Preface

Here are my online notes for my Calculus I course that I teach here at Lamar University. Despite the fact that these are my “class notes” they should be accessible to anyone wanting to learn Calculus I or needing a refresher in some of the early topics in calculus.

I’ve tried to make these notes as self contained as possible and so all the information needed to read through them is either from an Algebra or Trig class or contained in other sections of the notes.

Here are a couple of warnings to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn calculus I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn’t covered in class.

2. Because I want these notes to provide some more examples for you to read through, I don’t always work the same problems in class as those given in the notes. Likewise, even if I do work some of the problems in here I may work fewer problems in class than are presented here.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible when writing these up, but the reality is that I can’t anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I’ve not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.

Derivatives
**Derivatives of Hyperbolic Functions**

The last set of functions that we’re going to be looking in this chapter at are the hyperbolic functions. In many physical situations combinations of $e^x$ and $e^{-x}$ arise fairly often. Because of this these combinations are given names. There are six hyperbolic functions and they are defined as follows.

\[
\begin{align*}
\sinh x &= \frac{e^x - e^{-x}}{2} \\
\cosh x &= \frac{e^x + e^{-x}}{2} \\
\tanh x &= \frac{\sinh x}{\cosh x} \\
\coth x &= \frac{\cosh x}{\sinh x} = \frac{1}{\tanh x} \\
\text{sech } x &= \frac{1}{\cosh x} \\
\text{csch } x &= \frac{1}{\sinh x}
\end{align*}
\]

Here are the graphs of the three main hyperbolic functions.

\[
\begin{align*}
y &= \cosh x \\
y &= \sinh x \\
y &= \tanh x
\end{align*}
\]

We also have the following facts about the hyperbolic functions.

\[
\begin{align*}
\sinh (-x) &= -\sinh (x) & \cosh (-x) &= \cosh (x) \\
\cosh^2 (x) - \sinh^2 (x) &= 1 & 1 - \tanh^2 (x) &= \text{sech}^2 (x)
\end{align*}
\]

You’ll note that these are similar, but not quite the same, to some of the more common trig identities so be careful to not confuse the identities here with those of the standard trig functions.
Because the hyperbolic functions are defined in terms of exponential functions finding their derivatives is fairly simple provided you’ve already read through the next section. We haven’t however so we’ll need the following formula that can be easily proved after we’ve covered the next section.

\[
\frac{d}{dx}(e^{-x}) = -e^{-x}
\]

With this formula we’ll do the derivative for hyperbolic sine and leave the rest to you as an exercise.

\[
\frac{d}{dx}(\sinh x) = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^x - (-e^{-x})}{2} = \frac{e^x + e^{-x}}{2} = \cosh x
\]

For the rest we can either use the definition of the hyperbolic function and/or the quotient rule. Here are all six derivatives.

\[
\begin{align*}
\frac{d}{dx}(\sinh x) &= \cosh x \\
\frac{d}{dx}(\cosh x) &= \sinh x \\
\frac{d}{dx}(\tanh x) &= \text{sech}^2 x \\
\frac{d}{dx}(\coth x) &= -\text{csch}^2 x \\
\frac{d}{dx}(\text{sech} x) &= -\text{sech} x \tanh x \\
\frac{d}{dx}(\text{csch} x) &= -\text{csch} x \coth x
\end{align*}
\]

Here are a couple of quick derivatives using hyperbolic functions.

**Example 1** Differentiate each of the following functions.

(a) \( f(x) = 2x^5 \cosh x \)

(b) \( h(t) = \frac{\sinh t}{t+1} \)

*Solution*

(a) \( f''(x) = 10x^4 \cosh x + 2x^5 \sinh x \)

(b) \( h'(t) = \frac{(t+1)\cosh t - \sinh t}{(t+1)^2} \)