Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

**Review : Trig Functions**

1. Determine the exact value of $\cos \left( \frac{5\pi}{6} \right)$ without using a calculator.

   **Hint 1 :** Sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

   **Step 1**

   First we can notice that $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ and so the terminal line for $\frac{5\pi}{6}$ will form an angle of $\frac{\pi}{6}$ with the negative x-axis in the second quadrant and we’ll have the following unit circle for this problem.
Hint 2: Given the obvious symmetry in the unit circle relate the coordinates of the line representing \(\frac{5\pi}{6}\) to the coordinates of the line representing \(\frac{\pi}{6}\) and use those to answer the question.

Step 2

The coordinates of the line representing \(\frac{5\pi}{6}\) will be the same as the coordinates of the line representing \(\frac{\pi}{6}\) except that the \(x\) coordinate will now be negative. So, our new coordinates will then be \(\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)\) and so the answer is,

\[
\cos \left( \frac{5\pi}{6} \right) = -\frac{\sqrt{3}}{2}
\]
2. Determine the exact value of \( \sin \left( -\frac{4\pi}{3} \right) \) without using a calculator.

Hint 1: Sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

Step 1
First we can notice that \(-\pi - \frac{\pi}{3} = -\frac{4\pi}{3}\) and so (remembering that negative angles are rotated clockwise) we can see that the terminal line for \(-\frac{4\pi}{3}\) will form an angle of \(\frac{\pi}{3}\) with the negative x-axis in the second quadrant and we’ll have the following unit circle for this problem.
Hint 2: Given the obvious symmetry in the unit circle relate the coordinates of the line representing \(-\frac{4\pi}{3}\) to the coordinates of the line representing \(\frac{\pi}{3}\) and use those to answer the question.

Step 2
The coordinates of the line representing \(-\frac{4\pi}{3}\) will be the same as the coordinates of the line representing \(\frac{\pi}{3}\) except that the x coordinate will now be negative. So, our new coordinates will then be \((-\frac{1}{2}, \frac{\sqrt{3}}{2})\) and so the answer is,

\[
\sin\left(-\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2}
\]

3. Determine the exact value of \(\sin\left(\frac{7\pi}{4}\right)\) without using a calculator.

Hint 1: Sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

Step 1
First we can notice that \(2\pi - \frac{\pi}{4} = \frac{7\pi}{4}\) and so the terminal line for \(\frac{7\pi}{4}\) will form an angle of \(\frac{\pi}{4}\) with the positive x-axis in the fourth quadrant and we’ll have the following unit circle for this problem.
Hint 2 : Given the obvious symmetry in the unit circle relate the coordinates of the line representing $\frac{7\pi}{4}$ to the coordinates of the line representing $\frac{\pi}{4}$ and use those to answer the question.

Step 2

The coordinates of the line representing $\frac{7\pi}{4}$ will be the same as the coordinates of the line representing $\frac{\pi}{4}$ except that the $y$ coordinate will now be negative. So, our new coordinates will then be $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ and so the answer is,

$$\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$
4. Determine the exact value of \( \cos\left(-\frac{2\pi}{3}\right) \) without using a calculator.

**Hint 1:** Sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

**Step 1**

First we can notice that \(-\pi + \frac{\pi}{3} = -\frac{2\pi}{3}\) so (recalling that negative angles rotate clockwise and positive angles rotation counter clockwise) the terminal line for \(-\frac{2\pi}{3}\) will form an angle of \(\frac{\pi}{3}\) with the negative x-axis in the third quadrant and we’ll have the following unit circle for this problem.
Hint 2 : Given the obvious symmetry in the unit circle relate the coordinates of the line representing \(-\frac{2\pi}{3}\) to the coordinates of the line representing \(\frac{\pi}{3}\) and use those to answer the question.

