Preface

Here are my online notes for my Calculus I course that I teach here at Lamar University. Despite the fact that these are my “class notes”, they should be accessible to anyone wanting to learn Calculus I or needing a refresher in some of the early topics in calculus.

I’ve tried to make these notes as self contained as possible and so all the information needed to read through them is either from an Algebra or Trig class or contained in other sections of the notes.

Here are a couple of warnings to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn calculus I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn’t covered in class.

2. Because I want these notes to provide some more examples for you to read through, I don’t always work the same problems in class as those given in the notes. Likewise, even if I do work some of the problems in here I may work fewer problems in class than are presented here.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible when writing these up, but the reality is that I can’t anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I’ve not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.
Proof of Various Derivative Facts/Formulas/Properties

In this section we’re going to prove many of the various derivative facts, formulas and/or properties that we encountered in the early part of the Derivatives chapter. Not all of them will be proved here and some will only be proved for special cases, but at least you’ll see that some of them aren’t just pulled out of the air.

**Theorem, from Definition of Derivative**

If \( f(x) \) is differentiable at \( x = a \) then \( f(x) \) is continuous at \( x = a \).

**Proof**

Because \( f(x) \) is differentiable at \( x = a \) we know that

\[
  f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}
\]

exists. We’ll need this in a bit.

If we next assume that \( x \neq a \) we can write the following,

\[
  f(x) - f(a) = \frac{f(x) - f(a)}{x - a} (x - a)
\]

Then basic properties of limits tells us that we have,

\[
  \lim_{x \to a} (f(x) - f(a)) = \lim_{x \to a} \left[ \frac{f(x) - f(a)}{x - a} (x - a) \right]
  = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \lim_{x \to a} (x - a)
\]

The first limit on the right is just \( f'(a) \) as we noted above and the second limit is clearly zero and so,

\[
  \lim_{x \to a} (f(x) - f(a)) = f'(a) \cdot 0 = 0
\]

Okay, we’ve managed to prove that \( \lim_{x \to a} (f(x) - f(a)) = 0 \). But just how does this help us to prove that \( f(x) \) is continuous at \( x = a \)?

Let’s start with the following.

\[
  \lim_{x \to a} f(x) = \lim_{x \to a} \left[ f(x) + f(a) - f(a) \right]
\]
Note that we’ve just added in zero on the right side. A little rewriting and the use of limit properties gives,

\[
\lim_{x \to a} f(x) = \lim_{x \to a} \left[ f(a) + f(x) - f(a) \right] = \lim_{x \to a} f(a) + \lim_{x \to a} \left[ f(x) - f(a) \right]
\]

Now, we just proved above that \( \lim_{x \to a} (f(x) - f(a)) = 0 \) and because \( f(a) \) is a constant we also know that \( \lim_{x \to a} f(a) = f(a) \) and so this becomes,

\[
\lim_{x \to a} f(x) = \lim_{x \to a} f(a) + 0 = f(a)
\]

Or, in other words, \( \lim_{x \to a} f(x) = f(a) \) but this is exactly what it means for \( f(x) \) is continuous at \( x = a \) and so we’re done.

---

**Proof of Sum/Difference of Two Functions**:

\[
(f(x) \pm g(x))' = f'(x) \pm g'(x)
\]

This is easy enough to prove using the definition of the derivative. We’ll start with the sum of two functions. First plug the sum into the definition of the derivative and rewrite the numerator a little.

\[
(f(x) + g(x))' = \lim_{h \to 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h} = \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}
\]

Now, break up the fraction into two pieces and recall that the limit of a sum is the sum of the limits. Using this fact we see that we end up with the definition of the derivative for each of the two functions.

\[
(f(x) + g(x))' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = f'(x) + g'(x)
\]

The proof of the difference of two functions in nearly identical so we’ll give it here without any explanation.
\[(f(x) - g(x))' = \lim_{h \to 0} \frac{f(x+h) - g(x+h) - (f(x) - g(x))}{h} \]

\[= \lim_{h \to 0} \frac{f(x+h) - f(x) - (g(x+h) - g(x))}{h} \]

\[= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} - \frac{g(x+h) - g(x)}{h} \]

\[= f'(x) - g'(x) \]

**Proof of Constant Times a Function**: \((cf(x))' = cf'(x)\)

This property is very easy to prove using the definition provided you recall that we can factor a constant out of a limit. Here’s the work for this property.

