Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Indeterminate Forms and L'Hospital's Rule

1. Use L’Hospital’s Rule to evaluate \( \lim_{x \to 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6} \).

Step 1
The first step we should really do here is verify that L’Hospital’s Rule can in fact be used on this limit.

This may seem like a silly step given that we are told to use L’Hospital’s Rule. However, in later sections we won’t be told to use it when/if it can be used. Therefore, we really need to get in the habit of checking that it can be used before applying it just to make sure that we can. If we apply L’Hospital’s Rule to a problem that it can’t be applied to then it’s is almost assured that we will get the wrong answer (it’s always possible you might get lucky and get the correct answer, but we will only be very lucky if it does).

So a quick check shows us that,

\[ \text{as } x \to 2 \quad \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6} \to 0 \]

and so this is a form that allows the use of L’Hospital’s Rule.
Step 2
So, at this point let’s just apply L’Hospital’s Rule.

\[
\lim_{x \to 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6} = \lim_{x \to 2} \frac{3x^2 - 14x + 10}{2x + 1}
\]

Step 3
At this point all we need to do is try the limit and see if it can be done.

\[
\lim_{x \to 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6} = \lim_{x \to 2} \frac{3x^2 - 14x + 10}{2x + 1} = \frac{-6}{5}
\]

So, the limit can be done and we done with the problem! The limit is then,

\[
\lim_{x \to 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6} = \frac{-6}{5}
\]

2. Use L’Hospital’s Rule to evaluate \( \lim_{w \to 4} \frac{\sin(\pi w)}{w^2 - 16} \).

Step 1
The first step we should really do here is verify that L’Hospital’s Rule can in fact be used on this limit.

This may seem like a silly step given that we are told to use L’Hospital’s Rule. However, in later sections we won’t be told to use it when/if it can be used. Therefore, we really need to get in the habit of checking that it can be used before applying it just to make sure that we can. If we apply L’Hospital’s Rule to a problem that it can’t be applied to then it’s is almost assured that we will get the wrong answer (it’s always possible you might get lucky and get the correct answer, but we will only be very lucky if it does).

So a quick check shows us that,

\[
\text{as } w \to 4 \quad \frac{\sin(\pi w)}{w^2 - 16} \to 0
\]

and so this is a form that allows the use of L’Hospital’s Rule.

Step 2
So, at this point let’s just apply L’Hospital’s Rule.
Step 3
At this point all we need to do is try the limit and see if it can be done.

\[
\lim_{w \to -4} \frac{\sin(\pi w)}{w^2 - 16} = \lim_{w \to -4} \frac{\pi \cos(\pi w)}{2w} = \pi \cos(-4\pi) / -8 = \pi / -8
\]

So, the limit can be done and we done with the problem! The limit is then,

\[
\lim_{w \to -4} \frac{\sin(\pi w)}{w^2 - 16} = -\frac{\pi}{8}
\]

3. Use L’Hospital’s Rule to evaluate \(\lim_{t \to \infty} \frac{\ln(3t)}{t^2}\).

Step 1
The first step we should really do here is verify that L’Hospital’s Rule can in fact be used on this limit.

This may seem like a silly step given that we are told to use L’Hospital’s Rule. However, in later sections we won’t be told to use it when/if it can be used. Therefore, we really need to get in the habit of checking that it can be used before applying it just to make sure that we can. If we apply L’Hospital’s Rule to a problem that it can’t be applied to then it’s is almost assured that we will get the wrong answer (it’s always possible you might get lucky and get the correct answer, but we will only be very lucky if it does).

So a quick check shows us that,

\[
\text{as } t \to \infty \quad \frac{\ln(3t)}{t^2} \to \infty
\]

and so this is a form that allows the use of L’Hospital’s Rule.

Step 2
So, at this point let’s just apply L’Hospital’s Rule.

\[
\lim_{t \to \infty} \frac{\ln(3t)}{t^2} = \lim_{t \to \infty} \frac{\frac{1}{t}}{2t} = \lim_{t \to \infty} \frac{1}{2t^2}
\]
Don’t forget to simplify after taking the derivatives. This can often be the difference between being able to do the problem or not.

