Here are my online notes for my Calculus I course that I teach here at Lamar University. Despite the fact that these are my “class notes” they should be accessible to anyone wanting to learn Calculus I or needing a refresher in some of the early topics in calculus.

I’ve tried to make these notes as self contained as possible and so all the information needed to read through them is either from an Algebra or Trig class or contained in other sections of the notes.

Here are a couple of warnings to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn calculus I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn’t covered in class.

2. Because I want these notes to provide some more examples for you to read through, I don’t always work the same problems in class as those given in the notes. Likewise, even if I do work some of the problems in here I may work fewer problems in class than are presented here.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible when writing these up, but the reality is that I can’t anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I’ve not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.
**Review: Exponential and Logarithm Equations**

In this section we’ll take a look at solving equations with exponential functions or logarithms in them.

We’ll start with equations that involve exponential functions. The main property that we’ll need for these equations is,

\[ \log_b b^x = x \]

**Example 1** Solve \( 7 + 15e^{1-3z} = 10 \).

**Solution**
The first step is to get the exponential all by itself on one side of the equation with a coefficient of one.

\[
\begin{align*}
7 + 15e^{1-3z} &= 10 \\
15e^{1-3z} &= 3 \\
e^{1-3z} &= \frac{1}{5}
\end{align*}
\]

Now, we need to get the \( z \) out of the exponent so we can solve for it. To do this we will use the property above. Since we have an \( e \) in the equation we’ll use the natural logarithm. First we take the logarithm of both sides and then use the property to simplify the equation.

\[
\ln\left(e^{1-3z}\right) = \ln\left(\frac{1}{5}\right)
\]

\[
1 - 3z = \ln\left(\frac{1}{5}\right)
\]

All we need to do now is solve this equation for \( z \).

\[
1 - 3z = \ln\left(\frac{1}{5}\right)
\]

\[
-3z = -1 + \ln\left(\frac{1}{5}\right)
\]

\[
z = \frac{1}{3}\left(-1 + \ln\left(\frac{1}{5}\right)\right) = 0.8698126372
\]

**Example 2** Solve \( 10^{3-r} = 100 \).

**Solution**
Now, in this case it looks like the best logarithm to use is the common logarithm since left hand side has a base of 10. There’s no initial simplification to do, so just take the log of both sides and simplify.
\[
\log_{10} t^2 \cdot t = \log_{10} 100
\]
\[
t^2 - t = 2
\]
At this point, we’ve just got a quadratic that can be solved
\[
t^2 - t - 2 = 0
\]
\[
(t - 2)(t + 1) = 0
\]
So, it looks like the solutions in this case are \( t = 2 \) and \( t = -1 \).

Now that we’ve seen a couple of equations where the variable only appears in the exponent we need to see an example with variables both in the exponent and out of it.

**Example 3** Solve \( x - xe^{5x + 2} = 0 \).

**Solution**
The first step is to factor an \( x \) out of both terms.

**DO NOT DIVIDE AN \( x \) FROM BOTH TERMS!!!!**

Note that it is very tempting to “simplify” the equation by dividing an \( x \) out of both terms. However, if you do that you’ll miss a solution as we’ll see.

\[
x - xe^{5x + 2} = 0
\]
\[
x(1 - e^{5x + 2}) = 0
\]

So, it’s now a little easier to deal with. From this we can see that we get one of two possibilities.

\[
x = 0 \quad \text{OR} \quad 1 - e^{5x + 2} = 0
\]

The first possibility has nothing more to do, except notice that if we had divided both sides by an \( x \) we would have missed this one so be careful. In the second possibility we’ve got a little more to do. This is an equation similar to the first two that we did in this section.

\[
e^{5x + 2} = 1
\]
\[
5x + 2 = \ln 1
\]
\[
5x + 2 = 0
\]
\[
x = -\frac{2}{5}
\]

Don’t forget that \( \ln 1 = 0 \)!

So, the two solutions are \( x = 0 \) and \( x = -\frac{2}{5} \).
The next equation is a more complicated (looking at least…) example similar to the previous one.

**Example 4** Solve \(5(x^2 - 4) = (x^2 - 4)e^{7-x} \).

**Solution**

As with the previous problem do NOT divide an \(x^2 - 4\) out of both sides. Doing this will lose solutions even though it “simplifies” the equation. Note however, that if you can divide a term out then you can also factor it out if the equation is written properly.

So, the first step here is to move everything to one side of the equation and then to factor out the \(x^2 - 4\).

\[
5(x^2 - 4) - (x^2 - 4)e^{7-x} = 0 \\
(x^2 - 4)(5 - e^{7-x}) = 0
\]

At this point all we need to do is set each factor equal to zero and solve each.

\[
x^2 - 4 = 0 \quad 5 - e^{7-x} = 0
\]

\[
x = \pm 2 \quad e^{7-x} = 5
\]

\[
7 - x = \ln(5) \\
x = 7 - \ln(5) = 5.390562088
\]

The three solutions are then \(x = \pm 2\) and \(x = 5.3906\).

As a final example let’s take a look at an equation that contains two different logarithms.

