Preface

Here are my online notes for my Calculus I course that I teach here at Lamar University. Despite the fact that these are my “class notes”, they should be accessible to anyone wanting to learn Calculus I or needing a refresher in some of the early topics in calculus.

I’ve tried to make these notes as self contained as possible and so all the information needed to read through them is either from an Algebra or Trig class or contained in other sections of the notes.

Here are a couple of warnings to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn calculus I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn’t covered in class.

2. Because I want these notes to provide some more examples for you to read through, I don’t always work the same problems in class as those given in the notes. Likewise, even if I do work some of the problems in here I may work fewer problems in class than are presented here.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible when writing these up, but the reality is that I can’t anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I’ve not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.
**Differentials**

In this section we’re going to introduce a notation that we’ll be seeing quite a bit in the next chapter. We will also look at an application of this new notation.

Given a function \( y = f(x) \) we call \( dy \) and \( dx \) differentials and the relationship between them is given by,

\[
dy = f'(x) \, dx
\]

Note that if we are just given \( f(x) \) then the differentials are \( df \) and \( dx \) and we compute them in the same manner.

\[
df = f'(x) \, dx
\]

Let’s compute a couple of differentials.

**Example 1** Compute the differential for each of the following.

(a) \( y = t^3 - 4t^2 + 7t \)

(b) \( w = x^2 \sin(2x) \)

(c) \( f(z) = e^{3-z^4} \)

**Solution**

Before working any of these we should first discuss just what we’re being asked to find here. We defined two differentials earlier and here we’re being asked to compute a differential.

So, which differential are we being asked to compute? In this kind of problem we’re being asked to compute the differential of the function. In other words, \( dy \) for the first problem, \( dw \) for the second problem and \( df \) for the third problem.

Here are the solutions. Not much to do here other than take a derivative and don’t forget to add on the second differential to the derivative.

(a) \( dy = (3t^2 - 8t + 7) \, dt \)

(b) \( dw = \left(2x \sin(2x) + 2x^2 \cos(2x)\right) \, dx \)

(c) \( df = -4z^3 e^{3-z^4} \, dz \)
There is a nice application to differentials. If we think of $\Delta x$ as the change in $x$ then $\Delta y = f(x + \Delta x) - f(x)$ is the change in $y$ corresponding to the change in $x$. Now, if $\Delta x$ is small we can assume that $\Delta y \approx dy$. Let’s see an illustration of this idea.

**Example 2** Compute $dy$ and $\Delta y$ if $y = \cos\left(x^2 + 1\right) - x$ as $x$ changes from $x = 2$ to $x = 2.03$.

**Solution**

First let’s compute actual the change in $y$, $\Delta y$.

$$\Delta y = \cos\left((2.03)^2 + 1\right) - 2.03 - \left(\cos\left(2^2 + 1\right) - 2\right) = 0.083581127$$

Now let’s get the formula for $dy$.

$$dy = \left(-2\sin\left(x^2 + 1\right) - 1\right)dx$$

Next, the change in $x$ from $x = 2$ to $x = 2.03$ is $\Delta x = 0.03$ and so we then assume that $dx \approx \Delta x = 0.03$. This gives an approximate change in $y$ of,

$$dy = \left(-2\left(2\sin\left(2^2 + 1\right) - 1\right)(0.03) = 0.085070913$$

We can see that in fact we do have that $\Delta y \approx dy$ provided we keep $\Delta x$ small.

We can use the fact that $\Delta y \approx dy$ in the following way.

**Example 3** A sphere was measured and its radius was found to be 45 inches with a possible error of no more that 0.01 inches. What is the maximum possible error in the volume if we use this value of the radius?

**Solution**

First, recall the equation for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

Now, if we start with $r = 45$ and use $dr \approx \Delta r = 0.01$ then $\Delta V \approx dV$ should give us maximum error.

So, first get the formula for the differential.

$$dV = 4\pi r^2 dr$$

Now compute $dV$.

$$\Delta V \approx dV = 4\pi \left(45\right)^2 \left(0.01\right) = 254.47 \text{ in}^3$$
The maximum error in the volume is then approximately 254.47 in$^3$.

Be careful to not assume this is a large error. On the surface it looks large, however if we compute the actual volume for $r = 45$ we get $V = 381,703.51$ in$^3$. So, in comparison the error in the volume is,

$$\frac{254.47}{381703.51} \times 100 = 0.067\%$$

That’s not much possible error at all!