Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Review: Common Graphs

1. Without using a graphing calculator sketch the graph of \( y = \frac{4}{3}x - 2 \).

Solution

This is just a line with slope \( \frac{4}{3} \) and y-intercept \( (0,-2) \) so here is the graph.

![Graph of the line](image)
2. Without using a graphing calculator sketch the graph of \( f(x) = |x - 3| \).

Hint: Recall that the graph of \( g(x + c) \) is simply the graph of \( g(x) \) shifted right by \( c \) units if \( c < 0 \) or shifted left by \( c \) units if \( c > 0 \).

Solution

Recall the basic Algebraic transformations. If we know the graph of \( g(x) \) then the graph of \( g(x + c) \) is simply the graph of \( g(x) \) shifted right by \( c \) units if \( c < 0 \) or shifted left by \( c \) units if \( c > 0 \).

So, in our case if \( g(x) = |x| \) we can see that,

\[
f(x) = |x - 3| = g(x - 3)
\]

and so the graph we’re being asked to sketch is the graph of the absolute value function shifted right by 3 units.

Here is the graph of \( f(x) = |x - 3| \) and note that to help see the transformation we have also sketched in the graph of \( g(x) = |x| \).

3. Without using a graphing calculator sketch the graph of \( g(x) = \sin(x) + 6 \).
Hint: Recall that the graph of \( f(x) + c \) is simply the graph of \( f(x) \) shifted down by \( c \) units if \( c < 0 \) or shifted up by \( c \) units if \( c > 0 \).

Solution
Recall the basic Algebraic transformations. If we know the graph of \( f(x) \) then the graph of \( f(x) + c \) is simply the graph of \( f(x) \) shifted down by \( c \) units if \( c < 0 \) or shifted up by \( c \) units if \( c > 0 \).

So, in our case if \( f(x) = \sin(x) \) we can see that,
\[
g(x) = \sin(x) + 6 = f(x) + 6
\]
and so the graph we’re being asked to sketch is the graph of the sine function shifted up by 6 units.

Here is the graph of \( g(x) = \sin(x) + 6 \) and note that to help see the transformation we have also sketched in the graph of \( f(x) = \sin(x) \).

4. Without using a graphing calculator sketch the graph of \( f(x) = \ln(x) - 5 \).

Hint: Recall that the graph of \( g(x) + c \) is simply the graph of \( g(x) \) shifted down by \( c \) units if \( c < 0 \) or shifted up by \( c \) units if \( c > 0 \).

Solution
Recall the basic Algebraic transformations. If we know the graph of $g(x)$ then the graph of $g(x)+c$ is simply the graph of $g(x)$ shifted down by $c$ units if $c < 0$ or shifted up by $c$ units if $c > 0$.

So, in our case if $g(x) = \ln(x)$ we can see that,

$$f(x) = \ln(x) - 5 = g(x) - 5$$

and so the graph we’re being asked to sketch is the graph of the natural logarithm function shifted down by 5 units.

Here is the graph of $f(x) = \ln(x) - 5$ and note that to help see the transformation we have also sketched in the graph of $g(x) = \ln(x)$.

5. Without using a graphing calculator sketch the graph of $h(x) = \cos \left( x + \frac{\pi}{2} \right)$.

Hint : Recall that the graph of $g(x+c)$ is simply the graph of $g(x)$ shifted right by $c$ units if $c < 0$ or shifted left by $c$ units if $c > 0$.

Solution
Recall the basic Algebraic transformations. If we know the graph of $g(x)$ then the graph of $g(x+c)$ is simply the graph of $g(x)$ shifted right by $c$ units if $c < 0$ or shifted left by $c$ units if $c > 0$. 

So, in our case if \( g(x) = \cos(x) \) we can see that,
\[
h(x) = \cos\left( x + \frac{\pi}{2} \right) = g\left( x + \frac{\pi}{2} \right)
\]
and so the graph we’re being asked to sketch is the graph of the cosine function shifted left by \( \frac{\pi}{2} \) units.

Here is the graph of \( h(x) = \cos\left( x + \frac{\pi}{2} \right) \) and note that to help see the transformation we have also sketched in the graph of \( g(x) = \cos(x) \).

