**Preface**

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

**Integrals**

**Indefinite Integrals**

1. Evaluate each of the following indefinite integrals.
   
   (a) \( \int 6x^5 - 18x^2 + 7 \, dx \)

   (b) \( \int 6x^5 \, dx - 18x^2 + 7 \)

   **Hint**: As long as you recall your derivative rules and the fact that all this problem is really asking is the for us to determine the function that we differentiated to get the integrand (\(i.e.\) the function inside the integral….) this problem shouldn’t be too difficult.

   (a) \( \int 6x^5 - 18x^2 + 7 \, dx \)

   All we are being asked to do here is “undo” a differentiation and if you recall the basic differentiation rules for polynomials this shouldn’t be too difficult. As we saw in the notes for
this section all we really need to do is increase the exponent by one (so upon differentiation we get the correct exponent) and then fix up the coefficient to make sure that we will get the correct coefficient upon differentiation.

Here is the answer for this part.

\[ \int 6x^5 - 18x^2 + 7 \, dx = x^6 - 6x^3 + 7x + c \]

Don’t forget the “+c”! Remember that the original function may have had a constant on it and the “+c” is there to remind us of that.

Also don’t forget that you can easily check your answer by differentiating your answer and making sure that the result is the same as the integrand.

\[ (b) \int 6x^5 \, dx - 18x^2 + 7 \]

This part is not really all that different from the first part. The only difference is the placement of the \( dx \). Recall that one of the things that the \( dx \) tells us where to end the integration. So, in the part we are only going to integrate the first term.

Here is the answer for this part.

\[ \int 6x^5 \, dx - 18x^2 + 7 = x^6 + c - 18x^2 + 7 \]

2. Evaluate each of the following indefinite integrals.

\( (a) \int 40x^3 + 12x^2 - 9x + 14 \, dx \)

\( (b) \int 40x^3 + 12x^2 - 9x \, dx + 14 \)

\( (c) \int 40x^3 + 12x^2 \, dx - 9x + 14 \)

Hint: As long as you recall your derivative rules and the fact that all this problem is really asking is for us to determine the function that we differentiated to get the integrand (i.e. the function inside the integral...) this problem shouldn’t be too difficult.

\( (a) \int 40x^3 + 12x^2 - 9x + 14 \, dx \)

All we are being asked to do here is “undo” a differentiation and if you recall the basic differentiation rules for polynomials this shouldn’t be too difficult. As we saw in the notes for this section all we really need to do is increase the exponent by one (so upon differentiation we
get the correct exponent) and then fix up the coefficient to make sure that we will get the correct coefficient upon differentiation.

Here is that answer for this part.

\[ \int 40x^3 + 12x^2 - 9x + 14 \, dx = 10x^4 + 4x^3 - \frac{9}{2}x^2 + 14x + c \]

Don’t forget the “+c”! Remember that the original function may have had a constant on it and the “+c” is there to remind us of that.

Also don’t forget that you can easily check your answer by differentiating your answer and making sure that the result is the same as the integrand.

(b) \[ \int 40x^3 + 12x^2 - 9x \, dx + 14 \]

This part is not really all that different from the first part. The only difference is the placement of the \( dx \). Recall that one of the things that the \( dx \) tells us where to end the integration. So, in the part we are only going to integrate the first term.

Here is the answer for this part.

\[ \int 40x^3 + 12x^2 - 9x \, dx + 14 = 10x^4 + 4x^3 - \frac{9}{2}x^2 + c + 14 \]

(c) \[ \int 40x^3 + 12x^2 \, dx - 9x + 14 \]

The only difference between this part and the previous part is that the location of the \( dx \) moved.

Here is the answer for this part.

\[ \int 40x^3 + 12x^2 \, dx - 9x + 14 = 10x^4 + 4x^3 + c - 9x + 14 \]

3. Evaluate \( \int 12t^7 - t^2 - t + 3 \, dt \).

Hint: As long as you recall your derivative rules and the fact that all this problem is really asking is the for us to determine the function that we differentiated to get the integrand (i.e. the function inside the integral….) this problem shouldn’t be too difficult.

