Preface

Here are the solutions to the practice problems for my Calculus II notes. Some solutions will have more or less detail than other solutions. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Dot Product

1. Determine the dot product, \( \vec{a} \cdot \vec{b} \) if \( \vec{a} = 9, 5, -4, 2 \) and \( \vec{b} = -3, -2, 7, -1 \).

Solution
Not really a whole lot to do here. We just need to run through the definition of the dot product.

\[
\vec{a} \cdot \vec{b} = (9)(-3) + (5)(-2) + (-4)(7) + (2)(-1) = -67
\]

2. Determine the dot product, \( \vec{a} \cdot \vec{b} \) if \( \vec{a} = 0, 4, -2 \) and \( \vec{b} = 2\vec{i} - \vec{j} + 7\vec{k} \).

Solution
Not really a whole lot to do here. We just need to run through the definition of the dot product and do not get excited about the “mixed” notation here. We know that they are equivalent notations!

\[
\vec{a} \cdot \vec{b} = (0)(2) + (4)(-1) + (-2)(7) = -18
\]
3. Determine the dot product, $\mathbf{a} \cdot \mathbf{b}$ if $\|\mathbf{a}\| = 5$, $\|\mathbf{b}\| = \frac{3}{7}$ and the angle between the two vectors is $\theta = \frac{\pi}{12}$.

Solution
Not really a whole lot to do here. We just need to run through the formula from the geometric interpretation of the dot product.

\[
\mathbf{a} \cdot \mathbf{b} = (5) \left( \frac{3}{7} \right) \cos \left( \frac{\pi}{12} \right) = 2.0698
\]

4. Determine the angle between $\mathbf{v} = \langle 1, 2, 3, 4 \rangle$ and $\mathbf{w} = \langle 0, -1, 4, -2 \rangle$.

Solution
Not really a whole lot to do here. All we really need to do is rewrite the formula from the geometric interpretation of the dot product as,

\[
\cos \theta = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\| \|\mathbf{w}\|}
\]

This will allow us to quickly determine the angle between the two vectors.

We'll first need the following quantities (we'll leave it to you to verify the arithmetic involved in these computations…).

\[
\mathbf{v} \cdot \mathbf{w} = 2 \quad \|\mathbf{v}\| = \sqrt{30} \quad \|\mathbf{w}\| = \sqrt{21}
\]

The angle between the vectors is then,

\[
\cos \theta = \frac{2}{\sqrt{30} \sqrt{21}} = 0.07968 \quad \Rightarrow \quad \theta = \cos^{-1} (0.07968) = 1.49103 \text{ radians}
\]

5. Determine the angle between $\mathbf{a} = \mathbf{i} + 3 \mathbf{j} - 2 \mathbf{k}$ and $\mathbf{b} = \langle -9, 1, -5 \rangle$.

Solution
Not really a whole lot to do here. All we really need to do is rewrite the formula from the geometric interpretation of the dot product as,
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\[ \cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} \]

This will allow us to quickly determine the angle between the two vectors.

We’ll first need the following quantities (we’ll leave it to you to verify the arithmetic involved in these computations…).

\[ \vec{a} \cdot \vec{b} = 4 \quad \|\vec{v}\| = \sqrt{14} \quad \|\vec{w}\| = \sqrt{107} \]

The angle between the vectors is then,

\[ \cos \theta = \frac{4}{\sqrt{14} \sqrt{107}} = 0.1033 \quad \Rightarrow \quad \theta = \cos^{-1}(0.1034) = 1.4673 \text{ radians} \]

6. Determine if \( \vec{q} = \langle 4, -2, 7 \rangle \) and \( \vec{p} = -3\vec{i} + \vec{j} + 2\vec{k} \) are parallel, orthogonal or neither.

Step 1
Based on a quick inspection of the components we can see that the first and second components of the two vectors have opposite signs and the third doesn’t. This means there is no possible way for these two vectors to be scalar multiples since there is no number that will change the sign on the first two components and leave the sign of the third component unchanged.

Therefore we can quickly see that the two vectors are **not parallel**.

Step 2
Let’s do a quick dot product on the two vectors next.

\[ \vec{q} \cdot \vec{p} = 0 \]

Okay, the dot product is zero and we know from the notes that this in turn means that the two vectors must be **orthogonal**.