**Step 2**

The line representing \(-\frac{2\pi}{3}\) is a mirror image of the line representing \(\frac{\pi}{3}\) and so the coordinates for \(-\frac{2\pi}{3}\) will be the same as the coordinates for \(\frac{\pi}{3}\) except that both coordinates will now be negative. So, our new coordinates will then be \((-\frac{1}{2}, -\frac{\sqrt{3}}{2})\) and so the answer is,

\[
\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}
\]

5. Determine the exact value of \(\tan\left(\frac{3\pi}{4}\right)\) without using a calculator.

**Hint 1** : Even though a unit circle only tells us information about sine and cosine it is still useful for tangents so sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

**Step 1**

First we can notice that \(\pi - \frac{\pi}{4} = \frac{3\pi}{4}\) and so (remembering that negative angles are rotated clockwise) we can see that the terminal line for \(\frac{3\pi}{4}\) will form an angle of \(\frac{\pi}{4}\) with the negative x-axis in the second quadrant and we’ll have the following unit circle for this problem.
Hint 2 : Given the obvious symmetry in the unit circle relate the coordinates of the line representing $\frac{3\pi}{4}$ to the coordinates of the line representing $\frac{\pi}{4}$ and then recall how tangent is defined in terms of sine and cosine to answer the question.

Step 2

The coordinates of the line representing $\frac{3\pi}{4}$ will be the same as the coordinates of the line representing $\frac{\pi}{4}$ except that the $x$ coordinate will now be negative. So, our new coordinates will then be $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and so the answer is,
6. Determine the exact value of $\sec\left(-\frac{11\pi}{6}\right)$ without using a calculator.

Hint 1: Even though a unit circle only tells us information about sine and cosine it is still useful for secant so sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

Step 1
First we can notice that $\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$ and so (remembering that negative angles are rotated clockwise) we can see that the terminal line for $-\frac{11\pi}{6}$ will form an angle of $\frac{\pi}{6}$ with the positive $x$-axis in the first quadrant. In other words $-\frac{11\pi}{6}$ and $\frac{\pi}{6}$ represent the same angle. So, we’ll have the following unit circle for this problem.
Hint 2: Given the obvious symmetry here use the definition of secant in terms of cosine to write down the solution.

Step 2
Because the two angles $\frac{11\pi}{6}$ and $\frac{\pi}{6}$ have the same coordinates the answer is,

$$\sec\left(-\frac{11\pi}{6}\right) = \frac{1}{\cos\left(-\frac{11\pi}{6}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

7. Determine the exact value of $\cos\left(\frac{8\pi}{3}\right)$ without using a calculator.

Hint 1: Sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.
Step 1

First we can notice that \( 2\pi + \frac{2\pi}{3} = \frac{8\pi}{3} \) and because \( 2\pi \) is one complete revolution the angles \( \frac{8\pi}{3} \) and \( \frac{2\pi}{3} \) are the same angle. Also, note that \( \pi - \frac{\pi}{3} = \frac{2\pi}{3} \) and so the terminal line for \( \frac{8\pi}{3} \) will form an angle of \( \frac{\pi}{3} \) with the negative \( x \)-axis in the second quadrant and we’ll have the following unit circle for this problem.

![Unit Circle Diagram](image)

Hint 2: Given the obvious symmetry in the unit circle relate the coordinates of the line representing \( \frac{8\pi}{3} \) to the coordinates of the line representing \( \frac{2\pi}{3} \) and use those to answer the question.

Step 2
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The coordinates of the line representing \( \frac{8\pi}{3} \) will be the same as the coordinates of the line representing \( \frac{\pi}{3} \) except that the \( x \) coordinate will now be negative. So, our new coordinates will then be \( \left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right) \) and so the answer is,

\[
\cos \left( \frac{8\pi}{3} \right) = -\frac{1}{2}
\]

8. Determine the exact value of \( \tan \left( -\frac{\pi}{3} \right) \) without using a calculator.

Hint 1: Even though a unit circle only tells us information about sine and cosine it is still useful for tangents so sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

Step 1

To do this problem all we need to notice is that \( -\frac{\pi}{3} \) will form an angle of \( \frac{\pi}{3} \) with the positive \( x \)-axis in the fourth quadrant and we’ll have the following unit circle for this problem.
Hint 2: Given the obvious symmetry in the unit circle relate the coordinates of the line representing $-\frac{\pi}{3}$ to the coordinates of the line representing $\frac{\pi}{3}$ and use the definition of tangent in terms of sine and cosine to answer the question.

