Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Differentials

1. Compute the differential for \( f(x) = x^2 - \sec(x) \).

Solution
There is not really a whole lot to this problem.

\[
\frac{df}{dx} = (2x - \sec(x)\tan(x))\,dx
\]

Don’t forget to tack on the \( dx \) at the end!

2. Compute the differential for \( w = e^{x^4 - x^2 + 4x} \).

Solution
There is not really a whole lot to this problem.
\[ dw = (4x^3 - 2x + 4)e^{x^4 - x^2 + 4x} \, dx \]

Don’t forget to tack on the \( dx \) at the end!

3. Compute the differential for \( h(z) = \ln(2z) \sin(2z) \).

Solution
There is not really a whole lot to this problem.

\[ dh = \left( \frac{1}{z} \sin(2z) + 2 \ln(2z) \cos(2z) \right) \, dz \]

Don’t forget to tack on the \( dz \) at the end!

4. Compute \( dy \) and \( \Delta y \) for \( y = e^{x^2} \) as \( x \) changes from 3 to 3.01.

Step 1
First let’s get the actual change, \( \Delta y \).

\[ \Delta y = e^{3.01^2} - e^{3^2} = 501.927 \]

Step 2
Next, we’ll need the differential.

\[ dy = 2x e^{x^2} \, dx \]

Step 3
As \( x \) changes from 3 to 3.01 we have \( \Delta x = 3.01 - 3 = 0.01 \) and we’ll assume that \( dx \approx \Delta x = 0.01 \). The approximate change, \( dy \), is then,

\[ dy = 2(3)e^{3.01^2}(0.01) = 486.185 \]

Don’t forget to use the “starting” value of \( x \) (i.e. \( x = 3 \)) for all the \( x \)’s in the differential.
5. Compute $dy$ and $\Delta y$ for $y = x^2 - 2x^3 + 7x$ as $x$ changes from 6 to 5.9.

Step 1
First let’s get the actual change, $\Delta y$.

$$\Delta y = \left(5.9^5 - 2(5.9^3) + 7(5.9)\right) - \left(6^5 - 2(6^3) + 7(6)\right) = -606.215$$

Step 2
Next, we’ll need the differential.

$$dy = \left(5x^4 - 6x^2 + 7\right)dx$$

Step 3
As $x$ changes from 6 to 5.9 we have $\Delta x = 5.9 - 6 = -0.1$ and we’ll assume that $dx \approx \Delta x = -0.1$. The approximate change, $dy$, is then,

$$dy = \left(5(6^4) - 6(6^2) + 7\right)(-0.1) = -627.1$$

Don’t forget to use the “starting” value of $x$ (i.e. $x = 6$) for all the $x$’s in the differential.

6. The sides of a cube are found to be 6 feet in length with a possible error of no more than 1.5 inches. What is the maximum possible error in the volume of the cube if we use this value of the length of the side to compute the volume?

Step 1
Let’s get everything set up first.

If we let the side of the cube be denoted by $x$ the volume is then,

$$V(x) = x^3$$

We are told that $x = 6$ and we can assume that $dx \approx \Delta x = \frac{1.5}{12} = 0.125$ (don’t forget to convert the inches to feet!).

Step 2
We want to estimate the maximum error in the volume and so we can again assume that $\Delta V \approx dV$. 

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The differential is then,

\[ dV = 3x^2 \, dx \]

The maximum error in the volume is then,

\[ \Delta V \approx dV = 3\left(6^2\right)(0.125) = 13.5 \text{ ft}^3 \]