\[(cf(x))' = \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h} = c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = cf'(x)\]

**Proof of the Derivative of a Constant**: \(\frac{d}{dx}(c) = 0\)

This is very easy to prove using the definition of the derivative so define \(f(x) = c\) and the use the definition of the derivative.

\[f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0 = 0\]

**Power Rule**: \(\frac{d}{dx}(x^n) = nx^{n-1}\)

There are actually three proofs that we can give here and we’re going to go through all three here so you can see all of them. However, having said that, for the first two we will need to restrict \(n\) to be a positive integer. At the time that the Power Rule was introduced only enough information
has been given to allow the proof for only integers. So, the first two proofs are really to be read at that point.

The third proof will work for any real number \( n \). However, it does assume that you’ve read most of the Derivatives chapter and so should only be read after you’ve gone through the whole chapter.

**Proof 1**

In this case as noted above we need to assume that \( n \) is a positive integer. We’ll use the definition of the derivative and the Binomial Theorem in this theorem. The Binomial Theorem tells us that,

\[
(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k
\]

\[
= a^n + \binom{n}{1} a^{n-1} b + \frac{n}{2} a^{n-2} b^2 + \frac{n}{3} a^{n-3} b^3 + \cdots + \binom{n}{n-1} a b^{n-1} + \binom{n}{n} b^n
\]

\[
= a^n + na^{n-1} b + \frac{n(n-1)}{2!} a^{n-2} b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3} b^3 + \cdots + n a b^{n-1} + b^n
\]

where,

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}
\]

are called the binomial coefficients and \( n! = n(n-1)(n-2)\cdots(2)(1) \) is the factorial.

So, let’s go through the details of this proof. First, plug \( f(x) = x^n \) into the definition of the derivative and use the Binomial Theorem to expand out the first term.

\[
f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}
\]

\[
= \lim_{h \to 0} \frac{\left(x^n + nx^{n-1}h + \frac{n(n-1)}{2!} x^{n-2}h^2 + \cdots + nxh^{n-1} + h^n\right) - x^n}{h}
\]

Now, notice that we can cancel an \( x^n \) and then each term in the numerator will have an \( h \) in them that can be factored out and then canceled against the \( h \) in the denominator. At this point we can evaluate the limit.

\[
f'(x) = \lim_{h \to 0} \frac{nx^{n-1}h + \frac{n(n-1)}{2!} x^{n-2}h^2 + \cdots + nxh^{n-1} + h^n}{h}
\]

\[
= \lim_{h \to 0} nx^{n-1} + \frac{n(n-1)}{2!} x^{n-2}h + \cdots + nxh^{n-2} + h^{n-1}
\]

\[
= nx^{n-1}
\]
Proof 2

For this proof we’ll again need to restrict \( n \) to be a positive integer. In this case if we define \( f(x) = x^n \) we know from the alternate limit form of the definition of the derivative that the derivative \( f'(a) \) is given by,

\[
f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{x^n - a^n}{x - a}
\]

Now we have the following formula,

\[
x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \cdots + a^{n-1}x^2 + a^{n-2}x + a^{n-1})
\]

You can verify this if you’d like by simply multiplying the two factors together. Also, notice that there are a total of \( n \) terms in the second factor (this will be important in a bit).

If we plug this into the formula for the derivative we see that we can cancel the \( x - a \) and then compute the limit.

\[
f'(a) = \lim_{x \to a} \frac{(x - a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \cdots + a^{n-1}x^2 + a^{n-2}x + a^{n-1})}{x - a}
\]

\[
= \lim_{x \to a} x^{n-1} + ax^{n-2} + a^2x^{n-3} + \cdots + a^{n-1}x^2 + a^{n-2}x + a^{n-1}
\]

\[
= a^{n-1} + a^1a^{n-2} + a^2a^{n-3} + \cdots + a^{n-1}a^2 + a^{n-2}a + a^{n-1}
\]

After combining the exponents in each term we can see that we get the same term. So, then recalling that there are \( n \) terms in second factor we can see that we get what we claimed it would be.

To completely finish this off we simply replace the \( a \) with an \( x \) to get,

\[
f'(x) = nx^{n-1}
\]

Proof 3

In this proof we no longer need to restrict \( n \) to be a positive integer. It can now be any real number. However, this proof also assumes that you’ve read all the way through the Derivative chapter. In particular it needs both Implicit Differentiation and Logarithmic Differentiation. If you’ve not read, and understand, these sections then this proof will not make any sense to you.

