Preface

Here are a set of problems for my Calculus I notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.

Implicit Differentiation

For problems 1 – 6 do each of the following.

(a) Find \( y' \) by solving the equation for \( y \) and differentiating directly.

(b) Find \( y' \) by implicit differentiation.

(c) Check that the derivatives in (a) and (b) are the same.

1. \( x^2 y^9 = 2 \)

2. \( \frac{6x}{y^7} = 4 \)

3. \( 1 = x^4 + 5y^3 \)

4. \( 8x - y^2 = 3 \)

5. \( 4x - 6y^2 = xy^2 \)

6. \( \ln(x y) = x \)

For problems 7 – 21 find \( y' \) by implicit differentiation.

7. \( y^2 - 12x^3 = 8y \)

8. \( 3y^7 + x^{10} = y^2 - 6x^3 + 2 \)

9. \( y^{-3} + 4x^{-1} = 8y^{-1} \)
10. $10x^4 - y^6 = 7y^3 + 4x^{-3}$

11. $\sin(x) + \cos(y) = e^{4y}$

12. $x + \ln(y) = \sec(y)$

13. $y^2 \left( 4 - x^2 \right) = y^7 + 9x$

14. $6x^2 - x^3 y^2 + 4x = 0$

15. $8xy + 2x^4 y^{-3} = x^3$

16. $y^3 \cos(x) \sin(y) = 7x$

17. $e^x \cos(y) + \sin(xy) = 9$

18. $x^2 + \sqrt{x^3 + 2y} = y^2$

19. $\tan(3x + 7y) = 6 - 4x^{-1}$

20. $e^{x^2 + y^2} = e^{x^2 y^2} + 1$

21. $\sin\left( \frac{x}{y} \right) + x^3 = 2 - y^4$

For problems 22 - 24 find the equation of the tangent line at the given point.

22. $3x + y^2 = x^2 - 19$ at $(-4, 3)$

23. $x^2 y = y^2 - 6x$ at $(2, 6)$

24. $2 \sin(x) \cos(y) = 1$ at $\left( \frac{\pi}{4}, -\frac{\pi}{4} \right)$

For problems 25 – 27 determine if the function is increasing, decreasing or not changing at the given point.
25. $x^2 - y^3 = 4y + 9$ at $\left(2,-1\right)$

26. $e^{1-x}e^{y^2} = x^3 + y$ at $\left(1,0\right)$

27. $\sin\left(\pi - x\right) + y^2 \cos\left(x\right) = y$ at $\left(\frac{\pi}{2},1\right)$

For problems 28 - 31 assume that $x = x(t)$, $y = y(t)$ and $z = z(t)$ and differentiate the given equation with respect to $t$.

28. $x^4 - 6z = 3 - y^2$

29. $xy^4 = y^2z^3$

30. $z^7e^{6y} = \left(y^2 - 8x\right)^{10} + z^{-4}$

31. $\cos\left(z^2x^3\right) + \sqrt{y^2 + x^2} = 0$