On a side note an alternate method for working this problem is to find the angle between the two vectors and using that to determine the answer.

Depending on which method you find easiest either will get you the correct answer.

7. Determine if \( \vec{a} = \langle 3, 10 \rangle \) and \( \vec{b} = \langle 4, -1 \rangle \) are parallel, orthogonal or neither.

Step 1
Based on a quick inspection of the components we can see that the first components of the vectors have
the same sign and the second have opposite signs. This means there is no possible way for these two
vectors to be scalar multiples since there is no number that will change the sign on the second components
and leave the sign of the first component unchanged.

Therefore we can quickly see that the two vectors are not parallel.

Step 2
Let’s do a quick dot product on the two vectors next.

\[ \vec{a} \cdot \vec{b} = 2 \]

Okay, the dot product is not zero and we know from the notes that this in turn means that the two
vectors are not orthogonal.

The answer to the problem is therefore the two vectors are neither parallel or orthogonal.

On a side note an alternate method for working this problem is to find the angle between the two vectors
and using that to determine the answer.

Depending on which method you find easiest either will get you the correct answer.

8. Determine if \( \vec{w} = \vec{i} + 4\vec{j} - 2\vec{k} \) and \( \vec{v} = -3\vec{i} - 12\vec{j} + 6\vec{k} \) are parallel, orthogonal or neither.

Solution
Based on a quick inspection is seems (hopefully) fairly clear that we have,

\[ \vec{v} = -3\vec{w} \]

Therefore the two vectors are parallel.

On a side note an alternate method for working this problem is to find the angle between the two vectors
and using that to determine the answer.

Depending on which method you find easiest either will get you the correct answer.

9. Given \( \vec{a} = \langle -8, 2 \rangle \) and \( \vec{b} = \langle -1, -7 \rangle \) compute \( \text{proj}_a \vec{b} \).

Solution
All we really need to do here is use the formula from the notes. That will need the following quantities.

\[ \vec{a} \cdot \vec{b} = -6 \quad \|\vec{a}\|^2 = 68 \]

The projection is then,
\[ \text{proj}_a \vec{b} = \frac{-6}{68} \langle -8, 2 \rangle = \left\langle \frac{12}{17}, -\frac{3}{17} \right\rangle \]

10. Given \( \vec{u} = 7\vec{i} - \vec{j} + \vec{k} \) and \( \vec{w} = -2\vec{i} + 5\vec{j} - 6\vec{k} \) compute \( \text{proj}_w \vec{u} \).

Solution
All we really need to do here is use the formula from the notes. That will need the following quantities.

\[ \vec{u} \cdot \vec{w} = -25 \quad \|\vec{w}\|^2 = 65 \]

The projection is then,

\[ \text{proj}_w \vec{u} = \frac{-25}{65} (-2\vec{i} + 5\vec{j} - 6\vec{k}) = \left\langle \frac{10}{13} - \frac{25}{13}, \frac{30}{13} \right\rangle \]

11. Determine the direction cosines and direction angles for \( \vec{r} = -\vec{i} - \vec{j} + \vec{k} \).

Solution
All we really need to do here is use the formulas from the notes. That will need the following quantity.

\[ \|\vec{r}\| = \sqrt{\frac{161}{16}} = \frac{\sqrt{161}}{4} \]

The direction cosines and angles are then,

\[ \cos \alpha = \frac{-3}{\sqrt{161}/4} = \frac{-12}{\sqrt{161}} \quad \Rightarrow \quad \alpha = \cos^{-1} \left( \frac{-12}{\sqrt{161}} \right) = 2.8106 \text{ radians} \]

\[ \cos \beta = \frac{-1/4}{\sqrt{161}/4} = \frac{-1}{\sqrt{161}} \quad \Rightarrow \quad \beta = \cos^{-1} \left( \frac{-1}{\sqrt{161}} \right) = 1.6497 \text{ radians} \]

\[ \cos \gamma = \frac{1/4}{\sqrt{161}/4} = \frac{4}{\sqrt{161}} \quad \Rightarrow \quad \gamma = \cos^{-1} \left( \frac{4}{\sqrt{161}} \right) = 1.2501 \text{ radians} \]