So, to get set up for logarithmic differentiation let’s first define \( y = x^n \) then take the log of both sides, simplify the right side using logarithm properties and then differentiate using implicit differentiation.
\[
\ln y = \ln x^n \\
\ln y = n \ln x \\
y' = n \frac{1}{x}
\]

Finally, all we need to do is solve for \( y' \) and then substitute in for \( y \).

\[
y' = \frac{y}{x} = x^n \left( \frac{n}{x} \right) = nx^{n-1}
\]

Before moving onto the next proof, let’s notice that in all three proofs we did require that the exponent, \( n \), be a number (integer in the first two, any real number in the third). In the first proof we couldn’t have used the Binomial Theorem if the exponent wasn’t a positive integer. In the second proof we couldn’t have factored \( x^n - a^n \) if the exponent hadn’t been a positive integer. Finally, in the third proof we would have gotten a much different derivative if \( n \) had not been a constant.

This is important because people will often misuse the power rule and use it even when the exponent is not a number and/or the base is not a variable.

\[
\quad
\]

**Product Rule**: \( (fg)' = f'g + fg' \)

As with the Power Rule above, the Product Rule can be proved either by using the definition of the derivative or it can be proved using Logarithmic Differentiation. We’ll show both proofs here.

**Proof 1**

This proof can be a little tricky when you first see it so let’s be a little careful here. We’ll first use the definition of the derivative on the product.

\[
(fg)' = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}
\]

On the surface this appears to do nothing for us. We’ll first need to manipulate things a little to get the proof going. What we’ll do is subtract out and add in \( f(x+h)g(x) \) to the numerator. Note that we’re really just adding in a zero here since these two terms will cancel. This will give us,
\[(fg)' = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}\]

Notice that we added the two terms into the middle of the numerator. As written we can break up the limit into two pieces. From the first piece we can factor a \(f(x+h)\) out and we can factor a \(g(x)\) out of the second piece. Doing this gives,

\[(fg)' = \lim_{h \to 0} \frac{f(x+h)(g(x+h) - g(x))}{h} + \lim_{h \to 0} \frac{g(x)(f(x+h) - f(x))}{h}\]

At this point we can use limit properties to write,

\[(fg)' = \left(\lim_{h \to 0} f(x+h)\right) \left(\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}\right) + \left(\lim_{h \to 0} g(x)\right) \left(\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\right)\]

The individual limits in here are,

\[\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = g'(x)\]
\[\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)\]
\[\lim_{h \to 0} g(x) = g(x)\]
\[\lim_{h \to 0} f(x+h) = f(x)\]

The two limits on the left are nothing more than the definition the derivative for \(g(x)\) and \(f(x)\) respectively. The upper limit on the right seems a little tricky, but remember that the limit of a constant is just the constant. In this case since the limit is only concerned with allowing \(h\) to go to zero. The key here is to recognize that changing \(h\) will not change \(x\) and so as far as this limit is concerned \(g(x)\) is a constant. Note that the function is probably not a constant, however as far as the limit is concerned the function can be treated as a constant. We get the lower limit on the right we get simply by plugging \(h = 0\) into the function

Plugging all these into the last step gives us,

\[(fg)' = f(x)g'(x) + g(x)f'(x)\]
**Proof 2**

This is a much quicker proof but does presuppose that you’ve read and understood the _Implicit Differentiation_ and _Logarithmic Differentiation_ sections. If you haven’t then this proof will not make a lot of sense to you.

First write call the product $y$ and take the log of both sides and use a property of logarithms on the right side.

\[
y = f(x)g(x)
\]
\[
\ln(y) = \ln(f(x)g(x)) = \ln(f(x)) + \ln(g(x))
\]

Next, we take the derivative of both sides and solve for $y'$.

\[
\frac{y'}{y} = \frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)} \quad \Rightarrow \quad y' = y \left( \frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)} \right)
\]

Finally, all we need to do is plug in for $y$ and then multiply this through the parenthesis and we get the Product Rule.

\[
y = f(x)g(x) \left( \frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)} \right) \quad \Rightarrow \quad (fg)' = g(x)f'(x) + f(x)g'(x)
\]

\[\blacksquare\]

**Quotient Rule**: \[
\left( \frac{f}{g} \right)' = \frac{f'g - fg'}{g^2}
\]

Again, we can do this using the definition of the derivative or with Logarithmic Definition.