Step 3
At this point all we need to do is try the limit and see if it can be done.

\[ \lim_{t \to \infty} \frac{\ln(3t)}{t^2} = \lim_{t \to \infty} \frac{1}{2t} = 0 \]

So, the limit can be done and we done with the problem! The limit is then,

\[ \lim_{t \to \infty} \frac{\ln(3t)}{t^2} = 0 \]

4. Use L’Hospital’s Rule to evaluate \( \lim_{z \to 0} \frac{\sin(2z) + 7z^2 - 2z}{z^2 (z + 1)^2} \).

Step 1
The first step we should really do here is verify that L’Hospital’s Rule can in fact be used on this limit.

This may seem like a silly step given that we are told to use L’Hospital’s Rule. However, in later sections we won’t be told to use it when/if it can be used. Therefore, we really need to get in the habit of checking that it can be used before applying it just to make sure that we can. If we apply L’Hospital’s Rule to a problem that it can’t be applied to then it’s is almost assured that we will get the wrong answer (it’s always possible you might get lucky and get the correct answer, but we will only be very lucky if it does).

So a quick check shows us that,

\[ \text{as } z \to 0 \quad \frac{\sin(2z) + 7z^2 - 2z}{z^2 (z + 1)^2} \to 0 \]

and so this is a form that allows the use of L’Hospital’s Rule.

Step 2
Before actually using L’Hospital’s Rule it might be better if we multiply out the denominator to make the derivative (and later steps a little easier). Doing this gives,

\[ \lim_{z \to 0} \frac{\sin(2z) + 7z^2 - 2z}{z^2 (z + 1)^2} = \lim_{z \to 0} \frac{\sin(2z) + 7z^2 - 2z}{z^4 + 2z^3 + z^2} \]
Now let’s apply L’Hospital’s Rule.

\[
\lim_{z \to 0} \frac{\sin(2z) + 7z^2 - 2z}{z^2(z + 1)^2} = \lim_{z \to 0} \frac{2\cos(2z) + 14z - 2}{4z^3 + 6z^2 + 2z}
\]

**Step 3**

At this point let’s try the limit and see if it can be done. However, in this case, we can see that,

\[
as \ z \to 0 \quad \frac{2\cos(2z) + 14z - 2}{4z^3 + 6z^2 + 2z} \to 0
\]

**Step 4**

So, using L’Hospital’s Rule doesn’t give us a limit that we can do. However, the new limit is one that can use L’Hospital’s Rule on so let’s do that.

\[
\lim_{z \to 0} \frac{\sin(2z) + 7z^2 - 2z}{z^2(z + 1)^2} = \lim_{z \to 0} \frac{-4\sin(2z) + 14}{12z^2 + 12z + 2} = \frac{14}{2}
\]

Okay, the second L’Hospital’s Rule gives us a limit we can do and so the answer is,

\[
\lim_{z \to 0} \frac{\sin(2z) + 7z^2 - 2z}{z^2(z + 1)^2} = 7
\]

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5. Use L’Hospital’s Rule to evaluate \( \lim_{x \to -\infty} \frac{x^2}{e^{-x}} \).

**Step 1**

The first step we should really do here is verify that L’Hospital’s Rule can in fact be used on this limit.

This may seem like a silly step given that we are told to use L’Hospital’s Rule. However, in later sections we won’t be told to use it when/if it can be used. Therefore, we really need to get in the habit of checking that it can be used before applying it just to make sure that we can. If we apply L’Hospital’s Rule to a problem that it can’t be applied to then it’s is almost assured that we will get the wrong answer (it’s always possible you might get lucky and get the correct answer, but we will only be very lucky if it does).
So a quick check shows us that,

\[
\lim_{x \to -\infty} \frac{x^2}{e^{1-x}} = \infty
\]

and so this is a form that allows the use of L'Hospital's Rule.

