Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

The Limit

1. For the function \( f(x) = \frac{8-x^3}{x^2-4} \) answer each of the following questions.

   (a) Evaluate the function the following values of \( x \) compute (accurate to at least 8 decimal places).

   \[
   \begin{align*}
   & (i) \ 2.5 \quad (ii) \ 2.1 \quad (iii) \ 2.01 \quad (iv) \ 2.001 \quad (v) \ 2.0001 \\
   & (vi) \ 1.5 \quad (vii) \ 1.9 \quad (viii) \ 1.99 \quad (ix) \ 1.999 \quad (x) \ 1.9999
   \end{align*}
   \]

   (b) Use the information from (a) to estimate the value of \( \lim_{x \to 2} \frac{8-x^3}{x^2-4} \).

   (a) Evaluate the function the following values of \( x \) compute (accurate to at least 8 decimal places).

   \[
   \begin{align*}
   & (i) \ 2.5 \quad (ii) \ 2.1 \quad (iii) \ 2.01 \quad (iv) \ 2.001 \quad (v) \ 2.0001 \\
   & (vi) \ 1.5 \quad (vii) \ 1.9 \quad (viii) \ 1.99 \quad (ix) \ 1.999 \quad (x) \ 1.9999
   \end{align*}
   \]

   [Solution]

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http://tutorial.math.lamar.edu/terms.aspx
Here is a table of values of the function at the given points accurate to 8 decimal places.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$m_{PQ}$</th>
<th>$x$</th>
<th>$m_{PQ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>-3.38888889</td>
<td>1.5</td>
<td>-2.64285714</td>
</tr>
<tr>
<td>2.1</td>
<td>-3.07560976</td>
<td>1.9</td>
<td>-2.92564103</td>
</tr>
<tr>
<td>2.01</td>
<td>-3.00750623</td>
<td>1.99</td>
<td>-2.99250627</td>
</tr>
<tr>
<td>2.001</td>
<td>-3.00075006</td>
<td>1.999</td>
<td>-2.99925006</td>
</tr>
<tr>
<td>2.0001</td>
<td>-3.00007500</td>
<td>1.9999</td>
<td>-2.99992500</td>
</tr>
</tbody>
</table>

(b) Use the information from (a) to estimate the value of $\lim_{x \to 2} \frac{8-x^3}{x^2-4}$.

[Solution]
From the table of values above it looks like we can estimate that,

$$\lim_{x \to 2} \frac{8-x^3}{x^2-4} = -3$$

2. For the function $R(t) = 2 - \sqrt{t^2 + 3} \over t + 1$ answer each of the following questions.

(a) Evaluate the function the following values of $t$ compute (accurate to at least 8 decimal places).

(i) -0.5  (ii) -0.9  (iii) -0.99  (iv) -0.999  (v) -0.9999
(vi) -1.5  (vii) -1.1  (viii) -1.01  (ix) -1.001  (x) -1.0001

(b) Use the information from (a) to estimate the value of $\lim_{t \to -1} \frac{2 - \sqrt{t^2 + 3}}{t + 1}$.

(a) Evaluate the function the following values of $t$ compute (accurate to at least 8 decimal places).

(i) -0.5  (ii) -0.9  (iii) -0.99  (iv) -0.999  (v) -0.9999
(vi) -1.5  (vii) -1.1  (viii) -1.01  (ix) -1.001  (x) -1.0001

[Solution]
Here is a table of values of the function at the given points accurate to 8 decimal places.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$m_{PQ}$</th>
<th>$x$</th>
<th>$m_{PQ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>0.39444872</td>
<td>-1.5</td>
<td>0.58257569</td>
</tr>
<tr>
<td>-0.9</td>
<td>0.48077870</td>
<td>-1.1</td>
<td>0.51828453</td>
</tr>
<tr>
<td>-0.99</td>
<td>0.49812031</td>
<td>-1.01</td>
<td>0.50187032</td>
</tr>
</tbody>
</table>
(b) Use the information from (a) to estimate the value of \( \lim_{t \to 1} \frac{2 - \sqrt{t^2 + 3}}{t + 1} \).

