Preface

Here are a set of problems for my Calculus I notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.

As with the set of practice problems I write these as I get the time and some sections will have only a few problems at this point and others won’t have any problems in them yet. Rest assured that I’m always trying to get more problems written but this site has been written and maintained in my spare time and so I usually cannot devote as much time as I’d like to the site.

Continuity

1. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.

2. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.
3. The graph of \( f(x) \) is given below. Based on this graph determine where the function is discontinuous.

For problems 4 – 13 using only Properties 1- 9 from the Limit Properties section, one-sided limit properties (if needed) and the definition of continuity determine if the given function is continuous or discontinuous at the indicated points.

4. \( f(x) = \frac{6 + 2x}{7x-14} \)
   (a) \( x = -3 \), (b) \( x = 0 \), (c) \( x = 2 \)

5. \( R(y) = \frac{2y}{y^2-25} \)
   (a) \( y = -5 \), (b) \( y = -1 \), (c) \( y = 3 \)

6. \( g(z) = \frac{5z-20}{z^2-12z} \)
(a) $z = -1$, (b) $z = 0$, (c) $z = 4$?

7. $W(x) = \frac{2 + x}{x^2 + 6x - 7}$
   (a) $x = -7$, (b) $x = 0$, (c) $x = 1$?

8. $h(z) = \begin{cases} 2z^2 & z < -1 \\ 4z + 6 & z \geq -1 \end{cases}$
   (a) $z = -6$, (b) $z = -1$?

9. $g(x) = \begin{cases} x + e^x & x < 0 \\ x^2 & x \geq 0 \end{cases}$
   (a) $x = 0$, (b) $x = 4$?

10. $Z(t) = \begin{cases} 8 & t < 5 \\ 1 - 6t & t \geq 5 \end{cases}$
    (a) $t = 0$, (b) $t = 5$?

11. $h(z) = \begin{cases} z + 2 & z < -4 \\ 0 & z = -4 \\ 18 - z^2 & z > -4 \end{cases}$
    (a) $z = -4$, (b) $z = 2$?

12. $f(x) = \begin{cases} 1 - x^2 & x < 2 \\ -3 & x = 2 \\ 2x - 7 & 2 < x < 7 \\ 0 & x = 7 \\ x^2 & x > 7 \end{cases}$
    (a) $x = 2$, (b) $x = 7$?

13. $g(w) = \begin{cases} 3w & w < 0 \\ 0 & w = 0 \\ w + 6 & 0 < w < 8 \\ 14 & w = 8 \\ 22 - w & w > 8 \end{cases}$
    (a) $w = 0$, (b) $w = 8$?
For problems 14 – 22 determine where the given function is discontinuous.

14. \( f(x) = \frac{11 - 2x}{2x^2 - 13x - 7} \)

15. \( Q(z) = \frac{3}{2z^2 + 3z - 4} \)

16. \( h(t) = \frac{t^2 - 1}{t^3 + 6t^2 + t} \)

17. \( f(z) = \frac{4z + 1}{5\cos\left(\frac{z}{7}\right) + 1} \)

18. \( h(x) = \frac{1 - x}{x \sin(x - 1)} \)

19. \( f(x) = \frac{3}{4e^{x-7} - 1} \)

20. \( R(w) = \frac{e^{w^2} + 1}{e^w - 2e^{1-w}} \)

21. \( g(x) = \cot(4x) \)

22. \( f(t) = \sec\left(\sqrt{t}\right) \)

For problems 23 – 27 use the Intermediate Value Theorem to show that the given equation has at least one solution in the indicated interval. Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.

23. \( x^4 + 7x^3 - x = 0 \) on \([4, 8]\)

24. \( z^2 + 11z = 3 \) on \([-15, -5]\)

25. \( \frac{t^3 + t - 15}{t - 8} = 0 \) on \([-5, 1]\)
26. \( \ln(2r^2 + 1) - \ln(r^2 - 4) = 0 \) on \([-1, 2]\)

27. \( 10 = w^3 + w^2 e^{-w} - 5 \) on \([0, 4]\)

For problems 28 – 33 assume that \( f(x) \) is continuous everywhere unless otherwise indicated in some way. From the given information is it possible to determine if there is a root of \( f(x) \) in the given interval?

If it is possible to determine that there is a root in the given interval clearly explain how you know that a root must exist. If it is not possible to determine if there is a root in the interval sketch a graph of two functions each of which meets the given information and one will have a root in the given interval and the other will not have a root in the given interval.

28. \( f(-5) = 12 \) and \( f(0) = -3 \) on the interval \([-5, 0]\).

29. \( f(1) = 30 \) and \( f(9) = 6 \) on the interval \([1, 9]\).

30. \( f(20) = -100 \) and \( f(40) = -100 \) on the interval \([20, 40]\).

31. \( f(-4) = -10, \quad f(5) = 17, \quad \lim_{x \to 1^-} f(x) = -2, \quad \lim_{x \to 1^+} f(x) = 4 \) on the interval \([-4, 5]\).

32. \( f(-8) = 2, \quad f(1) = 23, \quad \lim_{x \to -4^-} f(x) = 35, \quad \lim_{x \to -4^+} f(x) = 1 \) on the interval \([-8, 1]\).

33. \( f(0) = -1, \quad f(9) = 10, \quad \lim_{x \to 2^-} f(x) = -12, \quad \lim_{x \to 2^+} f(x) = -3 \) on the interval \([0,10]\).