Step 2
So, at this point let's just apply L'Hospital's Rule.

\[
\lim_{x \to -\infty} \frac{x^2}{e^{1-x}} = \lim_{x \to -\infty} \frac{2x}{-e^{1-x}}
\]

Step 3
At this point let's try the limit and see if it can be done. However, in this case, we can see that,

\[
\lim_{x \to -\infty} \frac{2x}{-e^{1-x}} = \infty
\]

Step 4
So, using L'Hospital's Rule doesn't give us a limit that we can do. However, the new limit is one that can use L'Hospital's Rule on so let's do that.

\[
\lim_{x \to -\infty} \frac{x^2}{e^{1-x}} = \lim_{x \to -\infty} \frac{2x}{-e^{1-x}} = \lim_{x \to -\infty} \frac{2}{-e^{1-x}} = 0
\]

Okay, the second L'Hospital's Rule gives us a limit we can do and so the answer is,

\[
\lim_{x \to -\infty} \frac{x^2}{e^{1-x}} = 0
\]

6. Use L'Hospital's Rule to evaluate \( \lim_{z \to \infty} \frac{z^2 + e^{4z}}{2z - e^z} \).

Step 1
The first step we should really do here is verify that L'Hospital's Rule can in fact be used on this limit.

This may seem like a silly step given that we are told to use L'Hospital's Rule. However, in later sections we won't be told to use it when/if it can be used. Therefore, we really need to get in the habit of checking that it can be used before applying it just to make sure that we can. If we apply L'Hospital's Rule to a problem that it can't be applied to then it's almost assured that we will get the wrong answer (it's always possible you might get lucky and get the correct answer, but we will only be very lucky if it does).
So a quick check shows us that,

\[
\text{as } z \to \infty \quad \frac{z^2 + e^{4z}}{2z - e^z} \to \frac{\infty}{-\infty}
\]

and so this is a form that allows the use of L’Hospital’s Rule.

Step 2
So, at this point let’s just apply L’Hospital’s Rule.

\[
\lim_{z \to \infty} \frac{z^2 + e^{4z}}{2z - e^z} = \lim_{z \to \infty} \frac{2z + 4e^{4z}}{2 - e^z}
\]

Step 3
At this point let’s try the limit and see if it can be done. However, in this case, we can see that,

\[
\text{as } z \to \infty \quad \frac{2z + 4e^{4z}}{2 - e^z} \to \frac{\infty}{-\infty}
\]

Step 4
So, using L’Hospital’s Rule doesn’t give us a limit that we can do. However, the new limit is one that can use L’Hospital’s Rule on so let’s do that.

\[
\lim_{z \to \infty} \frac{z^2 + e^{4z}}{2z - e^z} = \lim_{z \to \infty} \frac{2z + 4e^{4z}}{2 - e^z} = \lim_{z \to \infty} \frac{2 + 16e^{4z}}{-e^z}
\]

Step 5
Now, at this point we need to be careful. It looks like we are still in a case of an infinity divided by an infinity and that looks to continue forever if we keep applying L’Hospital’s Rule. However, do not forget to do some basic simplifications where you can.

If we simplify we get the following.

\[
\lim_{z \to \infty} \frac{z^2 + e^{4z}}{2z - e^z} = \lim_{z \to \infty} \left(2 + 16e^{4z}\right)\left(-e^{-z}\right) = \lim_{z \to \infty} \left(-2e^{-z} - 16e^{3z}\right)
\]

and this is something that we can take the limit of.

So the answer is,

\[
\lim_{z \to \infty} \frac{z^2 + e^{4z}}{2z - e^z} = \lim_{z \to \infty} \left(-2e^{-z} - 16e^{3z}\right) = \text{[\(-\infty\)]}
\]

Again, it cannot be stressed enough that you’ve got to do simplification where you can. For some of these problems that can mean the difference between being able to do the problem or not.
7. Use L’Hospital’s Rule to evaluate \( \lim_{t \to \infty} t \ln \left( 1 + \frac{3}{t} \right) \).

Step 1
The first thing to notice here is that is not in a form that allows L’Hospital’s Rule. L’Hospital’s Rule only works on a certain class of rational functions and this is clearly not a rational function.

Note however that it is in the following indeterminate form,

\[
\text{as } t \to \infty \quad t \ln \left( 1 + \frac{3}{t} \right) \to (\infty)(0)
\]

and as we discussed in the notes for this section we can always turn this kind of indeterminate form into a rational expression that will allow L’Hospital’s Rule to be applied.