6. Without using a graphing calculator sketch the graph of \( h(x) = (x - 3)^2 + 4 \).

Hint : The Algebraic transformations that we used to help us graph the first few graphs in this section can be used together to shift the graph of a function both up/down and right/left at the same time.

Solution
The Algebraic transformations we were using in the first few problems of this section can be combined to shift a graph up/down and right/left at the same time. If we know the graph of \( g(x) \) then the graph of \( g(x + c) + k \) is simply the graph of \( g(x) \) shifted right by \( c \) units if \( c < 0 \) or shifted left by \( c \) units if \( c > 0 \) and shifted up by \( k \) units if \( k > 0 \) or shifted down by \( k \) units if \( k < 0 \).
So, in our case if \( g(x) = x^2 \) we can see that,
\[
h(x) = (x - 3)^2 + 4 = g(x - 3) + 4
\]
and so the graph we’re being asked to sketch is the graph of \( g(x) = x^2 \) shifted right by 3 units and up by 4 units.

Here is the graph of \( h(x) = (x - 3)^2 + 4 \) and note that to help see the transformation we have also sketched in the graph of \( g(x) = x^2 \).

7. Without using a graphing calculator sketch the graph of \( W(x) = e^{x+2} - 3 \).

Hint : The Algebraic transformations that we used to help us graph the first few graphs in this section can be used together to shift the graph of a function both up/down and right/left at the same time.

Solution
The Algebraic transformations we were using in the first few problems of this section can be combined to shift a graph up/down and right/left at the same time. If we know the graph of \( g(x) \) then the graph of \( g(x + c) + k \) is simply the graph of \( g(x) \) shifted right by \( c \) units if \( c < 0 \) or shifted left by \( c \) units if \( c > 0 \) and shifted up by \( k \) units if \( k > 0 \) or shifted down by \( k \) units if \( k < 0 \).

So, in our case if \( g(x) = e^x \) we can see that,
\[
W(x) = e^{x+2} - 3 = g(x + 2) - 3
\]
and so the graph we’re being asked to sketch is the graph of $g(x) = e^x$ shifted left by 2 units and down by 3 units.

Here is the graph of $W(x) = e^{x+2} - 3$ and note that to help see the transformation we have also sketched in the graph of $g(x) = e^x$.

In this case the resulting sketch of $W(x)$ that we get by shifting the graph of $g(x)$ is not really the best, as it pretty much cuts off at $x = 0$ so in this case we should probably extend the graph of $W(x)$ a little. Here is a better sketch of the graph.

8. Without using a graphing calculator sketch the graph of $f(y) = (y - 1)^2 + 2$. 
Hint: The Algebraic transformations can also be used to help us sketch graphs of functions in the form \( x = f(y) \), but we do need to remember that we’re now working with functions in which the variables have been interchanged.

Solution

Even though our function is in the form \( x = f(y) \) we can still use the Algebraic transformations to help us sketch this graph. We do need to be careful however and remember that we’re working with interchanged variables and so the transformations will also switch.

In this case if we know the graph of \( h(y) \) then the graph of \( h(y + c) + k \) is simply the graph of \( h(x) \) shifted up by \( c \) units if \( c < 0 \) or shifted down by \( c \) units if \( c > 0 \) and shifted right by \( k \) units if \( k > 0 \) or shifted left by \( k \) units if \( k < 0 \).

So, in our case if \( h(y) = y^2 \) we can see that,

\[
f(y) = (y - 1)^2 + 2 = h(y - 1) + 2
\]

and so the graph we’re being asked to sketch is the graph of \( h(y) = y^2 \) shifted up by 1 units and right by 2 units.

Here is the graph of \( f(y) = (y - 1)^2 + 2 \) and note that to help see the transformation we have also sketched in the graph of \( h(y) = y^2 \).
9. Without using a graphing calculator sketch the graph of \( R(x) = -\sqrt{x} \).

Hint: Recall that the graph of \( -f(x) \) is the graph of \( f(x) \) reflected about the \( x \)-axis.

