Preface

Here are the solutions to the practice problems for my Calculus II notes. Some solutions will have more or less detail than other solutions. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Approximating Definite Integrals

1. Using \( n = 6 \) approximate the value of \( \int_{1}^{7} \frac{1}{x^3 + 1} \, dx \) using
   
   (a) the Midpoint Rule,
   (b) the Trapezoid Rule, and
   (c) Simpson’s Rule

Use at least 6 decimal places of accuracy for your work.

(a) Midpoint Rule
While it’s not really needed to do the problem here is a sketch of the graph.
We know that we need to divide the interval $[1, 7]$ into 6 subintervals each with width,

$$\Delta x = \frac{7-1}{6} = 1$$

The endpoints of each of these subintervals are represented by the dots on the $x$ axis on the graph above.

The tick marks between each dot represents the midpoint of each of the subintervals. The $x$-values of the midpoints for each of the subintervals are then,

$$\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{13}{2}$$

So, to use the Midpoint Rule to approximate the value of the integral all we need to do is plug into the formula. Doing this gives,

$$\int_1^7 \frac{1}{x^3+1} \, dx \approx \left(1\right) \left[ f\left(\frac{3}{2}\right) + f\left(\frac{5}{2}\right) + f\left(\frac{7}{2}\right) + f\left(\frac{9}{2}\right) + f\left(\frac{11}{2}\right) + f\left(\frac{13}{2}\right) \right]$$

$$= 0.33197137$$

(b) Trapezoid Rule

From the Midpoint Rule work we know that the width of each subinterval is $\Delta x = 1$ and for reference purposes the sketch of the graph along with the endpoints of each subinterval marked by the dots is shown below.

So, to use the Trapezoid Rule to approximate the value of the integral all we need to do is plug into the formula. Doing this gives,

$$\int_1^7 \frac{1}{x^3+1} \, dx \approx \left(\frac{1}{2}\right) \left[ f\left(1\right) + 2f\left(2\right) + 2f\left(3\right) + 2f\left(4\right) + 2f\left(5\right) + 2f\left(6\right) + f\left(7\right) \right]$$

$$= 0.42620830$$

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(b) Simpson’s Rule
From the Midpoint Rule work we know that the width of each subinterval is $\Delta x = 1$ and for reference purposes the sketch of the graph along with the endpoints of each subinterval marked by the dots is shown below.

As with the first two parts all we need to do is plug into the formula to use Simpson’s Rule to approximate value of the integral. Doing this gives,

$$
\int_1^7 \frac{1}{x^3+1} dx \approx \left(\frac{1}{3}\right) \left[ f(1) + 4f(2) + 2f(3) + 4f(4) + 2f(5) + 4f(6) + f(7) \right] = 0.37154155
$$

2. Using $n = 6$ approximate the value of $\int_{-1}^2 \sqrt{e^{-x^2} + 1} \, dx$ using
   (a) the Midpoint Rule,
   (b) the Trapezoid Rule, and
   (c) Simpson’s Rule

Use at least 6 decimal places of accuracy for your work.

(a) Midpoint Rule
While it’s not really needed to do the problem here is a sketch of the graph.
We know that we need to divide the interval $[-1, 2]$ into 6 subintervals each with width,

$$\Delta x = \frac{2 - (-1)}{6} = \frac{1}{2}$$

The endpoints of each of these subintervals are represented by the dots on the $x$ axis on the graph above.

The tick marks between each dot represents the midpoint of each of the subintervals. The $x$-values of the midpoints for each of the subintervals are then,

$$-\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}$$

So, to use the Midpoint Rule to approximate the value of the integral all we need to do is plug into the formula. Doing this gives,

$$\int_{-1}^{2} e^{-x^2} + 1 \, dx \approx \left(\frac{1}{2}\right)\left[ f\left(-\frac{3}{4}\right) + f\left(-\frac{1}{4}\right) + f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right) \right]$$

$$= 3.70700857$$

(b) Trapezoid Rule
From the Midpoint Rule work we know that the width of each subinterval is $\Delta x = \frac{1}{2}$ and for reference purposes the sketch of the graph along with the endpoints of each subinterval marked by the dots is shown below.
So, to use the Trapezoid Rule to approximate the value of the integral all we need to do is plug into the formula. Doing this gives,

\[
\int_{-1}^{2} e^{-x^2} + 1 \, dx \approx \left( \frac{1}{2} \right) \left[ f(-1) + 2f\left( \frac{-1}{2} \right) + 2f(0) + 2f\left( \frac{1}{2} \right) + 2f(1) + 2f\left( \frac{3}{2} \right) + f(2) \right]
\]

\[= 3.69596543\]