Step 2
The real question is do we move the first term or the second term to the denominator. From the looks of things it appears that it would be best to move the first term to the denominator.

\[
\lim_{t \to \infty} \left[ t \ln \left( 1 + \frac{3}{t} \right) \right] = \lim_{t \to \infty} \frac{\ln \left( 1 + \frac{3}{t} \right)}{\frac{1}{t}}
\]

Notice as well that,

\[
\text{as } t \to \infty \quad \frac{\ln \left( 1 + \frac{3}{t} \right)}{\frac{1}{t}} \to 0
\]

and we can use L’Hospital’s Rule on this.

Step 3
Applying L’Hospital’s Rule gives,

\[
\lim_{t \to \infty} \left[ t \ln \left( 1 + \frac{3}{t} \right) \right] = \ln \left( 1 + \frac{3}{t} \right) = \lim_{t \to \infty} \frac{-3}{t^2} = \frac{-3}{t^2}
\]
Can you see why we chose to move the $t$ to the denominator? Moving the logarithm would have left us with a very messy derivative to take! It might have ended up working okay for us, but the work would be greatly increased.

Step 4
Do not forget to simplify after we’ve taken the derivative. This problem becomes very simple if we do that.

$$\lim_{t \to \infty} \left[ t \ln \left( 1 + \frac{3}{t} \right) \right] = \lim_{t \to \infty} \frac{\ln \left( 1 + \frac{3}{t} \right)}{1/t} = \lim_{t \to \infty} \frac{3}{1 + \frac{3}{t}} = \boxed{3}$$

8. Use L’Hospital’s Rule to evaluate $\lim_{w \to 0^+} \left[ w^2 \ln \left( 4w^2 \right) \right]$.

Step 1
The first thing to notice here is that is not in a form that allows L’Hospital’s Rule. L’Hospital’s Rule only works on a certain class of rational functions and this is clearly not a rational function.

Note however that it is in the following indeterminate form,

$$\text{as } w \to 0^+ \quad w^2 \ln \left( 4w^2 \right) \to (0)(-\infty)$$

and as we discussed in the notes for this section we can always turn this kind of indeterminate form into a rational expression that will allow L’Hospital’s Rule to be applied.

Step 2
The real question is do we move the first term or the second term to the denominator. From the looks of things it appears that it would be best to move the first term to the denominator.

$$\lim_{w \to 0^+} \left[ w^2 \ln \left( 4w^2 \right) \right] = \lim_{w \to 0^+} \frac{\ln \left( 4w^2 \right)}{1/w^2}$$

Notice as well that,

$$\text{as } w \to 0^+ \quad \frac{\ln \left( 4w^2 \right)}{1/w^2} \to -\infty$$

and we can use L’Hospital’s Rule on this.
Step 3
Applying L’Hospital’s Rule gives,
\[
\lim_{w \to 0^+} \left[ w^2 \ln(4w^2) \right] = \lim_{w \to 0^+} \frac{\ln(4w^2)}{1/w^2} = \lim_{w \to 0^+} \frac{2/w}{-2/w^3} = \lim_{w \to 0^+} \frac{2}{w}
\]

Can you see why we chose to move the first term to the denominator? Moving the logarithm would have left us with a very messy derivative to take! It might have ended up working okay for us, but the work would be greatly increased.

Step 4
Do not forget to simplify after we’ve taken the derivative. This problem becomes very simple if we do that. In fact, it is the only way to actually get an answer for this problem. If we do not simplify will get stuck in a never ending chain of infinity divided by infinity forms no matter how many times we apply L’Hospital’s Rule.

\[
\lim_{w \to 0^+} \left[ w^2 \ln(4w^2) \right] = \lim_{w \to 0^+} \frac{\ln(4w^2)}{1/w^2} = \lim_{w \to 0^+} \left(-w^2\right) = 0
\]

9. Use L’Hospital’s Rule to evaluate \( \lim_{x \to 1^+} \left[ (x - 1) \tan \left( \frac{\pi}{4} x \right) \right] \).

Step 1
The first thing to notice here is that is not in a form that allows L’Hospital’s Rule. L’Hospital’s Rule only works on a certain class of rational functions and this is clearly not a rational function.