Solution
All we are being asked to do here is “undo” a differentiation and if you recall the basic differentiation rules for polynomials this shouldn’t be too difficult. As we saw in the notes for
this section all we really need to do is increase the exponent by one (so upon differentiation we get the correct exponent) and then fix up the coefficient to make sure that we will get the correct coefficient upon differentiation.

Here is the answer.

\[ \int 12t^7 - t^2 - t + 3 \, dt = \frac{1}{8}t^8 - \frac{1}{3}t^3 - \frac{1}{2}t^2 + 3t + c \]

Don’t forget the “+c”! Remember that the original function may have had a constant on it and the “+c” is there to remind us of that.

Also don’t forget that you can easily check your answer by differentiating your answer and making sure that the result is the same as the integrand.

4. Evaluate \( \int 10w^4 + 9w^3 + 7w \, dw \).

Hint: As long as you recall your derivative rules and the fact that all this problem is really asking is the for us to determine the function that we differentiated to get the integrand (i.e. the function inside the integral…) this problem shouldn’t be too difficult.

Solution
All we are being asked to do here is “undo” a differentiation and if you recall the basic differentiation rules for polynomials this shouldn’t be too difficult. As we saw in the notes for this section all we really need to do is increase the exponent by one (so upon differentiation we get the correct exponent) and then fix up the coefficient to make sure that we will get the correct coefficient upon differentiation.

Here is the answer.

\[ \int 10w^4 + 9w^3 + 7w \, dw = 2w^5 + \frac{9}{4}w^4 + \frac{7}{2}w^2 + c \]

Don’t forget the “+c”! Remember that the original function may have had a constant on it and the “+c” is there to remind us of that.

Also don’t forget that you can easily check your answer by differentiating your answer and making sure that the result is the same as the integrand.
5. Evaluate \( \int (z^6 + 4z^4 - z^2) \, dz \).

Hint: As long as you recall your derivative rules and the fact that all this problem is really asking is the for us to determine the function that we differentiated to get the integrand (i.e. the function inside the integral…) this problem shouldn’t be too difficult.

Solution
All we are being asked to do here is “undo” a differentiation and if you recall the basic differentiation rules for polynomials this shouldn’t be too difficult. As we saw in the notes for this section all we really need to do is increase the exponent by one (so upon differentiation we get the correct exponent) and then fix up the coefficient to make sure that we will get the correct coefficient upon differentiation.

Here is the answer.

\[
\int (z^6 + 4z^4 - z^2) \, dz = \frac{1}{7}z^7 + \frac{4}{5}z^5 - \frac{1}{3}z^3 + c
\]

Don’t forget the “+c”! Remember that the original function may have had a constant on it and the “+c” is there to remind us of that.

Also don’t forget that you can easily check your answer by differentiating your answer and making sure that the result is the same as the integrand.

6. Determine \( f(x) \) given that \( f'(x) = 6x^8 - 20x^4 + x^2 + 9 \).

Hint: Remember that all indefinite integrals are asking us to do is “undo” a differentiation.

Solution
We know that indefinite integrals are asking us to undo a differentiation to so all we are really being asked to do here is evaluate the following indefinite integral.

\[
f(x) = \int f'(x) \, dx = \int (6x^8 - 20x^4 + x^2 + 9) \, dx = \frac{2}{9}x^9 - \frac{20}{5}x^5 + \frac{1}{3}x^3 + 9x + c
\]

Don’t forget the “+c”! Remember that the original function may have had a constant on it and the “+c” is there to remind us of that.

7. Determine \( h(t) \) given that \( h'(t) = t^4 - t^3 + t^2 + t - 1 \).
Hint: Remember that all indefinite integrals are asking us to do is “undo” a differentiation.

Solution
We know that indefinite integrals are asking us to undo a differentiation to so all we are really being asked to do here is evaluate the following indefinite integral.

\[
h(t) = \int h'(t)\, dt = \int (t^4 - t^3 + t^2 + t - 1)\, dt = \frac{1}{5}t^5 - \frac{1}{4}t^4 + \frac{1}{3}t^3 + \frac{1}{2}t^2 - t + c\\
\]

Don’t forget the “+c”! Remember that the original function may have had a constant on it and the “+c” is there to remind us of that.