Preface

Here are a set of problems for my Calculus I notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.

Applications of Integrals

Introduction

Here are a set of problems for which no solutions are available. The main intent of these problems is to have a set of problems available for any instructors who are looking for some extra problems.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have problems written for them.

Arc Length
Surface Area
Center of Mass
Hydrostatic Pressure and Force
Probability

Arc Length
1. Set up, but do not evaluate, an integral for the length of \( y = 14 - 9x \), \(-22 \leq y \leq 31\) using,

(a) \( ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \)

(b) \( ds = \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy \)

2. Set up, but do not evaluate, an integral for the length of \( x = e^{2y} \), \(-1 \leq y \leq 0\) using,

(a) \( ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \)

(b) \( ds = \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy \)

3. Set up, but do not evaluate, an integral for the length of \( y = \tan(2x) \), \(0 \leq x \leq \frac{\pi}{4}\) using,

(a) \( ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \)

(b) \( ds = \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy \)

4. Set up, but do not evaluate, an integral for the length of \( \frac{x^2}{16} + 9y^2 = 1 \) .

5. For \( x = 6y + 1 \), \(-2 \leq y \leq 8\)
   (a) Use an integral to find the length of the curve.
   (b) Verify your answer from part (a) geometrically.

6. Determine the length of \( y = \frac{4}{3}x + 2 \), \(0 \leq x \leq 9\) .

7. Determine the length of \( y = \left(8x + 3\right)^{\frac{3}{2}} \), \(11^{\frac{1}{2}} \leq y \leq 27^{\frac{1}{2}}\) .

8. Determine the length of \( x = \left(10 - 2y\right)^{\frac{3}{2}} \), \(-1 \leq y \leq 2\) .

9. Determine the length of \( x = 2 + \left(y - 1\right)^2 \), \(2 \leq y \leq 5\) .

10. Determine the length of \( y = \left(3x + 2\right)^{\frac{3}{2}} \), \(1 \leq x \leq 4\) .


**Surface Area**

1. Set up, but do not evaluate, an integral for the surface area of the object obtained by rotating $y = 7x + 2$, $-5 \leq y \leq 0$ about the $x$-axis using,

   (a) $ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx$

   (b) $ds = \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy$

2. Set up, but do not evaluate, an integral for the surface area of the object obtained by rotating $y = 1 + 2x^5$, $0 \leq x \leq 1$ about the $x$-axis using,

   (a) $ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx$

   (b) $ds = \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy$

3. Set up, but do not evaluate, an integral for the surface area of the object obtained by rotating $x = e^{2y}$, $-1 \leq y \leq 0$ about the $y$-axis using,

   (a) $ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx$

   (b) $ds = \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy$

4. Set up, but do not evaluate, an integral for the surface area of the object obtained by rotating $y = \cos \left( \frac{1}{2} x \right)$, $0 \leq x \leq \pi$ about

   (a) the $x$-axis

   (b) the $y$-axis.

5. Set up, but do not evaluate, an integral for the surface area of the object obtained by rotating $x = \sqrt{3 + 7y}$, $0 \leq y \leq 1$ about

   (a) the $x$-axis

   (b) the $y$-axis.

6. Find the surface area of the object obtained by rotating $y = \frac{1}{3} \sqrt{6x + 2}$, $\frac{5}{2} \leq y \leq \frac{6}{2}$ about the $x$-axis.

7. Find the surface area of the object obtained by rotating $y = 4 - x$, $1 \leq x \leq 6$ about the $y$-axis.
8. Find the surface area of the object obtained by rotating \( x = 2y^2 + 5 \), \(-1 \leq x \leq 2\) about the \(y\)-axis.

9. Find the surface area of the object obtained by rotating \( x = 1 - y^2 \), \(0 \leq y \leq 3\) about the \(x\)-axis.

10. Find the surface area of the object obtained by rotating \( x = e^{2y} \), \(-1 \leq y \leq 0\) about the \(y\)-axis.

11. Find the surface area of the object obtained by rotating \( y = \cos\left(\frac{1}{2}x\right) \), \(0 \leq x \leq \pi\) about the \(x\)-axis.

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**Center of Mass**

Find the center of mass for each of the following regions.