Step 2

The coordinates of the line representing $-\frac{\pi}{3}$ will be the same as the coordinates of the line representing $\frac{\pi}{3}$ except that the $y$ coordinate will now be negative. So, our new coordinates will then be $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ and so the answer is,
9. Determine the exact value of \( \tan\left(\frac{15\pi}{4}\right) \) without using a calculator.

Hint 1: Even though a unit circle only tells us information about sine and cosine it is still useful for tangents so sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

Step 1
First we can notice that \( 4\pi - \frac{\pi}{4} = \frac{15\pi}{4} \) and also note that \( 4\pi \) is two complete revolutions so the terminal line for \( \frac{15\pi}{4} \) and \(-\frac{\pi}{4}\) represent the same angle. Also note that \(-\frac{\pi}{4}\) will form an angle of \(-\frac{\pi}{4}\) with the positive \(x\)-axis in the fourth quadrant and we’ll have the following unit circle for this problem.
Hint 2 : Given the obvious symmetry in the unit circle relate the coordinates of the line representing $\frac{15\pi}{4}$ to the coordinates of the line representing $\frac{\pi}{4}$ and the definition of tangent in terms of sine and cosine to answer the question.

Step 2
The coordinates of the line representing $\frac{15\pi}{4}$ will be the same as the coordinates of the line representing $\frac{\pi}{4}$ except that the y coordinate will now be negative. So, our new coordinates will then be $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ and so the answer is,
10. Determine the exact value of \( \sin \left( -\frac{11\pi}{3} \right) \) without using a calculator.

Hint 1 : Sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

Step 1

First we can notice that \( \frac{\pi}{3} - 4\pi = -\frac{11\pi}{3} \) and note that \( 4\pi \) is two complete revolutions (also, remembering that negative angles are rotated clockwise) we can see that the terminal line for \( -\frac{11\pi}{3} \) and \( \frac{\pi}{3} \) are the same angle and so we’ll have the following unit circle for this problem.
Hint 2 : Given the very obvious symmetry here write down the answer to the question.

Step 2
Because \(-\frac{11\pi}{3}\) and \(\frac{\pi}{3}\) are the same angle the answer is,

\[
\sin \left( -\frac{11\pi}{3} \right) = \frac{\sqrt{3}}{2}
\]

11. Determine the exact value of \(\sec \left( \frac{29\pi}{4} \right)\) without using a calculator.

Hint 1 : Even though a unit circle only tells us information about sine and cosine it is still useful for secant so sketch a unit circle and relate the angle to one of the standard angles in the first quadrant.

Step 1
First we can notice that \(\frac{5\pi}{4} + 6\pi = \frac{25\pi}{4}\) and recalling that \(6\pi\) is three complete revolutions we can see that \(\frac{25\pi}{4}\) and \(\frac{5\pi}{4}\) represent the same angle. Next, note that \(\pi + \frac{\pi}{4} = \frac{5\pi}{4}\) and so the line representing \(\frac{5\pi}{4}\) will form an angle of \(\frac{\pi}{4}\) with the negative x-axis in the third quadrant and we’ll have the following unit circle for this problem.
Hint 2 : Given the obvious symmetry in the unit circle relate the coordinates of the line representing $\frac{25\pi}{4}$ to the coordinates of the line representing $\frac{\pi}{4}$ and the recall how secant is defined in terms of cosine to answer the question.

Step 2

The line representing $\frac{25\pi}{4}$ is a mirror image of the line representing $\frac{\pi}{4}$ and so the coordinates for $\frac{25\pi}{4}$ will be the same as the coordinates for $\frac{\pi}{4}$ except that both coordinates will now be negative. So, our new coordinates will then be $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ and so the answer is,
\[
\sec \left( \frac{29\pi}{4} \right) = \frac{1}{\sqrt{2}/2} = 2 = -\sqrt{2}
\]