Preface

Here are my online notes for my Calculus I course that I teach here at Lamar University. Despite the fact that these are my “class notes” they should be accessible to anyone wanting to learn Calculus I or needing a refresher in some of the early topics in calculus.

I’ve tried to make these notes as self contained as possible and so all the information needed to read through them is either from an Algebra or Trig class or contained in other sections of the notes.

Here are a couple of warnings to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn calculus I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn’t covered in class.

2. Because I want these notes to provide some more examples for you to read through, I don’t always work the same problems in class as those given in the notes. Likewise, even if I do work some of the problems in here I may work fewer problems in class than are presented here.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible when writing these up, but the reality is that I can’t anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I’ve not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.
**Summation Notation**

In this section we need to do a brief review of summation notation or sigma notation. We’ll start out with two integers, \( n \) and \( m \), with \( n < m \) and a list of numbers denoted as follows,

\[
a_n, a_{n+1}, a_{n+2}, \ldots, a_{m-2}, a_{m-1}, a_m
\]

We want to add them up, in other words we want,

\[
a_n + a_{n+1} + a_{n+2} + \ldots + a_{m-2} + a_{m-1} + a_m
\]

For large lists this can be a fairly cumbersome notation so we introduce summation notation to denote these kinds of sums. The case above is denoted as follows.

\[
\sum_{i=n}^{m} a_i = a_n + a_{n+1} + a_{n+2} + \ldots + a_{m-2} + a_{m-1} + a_m
\]

The \( i \) is called the index of summation. This notation tells us to add all the \( a_i \)'s up for all integers starting at \( n \) and ending at \( m \).

For instance,

\[
\sum_{i=1}^{4} \frac{i}{i+1} = \frac{0}{0+1} + \frac{1}{1+1} + \frac{2}{2+1} + \frac{3}{3+1} + \frac{4}{4+1} = \frac{163}{60} = 2.71666
\]

\[
\sum_{i=4}^{6} 2^i x^{2i+1} = 2^4 x^9 + 2^5 x^{11} + 2^6 x^{13} = 16x^9 + 32x^{11} + 64x^{13}
\]

\[
\sum_{i=1}^{4} f(x^*_i) = f(x^*_1) + f(x^*_2) + f(x^*_3) + f(x^*_4)
\]

**Properties**

Here are a couple of formulas for summation notation.

1. \( \sum_{i=1}^{n} c a_i = c \sum_{i=1}^{n} a_i \) where \( c \) is any number. So, we can factor constants out of a summation.

2. \( \sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i \) So we can break up a summation across a sum or difference.

Note that we started the series at \( i_0 \) to denote the fact that they can start at any value of \( i \) that we need them to. Also note that while we can break up sums and differences as we did in 2 above we can’t do the same thing for products and quotients. In other words,

\[
\sum_{i=i_0}^{n} (a_i b_i) \neq \left( \sum_{i=i_0}^{n} a_i \right) \left( \sum_{i=i_0}^{n} b_i \right)
\]

\[
\sum_{i=i_0}^{n} \frac{a_i}{b_i} \neq \frac{\sum_{i=i_0}^{n} a_i}{\sum_{i=i_0}^{n} b_i}
\]
Formulas

Here are a couple of nice formulas that we will find useful in a couple of sections. Note that these formulas are only true if starting at $i = 1$. You can, of course, derive other formulas from these for different starting points if you need to.

1. $\sum_{i=1}^{n} c = cn$
2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Here is a quick example on how to use these properties to quickly evaluate a sum that would not be easy to do by hand.

**Example 1** Using the formulas and properties from above determine the value of the following summation.

$$\sum_{i=1}^{100} (3-2i)^2$$

**Solution**

The first thing that we need to do is square out the stuff being summed and then break up the summation using the properties as follows,

$$\sum_{i=1}^{100} (3-2i)^2 = \sum_{i=1}^{100} 9 - 12i + 4i^2$$

$$= \sum_{i=1}^{100} 9 - \sum_{i=1}^{100} 12i + \sum_{i=1}^{100} 4i^2$$

$$= \sum_{i=1}^{100} 9 - 12\sum_{i=1}^{100} i + 4\sum_{i=1}^{100} i^2$$

Now, using the formulas, this is easy to compute,

$$\sum_{i=1}^{100} (3-2i)^2 = 9(100) - 12\left(\frac{100(101)}{2}\right) + 4\left(\frac{100(101)(201)}{6}\right)$$

$$= 1293700$$

Doing this by hand would definitely taken some time and there’s a good chance that we might have made a minor mistake somewhere along the line.