**Proof 1**

First plug the quotient into the definition of the derivative and rewrite the quotient a little.

\[
\left( \frac{f}{g} \right)' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{g(x+h) - g(x)}
\]
\[
= \lim_{h \to 0} \frac{1}{h} \frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)}
\]

To make our life a little easier we moved the $h$ in the denominator of the first step out to the front as a $\frac{1}{h}$. We also wrote the numerator as a single rational expression. This step is required to make this proof work.
Now, for the next step will need to subtract out and add in $f(x)g(x)$ to the numerator.

$$
\left( \frac{f}{g} \right)' = \lim_{h \to 0} \frac{1}{g(x+h)g(x)} \left( \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h} \right)
$$

The next step is to rewrite things a little,

$$
\left( \frac{f}{g} \right)' = \lim_{h \to 0} \frac{1}{g(x+h)g(x)} \left( \frac{f(x+h)g(x) - f(x)g(x)}{h} \right) + \frac{f(x)g(x) - f(x)g(x+h)}{h}
$$

Note that all we did was interchange the two denominators. Since we are multiplying the fractions we can do this.

Next, the larger fraction can be broken up as follows.

$$
\left( \frac{f}{g} \right)' = \lim_{h \to 0} \frac{1}{g(x+h)g(x)} \left( \frac{f(x+h)g(x) - f(x)g(x)}{h} \right) - \frac{f(x)g(x) - f(x)g(x+h)}{h}
$$

In the first fraction we will factor a $g(x)$ out and in the second we will factor a $-f(x)$ out.

This gives,

$$
\left( \frac{f}{g} \right)' = \lim_{h \to 0} \frac{1}{g(x+h)g(x)} \left( g(x) \frac{f(x+h) - f(x)}{h} - f(x) \frac{g(x+h) - g(x)}{h} \right)
$$

We can now use the basic properties of limits to write this as,

$$
\left( \frac{f}{g} \right)' = \lim_{h \to 0} \frac{1}{g(x+h)g(x)} \left( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right) - \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \left( \lim_{h \to 0} f(x) \right)
$$

The individual limits are,

$$
\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = g'(x) \quad \lim_{h \to 0} g(x+h) = g(x) \quad \lim_{h \to 0} g(x) = g(x)
$$

$$
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x) \quad \lim_{h \to 0} f(x) = f(x)
$$

The first two limits in each row are nothing more than the definition the derivative for $g(x)$ and
Plugging in the limits and doing some rearranging gives,

$$\left( \frac{f}{g} \right)' = -\frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

There's the quotient rule.

Proof 2

Now let's do the proof using Logarithmic Differentiation. We'll first call the quotient \( y \), take the log of both sides and use a property of logs on the right side.

$$y = \frac{f(x)}{g(x)}$$

$$\ln y = \ln \left( \frac{f(x)}{g(x)} \right) = \ln f(x) - \ln g(x)$$

Next, we take the derivative of both sides and solve for \( y' \).

$$\frac{y'}{y} = \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)} \Rightarrow y' = y \left( \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)} \right)$$

Next, plug in \( y \) and do some simplification to get the quotient rule.

$$y' = \frac{f(x)}{g(x)} \left( \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)} \right) = \frac{f'(x)}{g(x)} - \frac{g'(x)f(x)}{(g(x))^2} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

\[\blacksquare\]
Chain Rule

If $f(x)$ and $g(x)$ are both differentiable functions and we define $F(x) = (f \circ g)(x)$ then the derivative of $F(x)$ is $F'(x) = f'(g(x)) \cdot g'(x)$.