Solution
Recall the basic Algebraic transformations. If we know the graph of \( f(x) \) then the graph of \( -f(x) \) is simply the graph of \( f(x) \) reflected about the \( x \)-axis.

So, in our case if \( f(x) = \sqrt{x} \) we can see that,
\[
R(x) = -\sqrt{x} = -f(x)
\]
and so the graph we’re being asked to sketch is the graph of the square root function reflected about the \( x \)-axis.

Here is the graph of \( R(x) = -\sqrt{x} \) (the solid curve) and note that to help see the transformation we have also sketched in the graph of \( f(x) = \sqrt{x} \) (the dashed curve).

10. Without using a graphing calculator sketch the graph of \( g(x) = \sqrt{-x} \).

Hint: Recall that the graph of \( f(-x) \) is the graph of \( f(x) \) reflected about the \( y \)-axis.

Solution
First, do not get excited about the minus sign under the root. We all know that we won’t get real numbers if we take the square root of a negative number, but that minus sign doesn’t necessarily mean that we’ll be taking the square root of negative numbers. If we plug in positive value of \( x \)
then clearly we will be taking the square root of negative numbers, but if we plug in negative
values of \( x \) we will now be taking the square root of positive numbers and so there really is
nothing wrong with the function as written. We’ll just be using a different set of \( x \)’s than what we
may be used to working with when dealing with square roots.

Now, recall the basic Algebraic transformations. If we know the graph of \( f(x) \) then the graph
of \( f(-x) \) is simply the graph of \( f(x) \) reflected about the \( y \)-axis.

So, in our case if \( f(x) = \sqrt{x} \) we can see that,

\[
g(x) = \sqrt{-x} = f(-x)
\]

and so the graph we’re being asked to sketch is the graph of the square root function reflected
about the \( y \)-axis.

Here is the graph of \( g(x) = \sqrt{-x} \) and note that to help see the transformation we have also
sketched in the graph of \( f(x) = \sqrt{x} \).

11. Without using a graphing calculator sketch the graph of \( h(x) = 2x^2 - 3x + 4 \).

Hint : Recall that the graph of \( f(x) = ax^2 + bx + c \) is the graph of a parabola with vertex
\[
\left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right)
\]
that opens upwards if \( a > 0 \) and downwards if \( a < 0 \) and \( y \)-intercept at
\( (0,c) \).

Solution
We know that the graph of \( f(x) = ax^2 + bx + c \) will be a parabola that opens upwards if \( a > 0 \) and opens downwards if \( a < 0 \). We also know that its vertex is at, \[
\left( -\frac{b}{2a}, f\left( -\frac{b}{2a} \right) \right)
\]
The \( y \)-intercept of the parabola is the point \( (0, f(0)) = (0, c) \) and the \( x \)-intercepts (if any) are found by solving \( f(x) = 0 \)

So, in our case we know we have a parabola that opens upwards and that its vertex is at, \[
\left( -\frac{-3}{2(2)}, f\left( -\frac{-3}{2(2)} \right) \right) = \left( \frac{3}{4}, f\left( \frac{3}{4} \right) \right) = \left( \frac{3}{4}, \frac{23}{8} \right) = (0.75, 2.875)
\]

We can also see that the \( y \)-intercept is \( (0, 4) \). Because the vertex is above the \( x \)-axis and the parabola opens upwards we can see that there will be no \( x \)-intercepts.

It is usually best to have at least one point on either side of the vertex and we know that parabolas are symmetric about the vertical line running through the vertex. Therefore, because we know that the \( y \)-intercept is 0.75 units to the left of the vertex that we must also have a point that is 0.75 to the right of the vertex with the same \( y \)-value and this point is : \( (1.5, 4) \).

Here is a sketch of this parabola.

12. Without using a graphing calculator sketch the graph of \( f(y) = -4y^2 + 8y + 3 \).
Hint: Recall that the graph of $f(y) = ay^2 + by + c$ is the graph of a parabola with vertex
\[
\left(f\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)
\]
that opens towards the right if $a > 0$ and towards the left if $a < 0$ and $x$-intercept at $(c,0)$.

