Preface

Here are the solutions to the practice problems for my Calculus II notes. Some solutions will have more or less detail than other solutions. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Sequences

1. List the first 5 terms of the following sequence.

\[
\left\{ \frac{4n}{n^2 - 7} \right\}_{n=0}^{\infty}
\]

Solution

There really isn’t all that much to this problem. All we need to do is, starting at \( n = 0 \), plug in the first five values of \( n \) into the formula for the sequence terms. Doing that gives,

\[
\begin{align*}
n = 0: & \quad \frac{4(0)}{(0)^2 - 7} = 0 \\
n = 1: & \quad \frac{4(1)}{(1)^2 - 7} = \frac{4}{-6} = -\frac{2}{3} \\
n = 2: & \quad \frac{4(2)}{(2)^2 - 7} = \frac{8}{-3} = -\frac{8}{3} \\
n = 3: & \quad \frac{4(3)}{(3)^2 - 7} = \frac{12}{2} = 6 \\
n = 4: & \quad \frac{4(4)}{(4)^2 - 7} = \frac{16}{9}
\end{align*}
\]

So, the first five terms of the sequence are,
Note that we put the formal answer inside the braces to make sure that we don’t forget that we are dealing with a sequence and we made sure and included the “…” at the end to remind ourselves that there are more terms to this sequence that just the five that we listed out here.

2. List the first 5 terms of the following sequence.

\[
\left\{ \frac{(-1)^{n+1}}{2n + (-3)^n} \right\}_{n=2}^\infty
\]

Solution

There really isn’t all that much to this problem. All we need to do is, starting at \( n = 2 \), plug in the first five values of \( n \) into the formula for the sequence terms. Doing that gives,

\[
\begin{align*}
\text{n = 2: } & \quad \frac{(-1)^{2+1}}{2(2) + (-3)^2} = \frac{-1}{13} = -\frac{1}{13} \\
\text{n = 3: } & \quad \frac{(-1)^{3+1}}{2(3) + (-3)^3} = \frac{1}{-21} = -\frac{1}{21} \\
\text{n = 4: } & \quad \frac{(-1)^{4+1}}{2(4) + (-3)^4} = \frac{-1}{89} = -\frac{1}{89} \\
\text{n = 5: } & \quad \frac{(-1)^{5+1}}{2(5) + (-3)^5} = \frac{1}{-233} = -\frac{1}{233} \\
\text{n = 6: } & \quad \frac{(-1)^{6+1}}{2(6) + (-3)^6} = \frac{-1}{741} = -\frac{1}{741}
\end{align*}
\]

So, the first five terms of the sequence are,

\[
\left\{ \frac{1}{13}, -\frac{1}{21}, -\frac{1}{89}, -\frac{1}{233}, -\frac{1}{741}, \ldots \right\}
\]

Note that we put the formal answer inside the braces to make sure that we don’t forget that we are dealing with a sequence and we made sure and included the “…” at the end to remind ourselves that there are more terms to this sequence that just the five that we listed out here.
3. Determine if the given sequence converges or diverges. If it converges what is its limit?

\[
\left\{ \frac{n^2 - 7n + 3}{1 + 10n - 4n^2} \right\}_{n=3}^\infty
\]

Step 1
To answer this all we need is the following limit of the sequence terms.

\[
\lim_{n \to \infty} \frac{n^2 - 7n + 3}{1 + 10n - 4n^2} = -\frac{1}{4}
\]

You do recall how to take limits at infinity right? If not you should go back into the Calculus I material do some refreshing on limits at infinity as well at L’Hospital’s rule.

Step 2
We can see that the limit of the terms existed and was a finite number and so we know that the sequence converges and its limit is \(-\frac{1}{4}\).

4. Determine if the given sequence converges or diverges. If it converges what is its limit?

\[
\left\{ \frac{(-1)^{n-2} n^2}{4 + n^3} \right\}_{n=0}^\infty
\]

Step 1
To answer this all we need is the following limit of the sequence terms.

\[
\lim_{n \to \infty} \frac{(-1)^{n-2} n^2}{4 + n^3}
\]

However, because of the \((-1)^{n-2}\) we can’t compute this limit using our knowledge of computing limits from Calculus I.

Step 2
Recall however, that we had a nice Fact in the notes from this section that had us computing not the limit above but instead computing the limit of the absolute value of the sequence terms.

\[
\lim_{n \to \infty} \left| \frac{(-1)^{n-2} n^2}{4 + n^3} \right| = \lim_{n \to \infty} \frac{n^2}{4 + n^3} = 0
\]

This is a limit that we can compute because the absolute value got rid of the alternating sign, i.e. the \((-1)^{n-2}\).
Step 3
Now, by the Fact from class we know that because the limit of the absolute value of the sequence terms was zero (and recall that to use that fact the limit MUST be zero!) we also know the following limit.

\[
\lim_{n \to \infty} \frac{(-1)^{n-2} n^2}{4 + n^3} = 0
\]

Step 4
We can see that the limit of the terms existed and was a finite number and so we know that the sequence converges and its limit is zero.

5. Determine if the given sequence converges or diverges. If it converges what is its limit?

\[
\begin{align*}
\left\{ \frac{e^{5n}}{3 - e^{2n}} \right\}_{n=1}^\infty
\end{align*}
\]

Step 1
To answer this all we need is the following limit of the sequence terms.

\[
\lim_{n \to \infty} \frac{e^{5n}}{3 - e^{2n}} = \lim_{n \to \infty} \frac{5e^{5n}}{-2e^{2n}} = \lim_{n \to \infty} \frac{5}{-2} e^{3n} = \infty
\]

You do recall how to use L’Hospital’s rule to compute limits at infinity right? If not you should go back into the Calculus I material do some refreshing.

Step 2
We can see that the limit of the terms existed and was infinite and so we know that the sequence diverges.

6. Determine if the given sequence converges or diverges. If it converges what is its limit?

\[
\begin{align*}
\left\{ \frac{\ln(n+2)}{\ln(1+4n)} \right\}_{n=1}^\infty
\end{align*}
\]

Step 1
To answer this all we need is the following limit of the sequence terms.

\[
\lim_{n \to \infty} \frac{\ln(n+2)}{\ln(1+4n)} = \lim_{n \to \infty} \frac{1}{4} \frac{n+2}{1+4n} = \lim_{n \to \infty} \frac{1+4n}{4(n+2)} = 1
\]
You do recall how to use L’Hospital’s rule to compute limits at infinity right? If not you should go back into the Calculus I material do some refreshing.

Step 2
We can see that the limit of the terms existed and was a finite number and so we know that the sequence converges and its limit is one.