Proof

We’ll start off the proof by defining $u = g(x)$ and noticing that in terms of this definition what we’re being asked to prove is,

$$\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}$$

Let’s take a look at the derivative of $u(x)$ (again, remember we’ve defined $u = g(x)$ and so $u$ really is a function of $x$) which we know exists because we are assuming that $g(x)$ is differentiable. By definition we have,

$$u'(x) = \lim_{h \to 0} \frac{u(x+h) - u(x)}{h}$$

Note as well that,

$$\lim_{h \to 0} \left( \frac{u(x+h) - u(x)}{h} - u'(x) \right) = \lim_{h \to 0} \frac{u(x+h) - u(x)}{h} - \lim_{h \to 0} u'(x) = u'(x) - u'(x) = 0$$

Now, define,

$$v(h) = \begin{cases} \frac{u(x+h) - u(x)}{h} - u'(x) & \text{if } h \neq 0 \\ 0 & \text{if } h = 0 \end{cases}$$

and notice that $\lim_{h \to 0} v(h) = 0 = v(0)$ and so $v(h)$ is continuous at $h = 0$

Now if we assume that $h \neq 0$ we can rewrite the definition of $v(h)$ to get,

$$u(x+h) = u(x) + h(v(h)+u'(x)) \quad (1)$$

Now, notice that (1) is in fact valid even if we let $h = 0$ and so is valid for any value of $h$.

Next, since we also know that $f(x)$ is differentiable we can do something similar. However, we’re going to use a different set of letters/variables here for reasons that will be apparent in a bit.
So, define,

\[
  w(k) = \begin{cases} 
  \frac{f(z + k) - f(z)}{k} - f'(z) & \text{if } k \neq 0 \\ 
  0 & \text{if } k = 0 
  \end{cases}
\]

we can go through a similar argument that we did above so show that \( w(k) \) is continuous at \( k = 0 \) and that,

\[
  f(z + k) = f(z) + k(w(k) + f'(z))
\]  

Do not get excited about the different letters here all we did was use \( k \) instead of \( h \) and let \( x = z \). Nothing fancy here, but the change of letters will be useful down the road.

Okay, to this point it doesn’t look like we’ve really done anything that gets us even close to proving the chain rule. The work above will turn out to be very important in our proof however so let’s get going on the proof.

What we need to do here is use the definition of the derivative and evaluate the following limit.

\[
  \lim_{h \to 0} \frac{f[u(x + h)] - f[u(x)\]}{h}
\]  

Note that even though the notation is more than a little messy if we use \( u(x) \) instead of \( u \) we need to remind ourselves here that \( u \) really is a function of \( x \).

Let’s now use (1) to rewrite the \( u(x + h) \) and yes the notation is going to be unpleasant but we’re going to have to deal with it. By using (1), the numerator in the limit above becomes,

\[
  f[u(x + h)] - f[u(x)] = f[u(x) + h(v(h) + u'(x))] - f[u(x)]
\]

If we then define \( z = u(x) \) and \( k = h(v(h) + u'(x)) \) we can use (2) to further write this as,

\[
  f[u(x + h)] - f[u(x)] = f[u(x) + h(v(h) + u'(x))] - f[u(x)]
  = f[u(x)] + h(v(h) + u'(x))(w(k) + f'[u(x)]) - f[u(x)]
  = h(v(h) + u'(x))(w(k) + f'[u(x)])
\]

Notice that we were able to cancel a \( f[u(x)] \) to simplify things up a little. Also, note that the \( w(k) \) was intentionally left that way to keep the mess to a minimum here, just remember that \( k = h(v(h) + u'(x)) \) here as that will be important here in a bit. Let’s now go back and remember that all this was the numerator of our limit, (3). Plugging this into (3) gives,
\[
\frac{d}{dx} \left[ f(u(x)) \right] = \lim_{h \to 0} \frac{h(v(h) + u'(x))(w(k) + f'(u(x)))}{h}
= \lim_{h \to 0} \left( v(h) + u'(x) \right) \left( w(k) + f'(u(x)) \right)
\]

Notice that the $h$'s canceled out. Next, recall that $k = h(v(h) + u'(x))$ and so,
\[
\lim k = \lim_{h \to 0} h(v(h) + u'(x)) = 0
\]
But, if $\lim k = 0$, as we've defined $k$ anyway, then by the definition of $w$ and the fact that we know $w(k)$ is continuous at $k = 0$ we also know that,
\[
\lim_{h \to 0} w(k) = w\left( \lim_{h \to 0} k \right) = w(0) = 0
\]

Also, recall that $\lim_{h \to 0} v(h) = 0$. Using all of these facts our limit becomes,
\[
\frac{d}{dx} \left[ f(u(x)) \right] = \lim_{h \to 0} \left( v(h) + u'(x) \right) \left( w(k) + f'(u(x)) \right)
= u'(x) f'(u(x))
= f'(u(x)) \frac{du}{dx}
\]

This is exactly what we needed to prove and so we're done.