Preface

Here are the solutions to the practice problems for my Calculus II notes. Some solutions will have more or less detail than other solutions. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Quadric Surfaces

1. Sketch the following quadric surface.

\[
\frac{y^2}{9} + z^2 = 1
\]

Solution

This is a cylinder that is centered on the x-axis. The cross sections of the cylinder will be ellipses.

Make sure that you can “translate” the equations given in the notes to the other coordinate axes. Once you know what they look like when centered on one of the coordinates axes then a simple and predictable variable change will center them on the other coordinate axes.

Here are a couple of sketches of the region. We’ve given them with the more traditional axes as well as “boxed” axes to help visualize the surface.
2. Sketch the following quadric surface.
Solution  
This is an ellipsoid and because the numbers in the denominators of each of the terms are not the same we know that it won’t be a sphere.

Here are a couple of sketches of the region. We’ve given them with the more traditional axes as well as “boxed” axes to help visualize the surface.
3. Sketch the following quadric surface.

$$z = \frac{x^2}{4} + \frac{y^2}{4} - 6$$

Solution
This is an elliptic paraboloid that is centered on the $z$-axis. Because the $x$ and $y$ terms are positive we know that it will open upwards. The “-6” tells us that the surface will start at $z = -6$. We can also say that because the coefficients of the $x$ and $y$ terms are identical the cross sections of the surface will be circles.

Here are a couple of sketches of the region. We’ve given them with the more traditional axes as well as “boxed” axes to help visualize the surface.
4. Sketch the following quadric surface.

\[ y^2 = 4x^2 + 16z^2 \]

Solution
This is a cone that is centered on the y-axis and because the coefficients of the \( x \) and \( z \) terms are different the cross sections of the surface will be ellipses.

Make sure that you can “translate” the equations given in the notes to the other coordinate axes. Once you know what they look like when centered on one of the coordinates axes then a simple and predictable variable change will center them on the other coordinate axes.

Here are a couple of sketches of the region. We’ve given them with the more traditional axes as well as “boxed” axes to help visualize the surface.
5. Sketch the following quadric surface.

\[ x = 4 - 5y^2 - 9z^2 \]

**Solution**
This is an elliptic paraboloid that is centered on the \( x \)-axis. Because the \( y \) and \( z \) terms are negative we know that it will open in the negative \( x \) direction. The “4” tells us that the surface will start at \( x = 4 \). We can also say that because the coefficients of the \( y \) and \( z \) terms are different the cross sections of the surface will be ellipses.
Calculus II

Make sure that you can “translate” the equations given in the notes to the other coordinate axes. Once you know what they look like when centered on one of the coordinates axes then a simple and predictable variable change will center them on the other coordinate axes.

Here are a couple of sketches of the region. We’ve given them with the more traditional axes as well as “boxed” axes to help visualize the surface.