Note however that it is in the following indeterminate form,

\[
as \ x \to 1^+ \quad (x - 1) \tan \left( \frac{\pi}{4} x \right) \to (0)(\infty)
\]

and as we discussed in the notes for this section we can always turn this kind of indeterminate form into a rational expression that will allow L’Hospital’s Rule to be applied.

Step 2
The real question is do we move the first term or the second term to the denominator. At first glance it might appear that neither term will be particularly useful in the denominator. In particular, if we move the tangent to the denominator we would end up needing to differentiate a term in the form \( \frac{1}{\tan} \). That doesn’t look to be all that fun to differentiate and we’re liable to end up with a mess when we are done.
However, that is exactly the term we are going to move to the denominator for reasons that will quickly become apparent.

\[
\lim_{x \to 1^+} \left[ (x-1) \tan \left( \frac{\pi}{2} x \right) \right] = \lim_{x \to 1^+} \frac{x-1}{\tan \left( \frac{\pi}{2} x \right)} = \lim_{x \to 1^+} \frac{x-1}{\cot \left( \frac{\pi}{2} x \right)}
\]

Step 3
With a little simplification after moving the tangent to the denominator we ended up with something that doesn’t look all that bad. We’ll also see that the remainder of this problem is going to be quite simple.

Before we proceed however we should notice as well that,

as \( x \to 1^+ \) \[
\frac{x-1}{\cot \left( \frac{\pi}{2} x \right)} \to 0
\]

and we can use L’Hospital’s Rule on this.

Step 4
Applying L’Hospital’s Rule gives,

\[
\lim_{x \to 1^+} \left[ (x-1) \tan \left( \frac{\pi}{2} x \right) \right] = \lim_{x \to 1^+} \frac{x-1}{\cot \left( \frac{\pi}{2} x \right)} = \lim_{x \to 1^+} \frac{1}{\frac{\pi}{2} \csc^2 \left( \frac{\pi}{2} x \right)} = \frac{-2}{\pi}
\]

10. Use L’Hospital’s Rule to evaluate \( \lim_{{y \to 0^+}} \left[ \cos \left( \frac{2y}{y^2} \right) \right]^{\frac{1}{y^2}} \).

Step 1
The first thing to notice here is that is not in a form that allows L’Hospital’s Rule. L’Hospital’s Rule only works on certain classes of rational functions and this is clearly not a rational function.

Note however that it is in the following indeterminate form,

as \( y \to 0^+ \) \[
\left[ \cos \left( \frac{2y}{y^2} \right) \right]^{\frac{1}{y^2}} \to 1^\infty
\]

and as we discussed in the notes for this section we can do some manipulation on this to turn it into a problem that can be done with L’Hospital’s Rule.

Step 2
First, let’s define,
\[
z = \left[ \cos(2y) \right]^{y^2}\]

and take the log of both sides. We’ll also do a little simplification.

\[
\ln z = \ln \left( \left[ \cos(2y) \right]^{y^2} \right) = \frac{1}{y^2} \ln \left[ \cos(2y) \right] = \frac{\ln \left[ \cos(2y) \right]}{y^2}
\]

Step 3
We can now take the limit as \( y \to 0^+ \) of this.

\[
\lim_{y \to 0^+} \left[ \ln z \right] = \lim_{y \to 0^+} \left[ \frac{\ln \left[ \cos(2y) \right]}{y^2} \right]
\]

Before we proceed let’s notice that we have the following,

\[
as \ y \to 0^+ \quad \frac{\ln \left[ \cos(2y) \right]}{y^2} \to \frac{\ln (1)}{0} = \frac{0}{0}
\]

and we have a limit that we can use L’Hospital’s Rule on.

