Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Zeroes/Roots of Polynomials

1. List all of the zeros of the following polynomial and give their multiplicities.

\[ f(x) = 2x^2 + 13x - 7 \]

Step 1
For this problem we’ll first need to factor the polynomial.

\[ f(x) = 2x^2 + 13x - 7 = (2x - 1)(x + 7) \]

From this we see that we have the two zeroes/roots:

\[ x = \frac{1}{2} \text{ and } x = -7. \]

Step 2
For the multiplicities just remember that the multiplicity of the zero/root is simply the exponent on the term that produces the zero/root. Therefore the multiplicities of each zero/root is,

\[ x = \frac{1}{2} : \text{ multiplicity } 1 \]
\[ x = -7 : \text{ multiplicity } 1 \]

2. List all of the zeros of the following polynomial and give their multiplicities.

\[ g(x) = x^6 - 3x^5 - 6x^4 + 10x^3 + 21x^2 + 9x = x(x - 3)^2(x + 1)^3 \]

Step 1
For this problem the polynomial has already been factored and so all we need to do is get the zeroes/roots from the factored form.

The zeroes/roots of this polynomial are:

\[ x = 0, \quad x = 3 \quad \text{and} \quad x = -1. \]

Step 2
For the multiplicities just remember that the multiplicity of the zero/root is simply the exponent on the term that produces the zero/root. Therefore the multiplicities of each zero/root is,

\[ x = 0 : \text{ multiplicity } 1 \]
\[ x = 3 : \text{ multiplicity } 2 \]
\[ x = -1 : \text{ multiplicity } 3 \]

3. List all of the zeros of the following polynomial and give their multiplicities.
\[ A(x) = x^8 + 2x^7 - 29x^6 - 76x^5 + 199x^4 + 722x^3 + 261x^2 - 648x - 432 \]
\[ = (x+1)^2 (x-4)^2 (x-1)(x+3)^3 \]

Step 1
For this problem the polynomial has already been factored and so all we need to do is get the zeroes/roots from the factored form.

The zeroes/roots of this polynomial are: \( x = -1 \), \( x = 4 \), \( x = 1 \) and \( x = -3 \).

Step 2
For the multiplicities just remember that the multiplicity of the zero/root is simply the exponent on the term that produces the zero/root. Therefore the multiplicities of each zero/root is,

\[
\begin{align*}
x = -1: & \text{ multiplicity 2} \\
x = 4: & \text{ multiplicity 2} \\
x = 1: & \text{ multiplicity 1} \\
x = -3: & \text{ multiplicity 3}
\end{align*}
\]

4. \( x = r \) is a root of the following polynomial. Find the other two roots and write the polynomial in fully factored form.

\[ P(x) = x^3 - 6x^2 - 16x \ ; \ r = -2 \]

Step 1
We know that \( x = -2 \) is a root of the polynomial and so we know that we can write the polynomial as,

\[ P(x) = (x + 2)Q(x) \]

Step 2
To find \( Q(x) \) all we need to do is a quick synthetic division.

\[
\begin{array}{c|cccc}
-2 & 1 & -6 & -16 & 0 \\
& & -2 & 16 & 0 \\
\hline
1 & -8 & 0 & 0 & 0
\end{array}
\]

From this we see that,

\[ Q(x) = x^2 - 8x \]

Step 3
We can now write down \( P(x) \) and it is simple enough to factor \( Q(x) \).
Step 4

Finally, from the factored form of \( P(x) \) in the previous step we can see that the full list of roots/zeroes are:

\[
x = 0 \quad x = -2 \quad x = 8
\]

5. \( x = r \) is a root of the following polynomial. Find the other two roots and write the polynomial in fully factored form.

\[ P(x) = x^3 - 7x^2 - 6x + 72 \ ; \ r = 4 \]

Step 1

We know that \( x = 4 \) is a root of the polynomial and so we know that we can write the polynomial as,

\[ P(x) = (x - 4)Q(x) \]

Step 2

To find \( Q(x) \) all we need to do is a quick synthetic division.

\[
\begin{array}{c|ccccc}
4 & 1 & -7 & -6 & 72 \\
& 4 & -12 & -72 \\
\hline
& 1 & -3 & -18 & 0 \\
\end{array}
\]

From this we see that,

\[ Q(x) = x^2 - 3x - 18 \]

Step 3

We can now write down \( P(x) \) and it is simple enough to factor \( Q(x) \).

\[ P(x) = (x - 4)(x^2 - 3x - 18) = (x - 4)(x - 6)(x + 3) \]

Step 4

Finally, from the factored form of \( P(x) \) in the previous step we can see that the full list of roots/zeroes are:

\[
x = 4 \quad x = 6 \quad x = -3
\]
6. $x = r$ is a root of the following polynomial. Find the other two roots and write the polynomial in fully factored form.

$$P(x) = 3x^3 + 16x^2 - 33x + 14 ; \quad r = -7$$

**Step 1**
We know that $x = -7$ is a root of the polynomial and so we know that we can write the polynomial as,

$$P(x) = (x + 7)Q(x)$$

**Step 2**
To find $Q(x)$ all we need to do is a quick synthetic division.

\[
\begin{array}{c|cccc}
-7 & 3 & 16 & -33 & 14 \\
 & & -21 & 35 & -14 \\
 & 3 & -5 & 2 & 0 \\
\end{array}
\]

From this we see that,

$$Q(x) = 3x^2 - 5x + 2$$

**Step 3**
We can now write down $P(x)$ and it is simple enough to factor $Q(x)$.

$$P(x) = (x + 7)(3x^2 - 5x + 2) = (x + 7)(3x - 2)(x - 1)$$

**Step 4**
Finally, from the factored form of $P(x)$ in the previous step we can see that the full list of roots/zeroes are,

$$x = -7 \quad x = \frac{2}{3} \quad x = 1$$