Preface

Here are the solutions to the practice problems for my Calculus II notes. Some solutions will have more or less detail than other solutions. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Ratio Test

1. Determine if the following series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{3^{1-2n}}{n^2 + 1} \]

Step 1

We’ll need to compute \( L \).

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{1}{a_{n+1}} a_n = \lim_{n \to \infty} \frac{3^{1-2(n+1)}}{(n+1)^2 + 1} \frac{n^2 + 1}{3^{1-2n}}
\]

\[
= \lim_{n \to \infty} \left| \frac{3^{1-2n}}{(n+1)^2 + 1} \frac{n^2 + 1}{3^{1-2n}} \right| = \lim_{n \to \infty} \left| \frac{1}{(n+1)^2 + 1} \frac{n^2 + 1}{3} \right| = \lim_{n \to \infty} \left| \frac{n^2 + 1}{9(n+1)^2 + 1} \right| = \frac{1}{9}
\]

When computing \( a_{n+1} \) be careful to pay attention to any coefficients of \( n \) and powers of \( n \). Failure to properly deal with these is one of the biggest mistakes that students make in this computation and mistakes at that level often lead to the wrong answer!

Step 2

Okay, we can see that \( L = \frac{1}{9} < 1 \) and so by the Ratio Test the series converges.
2. Determine if the following series converges or diverges.

\[ \sum_{n=0}^{\infty} \frac{(2n)!}{5n+1} \]

Step 1
We’ll need to compute \( L \).

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{1}{n+1} \right| = \lim_{n \to \infty} \left| \frac{(2(n+1))!}{5(n+1)+1} \right| = \lim_{n \to \infty} \left| \frac{2n+2}{5n+6} \right| = \infty
\]

When computing \( a_{n+1} \) be careful to pay attention to any coefficients of \( n \) and powers of \( n \). Failure to properly deal with these is one of the biggest mistakes that students make in this computation and mistakes at that level often lead to the wrong answer!

Step 2
Okay, we can see that \( L = \infty > 1 \) and so by the Ratio Test the series diverges.

3. Determine if the following series converges or diverges.

\[ \sum_{n=2}^{\infty} \frac{(-2)^{1+3n}(n+1)}{n^2 5^{1+n}} \]

Step 1
We’ll need to compute \( L \).

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{1}{n+1} \right| = \lim_{n \to \infty} \left| \frac{(-2)^{1+3(n+1)}(n+1+1)}{(n+1)^2 5^{1+n+1}} \right| = \lim_{n \to \infty} \left| \frac{(-2)^{1+3n}(n+2)}{(n+1)^2 5^{1+n+1}} \right| = \lim_{n \to \infty} \left| \frac{(-2)^3(n+2)}{(n+1)^2(5)(n+1)} \right| = \frac{8}{5}
\]

When computing \( a_{n+1} \) be careful to pay attention to any coefficients of \( n \) and powers of \( n \). Failure to properly deal with these is one of the biggest mistakes that students make in this computation and mistakes at that level often lead to the wrong answer!
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Step 2
Okay, we can see that \( L = \frac{\frac{8}{5}}{1} > 1 \) and so by the Ratio Test the series diverges.

4. Determine if the following series converges or diverges.

\[ \sum_{n=3}^{\infty} \frac{e^{4n}}{(n-2)!} \]

Step 1
We’ll need to compute \( L \).

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{e^{4(n+1)}}{(n+1-2)!} \cdot \frac{(n-2)!}{e^{4n}} \right| \\
= \lim_{n \to \infty} \left| \frac{e^{4n+4}}{(n-1)!} \cdot \frac{(n-2)!}{e^{4n}} \right| = \lim_{n \to \infty} \left| \frac{e^{4(n+4)}}{(n-1)(n-2)!} \cdot \frac{e^{4n}}{e^{4n}} \right| = \lim_{n \to \infty} \left| \frac{e^4}{n-1} \right| = 0
\]

When computing \( a_{n+1} \) be careful to pay attention to any coefficients of \( n \) and powers of \( n \). Failure to properly deal with these is one of the biggest mistakes that students make in this computation and mistakes at that level often lead to the wrong answer!

Step 2
Okay, we can see that \( L = 0 < 1 \) and so by the Ratio Test the series converges.

5. Determine if the following series converges or diverges.

\[ \sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{6n + 7} \]

Step 1
We’ll need to compute \( L \).

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(-1)^{n+1}}{6(n+1) + 7} \cdot \frac{6n + 7}{(-1)^{n+1}} \right| \\
= \lim_{n \to \infty} \left| \frac{(-1)^{n+2}}{6n + 13} \cdot \frac{6n + 7}{(-1)^{n+1}} \right| = \lim_{n \to \infty} \left| \frac{6n + 7}{6n + 13} \right| = 1
\]
When computing $a_{n+1}$ be careful to pay attention to any coefficients of $n$ and powers of $n$. Failure to properly deal with these is one of the biggest mistakes that students make in this computation and mistakes at that level often lead to the wrong answer!

Step 2
Okay, we can see that $L = 1$ and so by the Ratio Test tells us nothing about this series.

Step 3
Just because the Ratio Test doesn’t tell us anything doesn’t mean we can’t determine if this series converges or diverges.

In fact, it’s actually quite simple to do in this case. This is an Alternating Series with,

$$b_n = \frac{1}{6n+7}$$

The $b_n$ are clearly positive and it should be pretty obvious (hopefully) that they also form a decreasing sequence. Finally we also can see that $\lim_{n \to \infty} b_n = 0$ and so by the Alternating Series Test this series will converge.

Note, that if this series were not in this section doing this as an Alternating Series from the start would probably have been the best way of approaching this problem.