[Solution]
From the table of values above it looks like we can estimate that,
\[
\lim_{t \to 1} \frac{2 - \sqrt{t^2 + 3}}{t + 1} = \frac{1}{2}
\]

3. For the function \( g(\theta) = \frac{\sin(7\theta)}{\theta} \) answer each of the following questions.

(a) Evaluate the function the following values of \( \theta \) compute (accurate to at least 8 decimal places). Make sure your calculator is set to radians for the computations.

(i) 0.5  (ii) 0.1  (iii) 0.01  (iv) 0.001  (v) 0.0001
(vi) -0.5  (vii) -0.1  (viii) -0.01  (ix) -0.001  (x) -0.0001

(b) Use the information from (a) to estimate the value of \( \lim_{\theta \to 0} \frac{\sin(7\theta)}{\theta} \).

(a) Evaluate the function the following values of \( x \) compute (accurate to at least 8 decimal places).

(i) 0.5  (ii) 0.1  (iii) 0.01  (iv) 0.001  (v) 0.0001
(vi) -0.5  (vii) -0.1  (viii) -0.01  (ix) -0.001  (x) -0.0001

[Solution]
Here is a table of values of the function at the given points accurate to 8 decimal places.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( m_{PQ} )</th>
<th>( x )</th>
<th>( m_{PQ} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-0.70156646</td>
<td>-0.5</td>
<td>-0.70156646</td>
</tr>
<tr>
<td>0.1</td>
<td>6.44217687</td>
<td>-0.1</td>
<td>6.44217687</td>
</tr>
<tr>
<td>0.01</td>
<td>6.99428473</td>
<td>-0.01</td>
<td>6.99428473</td>
</tr>
<tr>
<td>0.001</td>
<td>6.99994283</td>
<td>-0.001</td>
<td>6.99994283</td>
</tr>
<tr>
<td>0.0001</td>
<td>6.99999943</td>
<td>-0.0001</td>
<td>6.99999943</td>
</tr>
</tbody>
</table>

(b) Use the information from (a) to estimate the value of \( \lim_{\theta \to 0} \frac{\sin(7\theta)}{\theta} \).
[Solution]
From the table of values above it looks like we can estimate that,

$$\lim_{\theta \to 0} \frac{\sin(7\theta)}{\theta} = 7$$

4. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -3$  
(b) $a = -1$  
(c) $a = 2$  
(d) $a = 4$

(a) $a = -3$
From the graph we can see that,

$$f(-3) = 4$$

because the closed dot is at the value of $y = 4$.

We can also see that as we approach $x = -3$ from both sides the graph is approaching different values (4 from the left and -2 from the right). Because of this we get,

$$\lim_{x \to -3} f(x) \text{ does not exist}$$

Always recall that the value of a limit does not actually depend upon the value of the function at the point in question. The value of a limit only depends on the values of the function around the point in question. Often the two will be different.

(b) $a = -1$
From the graph we can see that,
Calculus I

\[ f(-1) = 3 \]
because the closed dot is at the value of \( y = 3 \).

We can also see that as we approach \( x = -1 \) from both sides the graph is approaching the same value, 1, and so we get,

\[ \lim_{x \to -1} f(x) = 1 \]

Always recall that the value of a limit does not actually depend upon the value of the function at the point in question. The value of a limit only depends on the values of the function around the point in question. Often the two will be different.

(c) \( a = 2 \)
Because there is no closed dot for \( x = 2 \) we can see that,

\[ f(2) \text{ does not exist} \]

We can also see that as we approach \( x = 2 \) from both sides the graph is approaching the same value, 1, and so we get,

\[ \lim_{x \to 2} f(x) = 1 \]

Always recall that the value of a limit does not actually depend upon the value of the function at the point in question. The value of a limit only depends on the values of the function around the point in question. Therefore, even though the function doesn’t exist at this point the limit can still have a value.

