cm} (d) \((g \cdot g)(t)\)

5. Given \(h(x) = x^2 + 1\) and \(g(x) = 6 - 4x\) compute each of the following.
(a) \((g \cdot h)(x)\)  \hspace{1cm} (b) \((g \circ h)(x)\)  \hspace{1cm} (c) \((h \circ g)(x)\)  \hspace{1cm} (d) \((h \circ h)(x)\)

6. Given \(A(w) = 2w^2 + 9\) and \(R(w) = 1 - 2w - w^2\) compute each of the following.
(a) \((A \circ R)(w)\)  \hspace{1cm} (b) \((A \cdot R)(w)\)  \hspace{1cm} (c) \((R \circ A)(w)\)  \hspace{1cm} (d) \((A \circ A)(w)\)

7. Given \(f(x) = 9x^2 + 10x + 12\) and \(g(x) = 2\) compute each of the following.
(a) \((f \cdot g)(x)\)  \hspace{1cm} (b) \((f \circ g)(x)\)  \hspace{1cm} (c) \((f \circ f)(x)\)  \hspace{1cm} (d) \((g \circ g)(x)\)

8. Given \(g(t) = t + 1\) and \(h(t) = \frac{2}{t-3}\) compute each of the following.
(a) \((g \cdot h)(t)\)  \hspace{1cm} (b) \((g \circ h)(t)\)  \hspace{1cm} (c) \((h \circ g)(t)\)  \hspace{1cm} (d) \((h \circ h)(t)\)

9. Given \(f(x) = \frac{1}{2}x - 3\) and \(g(x) = 2x + 6\), \(t \geq 0\) compute each of the following.
(a) \((f \circ g)(x)\)  \hspace{1cm} (b) \((g \circ f)(x)\)

10. Given \(h(w) = \frac{1}{w-3}\) and \(f(w) = \frac{1+3w}{w}\) compute each of the following.
(a) \((h \circ f)(w)\)  \hspace{1cm} (b) \((f \circ h)(w)\)

**Inverse Functions**

1. Given \(P(x) = 12x - 7\) find \(P^{-1}(x)\).

2. Given \(g(x) = 7x\) find \(g^{-1}(x)\).

3. Given \(h(x) = \frac{2}{4} - \frac{9}{7}x\) find \(h^{-1}(x)\).

4. Given \(A(x) = 4 - (x + 3)^5\) find \(A^{-1}(x)\).
5. Given \( f(x) = 2(1-4x)^3 + 1 \) find \( f^{-1}(x) \).

6. Given \( P(x) = \sqrt[3]{5-8x} \) find \( P^{-1}(x) \).

7. Given \( g(x) = 1 + \sqrt[3]{3x+4} \) find \( g^{-1}(x) \).

8. Given \( f(x) = \frac{10 - 3x}{8x} \) find \( f^{-1}(x) \).

9. Given \( g(x) = \frac{6x - 7}{4 + x} \) find \( g^{-1}(x) \).

10. Given \( f(x) = \frac{3 - x}{9 - 7x} \) find \( f^{-1}(x) \).