Here are the solutions to the practice problems for my Calculus II notes. Some solutions will have more or less detail than other solutions. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

**Tangents with Polar Coordinates**

1. Find the tangent line to \( r = \sin(4\theta)\cos(\theta) \) at \( \theta = \frac{\pi}{6} \).

   **Step 1**
   First, we’ll need to following derivative,
   \[
   \frac{dr}{d\theta} = 4\cos(4\theta)\cos(\theta) - \sin(4\theta)\sin(\theta)
   \]

   **Step 2**
   Next using the formula from the notes on this section we have,
   \[
   \frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}
   \]
   \[
   = \frac{(4\cos(4\theta)\cos(\theta) - \sin(4\theta)\sin(\theta))\sin \theta + (\sin(4\theta)\cos(\theta))\cos \theta}{(4\cos(4\theta)\cos(\theta) - \sin(4\theta)\sin(\theta))\cos \theta - (\sin(4\theta)\cos(\theta))\sin \theta}
   \]

   This is a very messy derivative (these often are) and, at least in this case, there isn’t a lot of simplification that we can do…

   **Step 3**
Next we’ll need to evaluate both the derivative from the previous step as well as \( r \) at \( \theta = \frac{\pi}{6} \).

\[
\left. \frac{dy}{dx} \right|_{\theta=\frac{\pi}{6}} = \frac{1}{3\sqrt{3}} \quad \quad r \bigg|_{\theta=\frac{\pi}{6}} = \frac{3}{4}
\]

You can see why we need both of these right?

**Step 4**
Lastly we need the \( x \) and \( y \) coordinate that we’ll be at when \( \theta = \frac{\pi}{6} \). These values are easy enough to find given that we know what \( r \) is at this point and we also know the polar to Cartesian coordinate conversion formulas. So,

\[
x = r \cos(\theta) = \frac{3}{4} \cos\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8} \quad \quad y = r \sin(\theta) = \frac{3}{4} \sin\left(\frac{\pi}{6}\right) = \frac{3}{8}
\]

Of course we also have the slope of the tangent line since it is just the value of the derivative we computed in the previous step.

**Step 5**
The tangent line is then,

\[
y = \frac{3}{8} + \frac{1}{3\sqrt{3}} \left( x - \frac{3\sqrt{3}}{8} \right) = \frac{1}{3\sqrt{3}} x + \frac{1}{4}
\]

2. Find the tangent line to \( r = \theta - \cos(\theta) \) at \( \theta = \frac{3\pi}{4} \).

**Step 1**
First, we’ll need to following derivative,

\[
\frac{dr}{d\theta} = 1 + \sin(\theta)
\]

**Step 2**
Next using the formula from the notes on this section we have,

\[
\frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta} = \frac{(1+\sin(\theta))\sin \theta + (\theta - \cos(\theta))\cos \theta}{(1+\sin(\theta))\cos \theta - (\theta - \cos(\theta))\sin \theta}
\]
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This is a somewhat messy derivative (these often are) and, at least in this case, there isn’t a lot of simplification that we can do…

Step 3
Next we’ll need to evaluate both the derivative from the previous step as well as \( r \) at \( \theta = \frac{3\pi}{4} \).

\[
\left. \frac{dy}{dx} \right|_{\theta = \frac{3\pi}{4}} = 0.2843 \quad r\left|_{\theta = \frac{3\pi}{4}} \right. = 3.0633
\]

You can see why we need both of these right?

Step 4
Lastly we need the \( x \) and \( y \) coordinate that we’ll be at when \( \theta = \frac{3\pi}{4} \). These values are easy enough to find given that we know what \( r \) is at this point and we also know the polar to Cartesian coordinate conversion formulas. So,

\[
x = r \cos(\theta) = 3.0633 \cos\left(\frac{3\pi}{4}\right) = -2.1661 \quad y = r \sin(\theta) = 3.0633 \sin\left(\frac{3\pi}{4}\right) = 2.1661
\]

Of course we also have the slope of the tangent line since it is just the value of the derivative we computed in the previous step.

Step 5
The tangent line is then,

\[
y = 2.1661 + 0.2843 \left( x + 2.1661 \right) = 0.2843x + 2.7819
\]