Preface

Here are a set of problems for my Calculus I notes. These problems do not have any solutions available on this site. These are intended mostly for instructors who might want a set of problems to assign for turning in. I try to put up both practice problems (with solutions available) and these problems at the same time so that both will be available to anyone who wishes to use them.

As with the set of practice problems I write these as I get the time and some sections will have only a few problems at this point and others won’t have any problems in them yet. Rest assured that I’m always trying to get more problems written but this site has been written and maintained in my spare time and so I usually cannot devote as much time as I’d like to the site.

The Mean Value Theorem

For problems 1 – 4 determine all the number(s) $c$ which satisfy the conclusion of Rolle’s Theorem for the given function and interval.

1. $f(x) = x^3 - 4x^2 + 3$ on $[0, 4]$

2. $Q(z) = 15 + 2z - z^2$ on $[-2, 4]$

3. $h(t) = 1 - e^{t^2 - 9}$ on $[-3, 3]$

4. $g(w) = 1 + \cos[\pi w]$ on $[5, 9]$

For problems 5 – 8 determine all the number(s) $c$ which satisfy the conclusion of the Mean Value Theorem for the given function and interval.

5. $f(x) = x^3 - x^2 + x + 8$ on $[-3, 4]$

6. $g(t) = 2t^3 + t^2 + 7t - 1$ on $[1, 6]$

7. $P(t) = e^{2t} - 6t - 3$
8. \( h(x) = 9x - 8\sin\left(\frac{x}{2}\right) \) on \([-3, -1]\)

9. Suppose we know that \( f(x) \) is continuous and differentiable on the interval \([-2, 5]\), that \( f(5) = 14 \) and that \( f''(x) \leq 10 \). What is the smallest possible value for \( f(-2) \)?

10. Suppose we know that \( f(x) \) is continuous and differentiable on the interval \([-6, -1]\), that \( f(-6) = -23 \) and that \( f''(x) \geq -4 \). What is the smallest possible value for \( f(-1) \)?

11. Suppose we know that \( f(x) \) is continuous and differentiable on the interval \([-3, 4]\), that \( f(-3) = 7 \) and that \( f''(x) \leq -17 \). What is the largest possible value for \( f(4) \)?

12. Suppose we know that \( f(x) \) is continuous and differentiable on the interval \([1, 9]\), that \( f(9) = 0 \) and that \( f''(x) \geq 8 \). What is the largest possible value for \( f(1) \)?

13. Show that \( f(x) = x^7 + 2x^5 + 3x^3 + 14x + 1 \) has exactly one real root.

14. Show that \( f(x) = 6x^3 - 2x^2 + 4x - 3 \) has exactly one real root.

15. Show that \( f(x) = 20x - e^{-4x} \) has exactly one real root.