Preface

Here are a set of problems for my Algebra notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.
Systems of Equations

Introduction

Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.

- Linear Systems with Two Variables
- Linear Systems with Three Variables
- Augmented Matrices
- More on the Augmented Matrix
- Nonlinear Systems

Linear Systems with Two Variables

For problems 1 – 5 use the Method of Substitution to find the solution to the given system or to determine if the system is inconsistent or dependent.

1. \[8x + y = 13\]
   \[3x + 4y = -6\]

2. \[x - 3y = 7\]
   \[-2x + 6y = 4\]

3. \[-12x + 6y = -12\]
   \[4x + 2y = -2\]

4. \[3x + 6y = 12\]
   \[-4x - 7y = -12\]
5. \(12x - 6y = 18\)  
   \(4x - 2y = 6\)

For problems 6 – 10 use the Method of Elimination to find the solution to the given system or to determine if the system is inconsistent or dependent.

6. \(-5x + 10y = 1\)  
   \(x - 2y = -8\)

7. \(7x + 6y = 0\)  
   \(2x + 3y = 0\)

8. \(-8x + 24y = 12\)  
   \(10x - 30y = -15\)

9. \(-2x + 3y = 24\)  
   \(3x - 8y = -57\)

10. \(6x + 4y = -20\)  
    \(7x + 3y = -35\)

**Linear Systems with Three Variables**

Find the solution to each of the following systems of equations.

1. \(-3x + 7y + 2z = -8\)  
   \(-2x + 5y - z = -10\)  
   \(8x - 2y + 3z = 38\)

2. \(6x + 4y - 8z = -56\)  
   \(-x - 4y + z = 5\)  
   \(3x + y + 9z = 10\)
3. \[
2x + 6y - z = 1 \\
-x + 2y + 9z = -19 \\
4x + 3y - 7z = 25
\]

**Augmented Matrices**

1. For the following augmented matrix perform the indicated elementary row operations.

\[
\begin{bmatrix}
9 & 0 & 7 & | & 4 \\
-3 & 2 & -1 & | & -7 \\
2 & 4 & 1 & | & 2
\end{bmatrix}
\]

(a) \(-4R_2\)  
(b) \(R_3 \leftrightarrow R_1\)  
(c) \(R_1 - 10R_3 \rightarrow R_1\)

2. For the following augmented matrix perform the indicated elementary row operations.

\[
\begin{bmatrix}
9 & 3 & 11 & | & 6 \\
-2 & 7 & 4 & | & -3 \\
1 & -1 & 1 & | & -1
\end{bmatrix}
\]

(a) \(5R_1\)  
(b) \(R_2 \leftrightarrow R_3\)  
(c) \(R_3 - 2R_2 \rightarrow R_3\)

3. For the following augmented matrix perform the indicated elementary row operations.

\[
\begin{bmatrix}
4 & 12 & -8 & | & 0 \\
-9 & -2 & 1 & | & 3 \\
1 & 5 & -1 & | & -10
\end{bmatrix}
\]

(a) \(\frac{1}{3}R_3\)  
(b) \(R_1 \leftrightarrow R_2\)  
(c) \(R_2 + \frac{5}{2}R_1 \rightarrow R_2\)

4. For the following augmented matrix perform the indicated elementary row operations.

\[
\begin{bmatrix}
1 & 5 & -6 & | & -2 \\
-3 & -15 & -18 & | & 3 \\
4 & -2 & 7 & | & 1
\end{bmatrix}
\]
Note: Problems using augmented matrices to solve systems of equations are in the next section.

More on the Augmented Matrix

For each of the following systems of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

1. \(8x + y = 13\) 
   \(3x + 4y = -6\)

2. \(x - 3y = 7\) 
   \(-2x + 6y = 4\)

3. \(-12x + 6y = -12\) 
   \(4x + 2y = -2\)

4. \(3x + 6y = 12\) 
   \(-4x - 7y = -12\)

5. \(12x - 6y = 18\) 
   \(4x - 2y = 6\)

6. \(-5x + 10y = 1\) 
   \(x - 2y = -8\)

7. \(7x + 6y = 0\) 
   \(2x + 3y = 0\)

8. \(-8x + 24y = 12\) 
   \(10x - 30y = -15\)
9. \(-2x + 3y = 24\)
   \(3x - 8y = -57\)

10. \(6x + 4y = -20\)
    \(7x + 3y = -35\)

11. \(-3x + 7y + 2z = -8\)
    \(-2x + 5y - z = -10\)
    \(8x - 2y + 3z = 38\)

12. \(6x + 4y - 8z = -56\)
    \(-x - 4y + z = 5\)
    \(3x + y + 9z = 10\)

13. \(2x + 6y - z = 1\)
    \(-x + 2y + 9z = -19\)
    \(4x + 3y - 7z = 25\)

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**Non-Linear Systems**

Find the solution to each of the following system of equations.

1. \(y = -x^2 + 5x + 16\)
   \(y = 7x - 8\)

2. \(y = 3 - x^2\)
   \(y = 8x^2 + 2\)

3. \(x^2 + \frac{y^2}{4} = 1\)
   \(y = 4 - 4x\)
4. \( x^2 + y^2 = 9 \)
   \[ y = 1 + \frac{x^2}{5} \]

5. \( x^2 + y^2 = 16 \)
   \[ y^2 - \frac{x^2}{15} = 1 \]

6. \( xy = -2 \)
   \[ x^2 + \frac{y^2}{25} = 1 \]

7. \( x^2 + y^2 = 1 \)
   \[ \frac{x^2}{4} + y^2 = 1 \]

8. \( x^2 + y^2 = 3 \)
   \[ \frac{x^2}{9} + y^2 = 1 \]