Step 4
Applying L’Hospital’s Rule gives,

\[
\lim_{y \to 0^+} \left[ \ln z \right] = \lim_{y \to 0^+} \left[ \frac{\ln \left[ \cos(2y) \right]}{y^2} \right] = \lim_{y \to 0^+} \frac{-2\sin(2y)}{\cos(2y)} = \lim_{y \to 0^+} \frac{-\tan(2y)}{2y}
\]

Step 5
We now have a limit that behaves like,

\[
as \ y \to 0^+ \quad \frac{-\tan(2y)}{y} \to \frac{0}{0}
\]

and so we can use L’Hospital’s Rule on this as well. Doing this gives,

\[
\lim_{y \to 0^+} \left[ \ln z \right] = \lim_{y \to 0^+} \frac{-\tan(2y)}{y} = \lim_{y \to 0^+} \frac{-2\sec^2(2y)}{1} = -2
\]

Step 6
Now all we need to do is recall that,
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\[ z = e^{\ln z} \]

This in turn means that we can do the original limit as follows,

\[
\lim_{{y \to 0^+}} \left[ \cos \left( 2y \right) \right]^{\frac{1}{y^2}} = \lim_{{y \to 0^+}} z = \lim_{{y \to 0^+}} e^{\ln z} = e^{\lim_{{y \to 0^+}} [\ln z]} = e^{-2}
\]

11. Use L’Hospital’s Rule to evaluate \( \lim_{{x \to \infty}} \left[ e^x + x \right]^{\frac{1}{x}} \).

Step 1
The first thing to notice here is that is not in a form that allows L’Hospital’s Rule. L’Hospital’s Rule only works on certain classes of rational functions and this is clearly not a rational function.

Note however that it is in the following indeterminate form,

\[
as \ x \to \infty \quad \left[ e^x + x \right]^{\frac{1}{x}} \to \infty^0
\]

and as we discussed in the notes for this section we can do some manipulation on this to turn it into a problem that can be done with L’Hospital’s Rule.

Step 2
First, let’s define,

\[ z = \left[ e^x + x \right]^{\frac{1}{x}} \]

and take the log of both sides. We’ll also do a little simplification.

\[
\ln z = \ln \left( \left[ e^x + x \right]^{\frac{1}{x}} \right) = \frac{1}{x} \ln \left[ e^x + x \right] = \frac{\ln \left[ e^x + x \right]}{x}
\]

Step 3
We can now take the limit as \( x \to \infty \) of this.

\[
\lim_{{x \to \infty}} \left[ \ln z \right] = \lim_{{x \to \infty}} \left[ \frac{\ln \left[ e^x + x \right]}{x} \right]
\]

Before we proceed let’s notice that we have the following,
as $x \to \infty$ \[ \frac{\ln\left(e^x + x\right)}{x} \to \infty \]

and we have a limit that we can use L’Hospital’s Rule on.

**Step 4**
Applying L’Hospital’s Rule gives,
\[
\lim_{x \to \infty} \ln \left[ \ln\left(\frac{e^x + x}{x}\right) \right] = \lim_{x \to \infty} \frac{e^x + 1}{e^x + x} = \lim_{x \to \infty} \frac{e^x + 1}{e^x + x} = \lim_{x \to \infty} \frac{e^x + 1}{e^x + x}
\]

**Step 5**
We now have a limit that behaves like,
as $x \to \infty$ \[ \frac{e^x + 1}{e^x + x} \to \infty \]

and so we can use L’Hospital’s Rule on this as well. Doing this gives,
\[
\lim_{x \to \infty} \ln z = \lim_{x \to \infty} \frac{e^x + 1}{e^x + x} = \lim_{x \to \infty} \frac{e^x}{e^x + 1} = \lim_{x \to \infty} \frac{e^x}{e^x} = \lim_{x \to \infty} (1) = 1
\]

Notice that we did have to use L’Hospital’s Rule twice here and we also made sure to do some simplification so we could actually take the limit.

**Step 6**
Now all we need to do is recall that,
\[ z = e^{\ln z} \]

This in turn means that we can do the original limit as follows,
\[
\lim_{x \to \infty} \left[ e^x + x \right]^{\frac{1}{x}} = \lim_{x \to \infty} z = \lim_{x \to \infty} e^{\ln z} = e^{\lim_{x \to \infty} \left[ \ln z \right]} = e
\]