1. The region bounded by \( y = x^3 \), \(x = -2\) and the \(x\)-axis.

2. The triangle with vertices (-2, -2), (4, -2) and (4,4).

3. The region bounded by \( y = (x - 2)^2 \) and \(y = 4\).

4. The region bounded by \( y = \cos(x) \) and the \(x\)-axis between \(-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\).

5. The region bounded by \( y = x^2 \) and \(y = x - 6\).

6. The region bounded by \( y = e^{2x} \) and the \(x\)-axis between \(-1 \leq x \leq 1\).

7. The region bounded by \( y = e^{2x} \) and \(y = -\cos(\pi x)\) between \(-\frac{1}{2} \leq x \leq \frac{1}{2}\).

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**Hydrostatic Pressure and Force**

Find the hydrostatic force on the following plates submerged in water as shown in each image. In each case consider the top of the blue “box” to be the surface of the water in which the plate is submerged. Note as well that the dimensions in many of the images will not be perfectly to scale in order to better fit the plate in the image. The lengths given in each image are in meters.

1.
2.

3.

4.
7.

**Probability**

1. Let,

\[ f(x) = \begin{cases} 
\frac{3}{4} (2x - x^2) & \text{if } 0 \leq x \leq 2 \\
0 & \text{otherwise}
\end{cases} \]

(a) Show that \( f(x) \) is a probability density function.

(b) Find \( P(X \leq 0.25) \).

(c) Find \( P(X \geq 1.4) \).

(d) Find \( P(0.1 \leq X \leq 1.2) \).

(e) Determine the mean value of \( X \).
2. Let,

\[
f(x) = \begin{cases} 
\frac{4}{\ln(3)(4x + x^2)} & \text{if } 1 \leq x \leq 6 \\
0 & \text{otherwise}
\end{cases}
\]

(a) Show that \( f(x) \) is a probability density function.
(b) Find \( P(X \leq 1) \).
(c) Find \( P(X \geq 5) \).
(d) Find \( P(1 \leq X \leq 5) \).
(e) Determine the mean value of \( X \).

3. Let,

\[
f(x) = \begin{cases} 
\frac{1}{10} \left(1 + \sin\left(\pi x - \frac{\pi}{2}\right)\right) & \text{if } 0 \leq x \leq 10 \\
0 & \text{otherwise}
\end{cases}
\]

(a) Show that \( f(x) \) is a probability density function.
(b) Find \( P(X \leq 3) \).
(c) Find \( P(X \geq 5) \).
(d) Find \( P(2.5 \leq X \leq 7) \).
(e) Determine the mean value of \( X \).

4. The probability density function of the life span of a battery is given by the function below, where \( t \) is in years.

\[
f(t) = \begin{cases} 
1.25e^{-0.8t} & \text{if } t \geq 0 \\
0 & \text{if } t < 0
\end{cases}
\]

(a) Verify that \( f(t) \) is a probability density function.
(b) What is the probability that a battery will have a life span less than 10 months?
(c) What is the probability that a battery will have a life span more than 2 years?
(d) What is the probability that a battery will have a life span between 1.5 and 4 years?
(e) Determine the mean value of the life span of the batteries.

5. The probability density function of the successful outcome from some experiment is given by the function below, where \( t \) is in minutes.
Calculus II

\[ f(t) = \begin{cases} 
36t e^{-6t} & \text{if } t \geq 0 \\
0 & \text{if } t < 0 
\end{cases} \]

(a) Verify that \( f(t) \) is a probability density function.
(b) What is the probability of a successful outcome happening in less than 12 minutes?
(c) What is the probability of a successful outcome happening after 25 minutes?
(d) What is the probability of a successful outcome happening between 10 and 75 minutes?
(e) What is the mean time of a successful outcome from the experiment?

6. Determine the value of \( c \) for which the function below will be a probability density function.

\[ f(x) = \begin{cases} 
 c \left(12x^4 - x^3\right) & \text{if } 0 \leq x \leq 12 \\
0 & \text{otherwise} 
\end{cases} \]

7. Use the function below for this problem.

\[ f(x) = \begin{cases} 
 c e^{-\frac{x}{a}} & x \geq 0 \\
0 & x < 0 
\end{cases} \]

(a) Determine the value of \( c \) for which this function will be a probability density function.
(b) Using the value of \( c \) found in the first part determine the mean value of the probability density function.