(b) Simpson’s Rule

From the Midpoint Rule work we know that the width of each subinterval is \( \Delta x = \frac{1}{2} \) and for reference purposes the sketch of the graph along with the endpoints of each subinterval marked by the dots is shown below.

As with the first two parts all we need to do is plug into the formula to use Simpson’s Rule to approximate value of the integral. Doing this gives,
3. Using \( n = 8 \) approximate the value of \( \int_0^4 \cos(1+\sqrt{x}) \, dx \) using

(a) the Midpoint Rule,
(b) the Trapezoid Rule, and
(c) Simpson’s Rule

Use at least 6 decimal places of accuracy for your work.

(a) Midpoint Rule
While it’s not really needed to do the problem here is a sketch of the graph.

We know that we need to divide the interval \([0, 4]\) into 8 subintervals each with width,

\[ \Delta x = \frac{4 - 0}{8} = \frac{1}{2} \]

The endpoints of each of these subintervals are represented by the dots on the \(x\) axis on the graph above.

The tick marks between each dot represents the midpoint of each of the subintervals. The \(x\)-values of the midpoints for each of the subintervals are then,

\[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}, \frac{13}{4}, \frac{15}{4} \]

So, to use the Midpoint Rule to approximate the value of the integral all we need to do is plug into the formula. Doing this gives,
\[
\int_0^4 \cos(1+\sqrt{x}) \, dx \approx \left( \frac{1}{2} \right) \left[ f\left( \frac{1}{4} \right) + f\left( \frac{3}{4} \right) + f\left( \frac{5}{4} \right) + f\left( \frac{7}{4} \right) + f\left( \frac{9}{4} \right) + f\left( \frac{11}{4} \right) + f\left( \frac{13}{4} \right) + f\left( \frac{15}{4} \right) \right]
\]
\[
= -2.51625938
\]

(b) Trapezoid Rule
From the Midpoint Rule work we know that the width of each subinterval is \( \Delta x = \frac{1}{2} \) and for reference purposes the sketch of the graph along with the endpoints of each subinterval marked by the dots is shown below.

So, to use the Trapezoid Rule to approximate the value of the integral all we need to do is plug into the formula. Doing this gives,
\[
\int_0^4 \cos(1+\sqrt{x}) \, dx \approx \left( \frac{\frac{1}{2}}{2} \right) \left[ f\left( \frac{0}{2} \right) + 2f\left( \frac{1}{2} \right) + 2f\left( \frac{3}{2} \right) + 2f\left( 2 \right) + 2f\left( \frac{5}{2} \right) + 2f\left( 3 \right) + 2f\left( \frac{7}{2} \right) + f\left( 4 \right) \right]
\]
\[
= -2.43000475
\]

(b) Simpson’s Rule
From the Midpoint Rule work we know that the width of each subinterval is \( \Delta x = \frac{1}{2} \) and for reference purposes the sketch of the graph along with the endpoints of each subinterval marked by the dots is shown below.
As with the first two parts all we need to do is plug into the formula to use Simpson’s Rule to approximate value of the integral. Doing this gives,

\[ \int_{0}^{4} \cos(1 + \sqrt{x}) \, dx \approx \left( \frac{1}{3} \right) \left[ f(0) + 4f\left( \frac{1}{2} \right) + 2f\left( \frac{1}{2} \right) + 4f\left( \frac{3}{2} \right) + 2f(2) + 4f\left( \frac{5}{2} \right) + 2f(3) + 4f\left( \frac{7}{2} \right) + f(4) \right] \]

\[ = -2.47160136 \]