**Example 5** Solve \(4e^{1+3x} - 9e^{5-2x} = 0\).

**Solution**

The first step here is to get one exponential on each side and then we’ll divide both sides by one of them (which doesn’t matter for the most part) so we’ll have a quotient of two exponentials. The quotient can then be simplified and we’ll finally get both coefficients on the other side. Doing all of this gives,

\[
4e^{1+3x} = 9e^{5-2x} \\
e^{1+3x} = \frac{9}{4}e^{5-2x} \\
e^{1+3x-(5-2x)} = \frac{9}{4} \\
e^{5x-4} = \frac{9}{4}
\]
Note that while we said that it doesn’t really matter which exponential we divide out by doing it the way we did here we’ll avoid a negative coefficient on the $x$. Not a major issue, but those minus signs on coefficients are really easy to lose on occasion.

This is now in a form that we can deal with so here’s the rest of the solution.

$$e^{5x-4} = \frac{9}{4}$$

$$5x - 4 = \ln\left(\frac{9}{4}\right)$$

$$5x = 4 + \ln\left(\frac{9}{4}\right)$$

$$x = \frac{1}{5}\left(4 + \ln\left(\frac{9}{4}\right)\right) = 0.9621860432$$

This equation has a single solution of $x = 0.9622$.

Now let’s take a look at some equations that involve logarithms. The main property that we’ll be using to solve these kinds of equations is, $b^{\log_b x} = x$

**Example 6** Solve $3 + 2 \ln\left(\frac{x}{7} + 3\right) = -4$.

**Solution**

This first step in this problem is to get the logarithm by itself on one side of the equation with a coefficient of 1.

$$2 \ln\left(\frac{x}{7} + 3\right) = -7$$

$$\ln\left(\frac{x}{7} + 3\right) = -\frac{7}{2}$$

Now, we need to get the $x$ out of the logarithm and the best way to do that is to “exponentiate” both sides using $e$. In other word,

$$e^{\ln\left(\frac{x}{7} + 3\right)} = e^{-\frac{7}{2}}$$

So using the property above with $e$, since there is a natural logarithm in the equation, we get,

$$\frac{x}{7} + 3 = e^{-\frac{7}{2}}$$

Now all that we need to do is solve this for $x$.

$$\frac{x}{7} + 3 = e^{-\frac{7}{2}}$$

$$\frac{x}{7} = -3 + e^{-\frac{7}{2}}$$

$$x = 7\left(-3 + e^{-\frac{7}{2}}\right) = -20.78861832$$
At this point we might be tempted to say that we’re done and move on. However, we do need to be careful. Recall from the previous section that we can’t plug a negative number into a logarithm. This, by itself, doesn’t mean that our answer won’t work since its negative. What we need to do is plug it into the logarithm and make sure that $\frac{x}{7} + 3$ will not be negative. I’ll leave it to you to verify that this is in fact positive upon plugging our solution into the logarithm and so $x = -20.78861832$ is in fact a solution to the equation.

Let’s now take a look at a more complicated equation. Often there will be more than one logarithm in the equation. When this happens we will need to use one or more of the following properties to combine all the logarithms into a single logarithm. Once this has been done we can proceed as we did in the previous example.

$$\log_b xy = \log_b x + \log_b y \quad \log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y \quad \log_b (x^r) = r \log_b x$$

**Example 7** Solve $2 \ln \left(\sqrt{x}\right) - \ln (1-x) = 2$.

**Solution**
First get the two logarithms combined into a single logarithm.

\[
2 \ln \left(\sqrt{x}\right) - \ln (1-x) = 2 \\
\ln \left(\left(\sqrt{x}\right)^2\right) - \ln (1-x) = 2 \\
\ln (x) - \ln (1-x) = 2 \\
\ln \left(\frac{x}{1-x}\right) = 2
\]

Now, exponentiate both sides and solve for $x$.

\[
\frac{x}{1-x} = e^2 \\
x = e^2 (1-x) \\
x = e^2 - e^2 x \\
x (1 + e^2) = e^2 \\
x = \frac{e^2}{1+e^2} = 0.8807970780
\]

Finally, we just need to make sure that the solution, $x = 0.8807970780$, doesn’t produce negative numbers in both of the original logarithms. It doesn’t, so this is in fact our solution to this problem.

Let’s take a look at one more example.
Example 8 Solve \( \log x + \log (x - 3) = 1 \).

Solution
As with the last example, first combine the logarithms into a single logarithm.

\[
\log x + \log (x - 3) = 1 \\
\log (x(x - 3)) = 1
\]

Now exponentiate, using 10 this time instead of \( e \) because we’ve got common logs in the equation, both sides.

\[
10^{\log (x^2 - 3x)} = 10^1 \\
x^2 - 3x = 10 \\
x^2 - 3x - 10 = 0 \\
(x - 5)(x + 2) = 0
\]

So, potential solutions are \( x = 5 \) and \( x = -2 \). Note, however that if we plug \( x = -2 \) into either of the two original logarithm numbers we would get negative numbers so this can’t be a solution. We can however, use \( x = 5 \).

Therefore, the solution to this equation is \( x = 5 \).

When solving equations with logarithms it is important to check your potential solutions to make sure that they don’t generate logarithms of negative numbers or zero. It is also important to make sure that you do the checks in the original equation. If you check them in the second logarithm above (after we’ve combined the two logs) both solutions will appear to work! This is because in combining the two logarithms we’ve actually changed the problem. In fact, it is this change that introduces the extra solution that we couldn’t use!

Also be careful in solving equations containing logarithms to not get locked into the idea that you will get two potential solutions and only one of these will work. It is possible to have problems where both are solutions and where neither are solutions.