Preface

Here are the solutions to the practice problems for my Calculus II notes. Some solutions will have more or less detail than other solutions. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Series – The Basics

1. Perform an index shift so that the following series starts at \( n = 3 \).

\[
\sum_{n=1}^{\infty} \left( n2^n - 3^{1-n} \right)
\]

Solution

There really isn’t all that much to this problem. Just remember that, in this case, we’ll need to increase the initial value of the index by two so it will start at \( n = 3 \) and this means all the \( n \)'s in the series terms will need to decrease by the same amount (two in this case…).

Doing this gives the following series.

\[
\sum_{n=1}^{\infty} \left( n2^n - 3^{1-n} \right) = \sum_{n=3}^{\infty} \left( (n-2)2^{n-2} - 3^{1-(n-2)} \right) = \sum_{n=3}^{\infty} \left( (n-2)2^{n-2} - 3^{1-n} \right)
\]

Be careful with parenthesis, exponents, coefficients and negative signs when “shifting” the \( n \)'s in the series terms. When replacing \( n \) with \( n - 2 \) make sure to add in parenthesis where needed to preserve coefficients and minus signs.

2. Perform an index shift so that the following series starts at \( n = 3 \).
Solution
There really isn’t all that much to this problem. Just remember that, in this case, we’ll need to decrease the initial value of the index by four so it will start at $n = 3$ and this means all the $n$’s in the series terms will need to increase by the same amount (four in this case…).

Doing this gives the following series.

\[
\sum_{n=7}^{\infty} \frac{4-n}{n^2 + 1} = \sum_{n=3}^{\infty} \frac{4-(n+4)}{(n+4)^2 + 1} = \sum_{n=3}^{\infty} \frac{-n}{(n+4)^2 + 1}
\]

Be careful with parenthesis, exponents, coefficients and negative signs when “shifting” the $n$’s in the series terms. When replacing $n$ with $n + 4$ make sure to add in parenthesis where needed to preserve coefficients and minus signs.

3. Perform an index shift so that the following series starts at $n = 3$.

\[
\sum_{n=2}^{\infty} \frac{(-1)^{n-3} (n+2)}{5^{1+2n}}
\]

Solution
There really isn’t all that much to this problem. Just remember that, in this case, we’ll need to increase the initial value of the index by one so it will start at $n = 3$ and this means all the $n$’s in the series terms will need to decrease by the same amount (one in this case…).

Doing this gives the following series.

\[
\sum_{n=2}^{\infty} \frac{(-1)^{n-3} (n+2)}{5^{1+2n}} = \sum_{n=3}^{\infty} \frac{(-1)^{n-1-3} (n-1+2)}{5^{1+2(n-1)}} = \sum_{n=3}^{\infty} \frac{(-1)^{n-4} (n+1)}{5^{2n-1}}
\]

Be careful with parenthesis, exponents, coefficients and negative signs when “shifting” the $n$’s in the series terms. When replacing $n$ with $n - 1$ make sure to add in parenthesis where needed to preserve coefficients and minus signs.

4. Strip out the first 3 terms from the series $\sum_{n=1}^{\infty} \frac{2^{-n}}{n^2 + 1}$.

Solution
Remember that when we say we are going to “strip out” terms from a series we aren’t really getting rid of them. All we are doing is writing the first few terms of the series as a summation in front of the series.

So, for this series stripping out the first three terms gives,

\[
\sum_{n=1}^{\infty} \frac{2^{-n}}{n^2 + 1} = \frac{2^{-1}}{1^2 + 1} + \frac{2^{-2}}{2^2 + 1} + \frac{2^{-3}}{3^2 + 1} + \sum_{n=4}^{\infty} \frac{2^{-n}}{n^2 + 1} \\
= \frac{1}{4} + \frac{1}{20} + \frac{1}{80} + \sum_{n=4}^{\infty} \frac{2^{-n}}{n^2 + 1} \\
= \frac{5}{16} + \sum_{n=4}^{\infty} \frac{2^{-n}}{n^2 + 1}
\]

This first step isn’t really all that necessary but was included here to make it clear that we were plugging in \( n = 1 \), \( n = 2 \) and \( n = 3 \) (i.e. the first three values of \( n \)) into the general series term. Also, don’t forget to change the starting value of \( n \) to reflect the fact that we’ve “stripped out” the first three values of \( n \) or terms.

5. Given that \( \sum_{n=0}^{\infty} \frac{1}{n^3 + 1} = 1.6865 \) determine the value of \( \sum_{n=2}^{\infty} \frac{1}{n^3 + 1} \).

Step 1
First notice that if we strip out the first two terms from the series that starts at \( n = 0 \) the result will involve a series that starts at \( n = 2 \).

Doing this gives,

\[
\sum_{n=0}^{\infty} \frac{1}{n^3 + 1} = \frac{1}{0^3 + 1} + \sum_{n=2}^{\infty} \frac{1}{n^3 + 1} = \frac{3}{2} + \sum_{n=2}^{\infty} \frac{1}{n^3 + 1}
\]

Step 2
Now, for this situation we are given the value of the series that starts at \( n = 0 \) and are asked to determine the value of the series that starts at \( n = 2 \). To do this all we need to do is plug in the known value of the series that starts at \( n = 0 \) into the “equation” above and “solve” for the value of the series that starts at \( n = 2 \).

This gives,

\[
1.6865 = \frac{3}{2} + \sum_{n=2}^{\infty} \frac{1}{n^3 + 1} \quad \Rightarrow \quad \sum_{n=2}^{\infty} \frac{1}{n^3 + 1} = 1.6865 - \frac{3}{2} = 0.1865
\]