(d) \( a = 4 \)
From the graph we can see that,

\[ f(4) = 5 \]
because the closed dot is at the value of \( y = 5 \).

We can also see that as we approach \( x = 4 \) from both sides the graph is approaching the same value, 5, and so we get,

\[ \lim_{x \to 4} f(x) = 5 \]

5. Below is the graph of \( f(x) \). For each of the given points determine the value of \( f(a) \) and \( \lim_{x \to a} f(x) \). If any of the quantities do not exist clearly explain why.
(a) $a = -8$

From the graph we can see that,

$$f(-8) = -3$$

because the closed dot is at the value of $y = -3$.

We can also see that as we approach $x = -8$ from both sides the graph is approaching the same value, -6, and so we get,

$$\lim_{{x \to -8}} f(x) = -6$$

Always recall that the value of a limit does not actually depend upon the value of the function at the point in question. The value of a limit only depends on the values of the function around the point in question. Often the two will be different.

(b) $a = -2$

From the graph we can see that,

$$f(-2) = 3$$

because the closed dot is at the value of $y = 3$.

We can also see that as we approach $x = -2$ from both sides the graph is approaching different values (3 from the left and doesn’t approach any value from the right). Because of this we get,

$$\lim_{{x \to -2}} f(x) \text{ does not exist}$$
Always recall that the value of a limit does not actually depend upon the value of the function at the point in question. The value of a limit only depends on the values of the function around the point in question. Often the two will be different.

(c) \( a = 6 \)
From the graph we can see that,
\[
\lim_{x \to 6} f(x) \text{ does not exist}
\]
because the closed dot is at the value of \( y = 5 \).

We can also see that as we approach \( x = 6 \) from both sides the graph is approaching different values (2 from the left and 5 from the right). Because of this we get,
\[
\lim_{x \to 6} f(x) \text{ does not exist}
\]

6. Below is the graph of \( f(x) \). For each of the given points determine the value of \( f(a) \) and \( \lim_{x \to a} f(x) \). If any of the quantities do not exist clearly explain why.

(a) \( a = -2 \)
(b) \( a = -1 \)
(c) \( a = 1 \)
(d) \( a = 3 \)
(a) $a = -2$
Because there is no closed dot for $x = -2$ we can see that,
\[
\not f(-2) \text{ does not exist}
\]
We can also see that as we approach $x = -2$ from both sides the graph is not approaching a value from either side and so we get,
\[
\lim_{x \to -2} f(x) \text{ does not exist}
\]

(b) $a = -1$
From the graph we can see that,
\[
f(-1) = 3
\]
because the closed dot is at the value of $y = 3$.

We can also see that as we approach $x = -1$ from both sides the graph is approaching the same value, 1, and so we get,
\[
\lim_{x \to -1} f(x) = 1
\]
Always recall that the value of a limit does not actually depend upon the value of the function at the point in question. The value of a limit only depends on the values of the function around the point in question. Often the two will be different.

(c) $a = 1$
Because there is no closed dot for $x = 1$ we can see that,
\[
f(1) \text{ does not exist}
\]
We can also see that as we approach \( x = 1 \) from both sides the graph is approaching the same value, -3, and so we get,

\[
\lim_{{x \to 1}} f(x) = -3
\]

Always recall that the value of a limit does not actually depend upon the value of the function at the point in question. The value of a limit only depends on the values of the function around the point in question. Therefore, even though the function doesn’t exist at this point the limit can still have a value.

(d) \( a = 3 \)

From the graph we can see that,

\[
f(3) = 4
\]

because the closed dot is at the value of \( y = 4 \).

We can also see that as we approach \( x = 3 \) from both sides the graph is approaching the same value, 4, and so we get,

\[
\lim_{{x \to 3}} f(x) = 4
\]