Here are my online notes for my Calculus II course that I teach here at Lamar University. Despite the fact that these are my "class notes", they should be accessible to anyone wanting to learn Calculus II or needing a refresher in some of the topics from the class.

These notes do assume that the reader has a good working knowledge of Calculus I topics including limits, derivatives and basic integration and integration by substitution.

Calculus II tends to be a very difficult course for many students. There are many reasons for this.

The first reason is that this course does require that you have a very good working knowledge of Calculus I. The Calculus I portion of many of the problems tends to be skipped and left to the student to verify or fill in the details. If you don’t have good Calculus I skills, and you are constantly getting stuck on the Calculus I portion of the problem, you will find this course very difficult to complete.

The second, and probably larger, reason many students have difficulty with Calculus II is that you will be asked to truly think in this class. That is not meant to insult anyone; it is simply an acknowledgment that you can’t just memorize a bunch of formulas and expect to pass the course as you can do in many math classes. There are formulas in this class that you will need to know, but they tend to be fairly general. You will need to understand them, how they work, and more importantly whether they can be used or not. As an example, the first topic we will look at is Integration by Parts. The integration by parts formula is very easy to remember. However, just because you’ve got it memorized doesn’t mean that you can use it. You’ll need to be able to look at an integral and realize that integration by parts can be used (which isn’t always obvious) and then decide which portions of the integral correspond to the parts in the formula (again, not always obvious).

Finally, many of the problems in this course will have multiple solution techniques and so you’ll need to be able to identify all the possible techniques and then decide which will be the easiest technique to use.

So, with all that out of the way let me also get a couple of warnings out of the way to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn calculus I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn’t covered in class.

2. In general I try to work problems in class that are different from my notes. However, with Calculus II many of the problems are difficult to make up on the spur of the moment and so in this class my class work will follow these notes fairly close as far as worked problems go. With that being said I will, on occasion, work problems off the top of my head when I can to provide more examples than just those in my notes. Also, I often
don’t have time in class to work all of the problems in the notes and so you will find that some sections contain problems that weren’t worked in class due to time restrictions.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible in writing these up, but the reality is that I can’t anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I’ve not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.
Quadric Surfaces

In the previous two sections we’ve looked at lines and planes in three dimensions (or \( \mathbb{R}^3 \)) and while these are used quite heavily at times in a Calculus class there are many other surfaces that are also used fairly regularly and so we need to take a look at those.

In this section we are going to be looking at quadric surfaces. Quadric surfaces are the graphs of any equation that can be put into the general form

\[ A x^2 + B y^2 + C z^2 + D x y + E x z + F y z + G x + H y + I z + J = 0 \]

where \( A, \ldots, J \) are constants.

There is no way that we can possibly list all of them, but there are some standard equations so here is a list of some of the more common quadric surfaces.

**Ellipsoid**

Here is the general equation of an ellipsoid.

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]

Here is a sketch of a typical ellipsoid.

If \( a = b = c \) then we will have a sphere.

Notice that we only gave the equation for the ellipsoid that has been centered on the origin. Clearly ellipsoids don’t have to be centered on the origin. However, in order to make the discussion in this section a little easier we have chosen to concentrate on surfaces that are “centered” on the origin in one way or another.

**Cone**

Here is the general equation of a cone.

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2} \]

Here is a sketch of a typical cone.
Calculus II

Note that this is the equation of a cone that will open along the $z$-axis. To get the equation of a cone that opens along one of the other axes all we need to do is make a slight modification of the equation. This will be the case for the rest of the surfaces that we'll be looking at in this section as well.

In the case of a cone the variable that sits by itself on one side of the equal sign will determine the axis that the cone opens up along. For instance, a cone that opens up along the $x$-axis will have the equation,

$$\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{x^2}{a^2}$$

For most of the following surfaces we will not give the other possible formulas. We will however acknowledge how each formula needs to be changed to get a change of orientation for the surface.

**Cylinder**

Here is the general equation of a cylinder.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

This is a cylinder whose cross section is an ellipse. If $a = b$ we have a cylinder whose cross section is a circle. We’ll be dealing with those kinds of cylinders more than the general form so the equation of a cylinder with a circular cross section is,

$$x^2 + y^2 = r^2$$

Here is a sketch of typical cylinder with an ellipse cross section.
The cylinder will be centered on the axis corresponding to the variable that does not appear in the equation.

Be careful to not confuse this with a circle. In two dimensions it is a circle, but in three dimensions it is a cylinder.

**Hyperboloid of One Sheet**
Here is the equation of a hyperboloid of one sheet.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1
\]

Here is a sketch of a typical hyperboloid of one sheet.

The variable with the negative in front of it will give the axis along which the graph is centered.

© 2007 Paul Dawkins

http://tutorial.math.lamar.edu/terms.aspx
Hyperboloid of Two Sheets
Here is the equation of a hyperboloid of two sheets.

\[-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\]

Here is a sketch of a typical hyperboloid of two sheets.

The variable with the positive in front of it will give the axis along which the graph is centered.

Notice that the only difference between the hyperboloid of one sheet and the hyperboloid of two sheets is the signs in front of the variables. They are exactly the opposite signs.

Elliptic Paraboloid
Here is the equation of an elliptic paraboloid.

\[\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z}{c} = 0\]

As with cylinders this has a cross section of an ellipse and if \(a = b\) it will have a cross section of a circle. When we deal with these we’ll generally be dealing with the kind that have a circle for a cross section.

Here is a sketch of a typical elliptic paraboloid.
In this case the variable that isn’t squared determines the axis upon which the paraboloid opens up. Also, the sign of $c$ will determine the direction that the paraboloid opens. If $c$ is positive then it opens up and if $c$ is negative then it opens down.

**Hyperbolic Paraboloid**
Here is the equation of a hyperbolic paraboloid.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$$

Here is a sketch of a typical hyperbolic paraboloid.

These graphs are vaguely saddle shaped and as with the elliptic paraboloid the sign of $c$ will determine the direction in which the surface “opens up”. The graph above is shown for $c$ positive.
With the both of the types of paraboloids discussed above the surface can be easily moved up or down by adding/subtracting a constant from the left side.

For instance

\[ z = -x^2 - y^2 + 6 \]

is an elliptic paraboloid that opens downward (be careful, the “-” is on the \( x \) and \( y \) instead of the \( z \)) and starts at \( z = 6 \) instead of \( z = 0 \).

Here are a couple of quick sketches of this surface.

Note that we’ve given two forms of the sketch here. The sketch on the left has the standard set of axes but it is difficult to see the numbers on the axis. The sketch on the right has been “boxed” and this makes it easier to see the numbers to give a sense of perspective to the sketch. In most sketches that actually involve numbers on the axis system we will give both sketches to help get a feel for what the sketch looks like.