Solution

We know that the graph of $f(y) = ay^2 + by + c$ will be a parabola that opens towards the right if $a > 0$ and opens towards the left if $a < 0$. We also know that its vertex is at,

\[
\left(f\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)
\]

The $x$-intercept of the parabola is the point $(f(0), 0) = (c,0)$ and the $x$-intercepts (if any) are found by solving $f(y) = 0$.

So, for our case we know we have a parabola that opens towards the left and that its vertex is at,

\[
\left(f\left(-\frac{8}{2(-4)}\right), -\frac{8}{2(-4)}\right) = (f(1), 1) = (7,1)
\]

We can also see that the $y$-intercept is $(3,0)$.

To find the $y$-intercepts all we need to do is solve: $-4y^2 + 8y + 3 = 0$.

\[
y = \frac{-8 \pm \sqrt{8^2 - 4(-4)(3)}}{2(-4)} = \frac{-8 \pm \sqrt{112}}{-8} = \frac{-8 \pm 4\sqrt{7}}{-8} = \frac{2 \pm \sqrt{7}}{2} = -0.3229, 2.3229
\]

So, the two $y$-intercepts are: $(0,-0.3229)$ and $(0,2.3229)$.

Here is a sketch of this parabola.
13. Without using a graphing calculator sketch the graph of \((x + 1)^2 + (y - 5)^2 = 9\).

Solution
This is just a circle in standard form and so we can see that it has a center of \((-1, 5)\) and a radius of 3. Here is a quick sketch of the circle.
14. Without using a graphing calculator sketch the graph of \( x^2 - 4x + y^2 - 6y - 87 = 0 \).

Hint: Complete the square a couple of times to put this into standard form. This will allow you to identify the type of graph this will be.

Solution
The first thing that we should do is complete the square on the \( x \)'s and the \( y \)'s to see what we’ve got here. This could be a circle, ellipse, or hyperbola and completing the square a couple of times will put it into standard form and we’ll be able to identify the graph at that point.

Here is the completing the square work.

\[
\begin{align*}
(x^2 - 4x + 4) + (y^2 - 6y + 9) & = 87 + 4 + 9 \\
(x - 2)^2 + (y - 3)^2 & = 100
\end{align*}
\]

So, we’ve got a circle with center \((2, 3)\) and radius 10. Here is a sketch of the circle.

![Circle Sketch]

15. Without using a graphing calculator sketch the graph of \( 25(x + 2)^2 + \frac{y^2}{16} = 1 \).

Solution
This is just an ellipse that is almost in standard form. With a little rewrite we can put it into standard form as follows,
We can now see that the ellipse has a center of \((-2,0)\) while the left/right most points will be \(\frac{1}{5} = 0.2\) units away from the center and the top/bottom most points will be 2 units away from the center. Here is a quick sketch of the ellipse.

16. Without using a graphing calculator sketch the graph of \(x^2 + \frac{(y-6)^2}{9} = 1\).

Solution

This is just an ellipse that is in standard form (if it helps rewrite the first term as \(\frac{x^2}{1}\)) and so we can see that it has a center of \((0,6)\) while the left/right most points will be 1 unit away from the center and the top/bottom most points will be 3 units away from the center.

Here is a quick sketch of the ellipse.
17. Without using a graphing calculator sketch the graph of \( \frac{x^2}{36} - \frac{y^2}{49} = 1 \).

Solution
This is a hyperbola in standard form with the minus sign in front of the \( y \) term and so will open right and left. The center of the hyperbola is at \((0,0)\), the two vertices are at \((-6,0)\) and \((6,0)\), and the slope of the two asymptotes are \(\pm \frac{7}{6}\).

Here is a quick sketch of the hyperbola.
18. Without using a graphing calculator sketch the graph of \((y + 2)^2 - \frac{(x + 4)^2}{16} = 1\).

Solution
This is a hyperbola in standard form with the minus sign in front of the \(x\) term and so will open up and down. The center of the hyperbola is at \((-4, -2)\), the two vertices are at \((-1, -1)\) and \((-1, -3)\), and the slope of the two asymptotes are \(\pm \frac{1}{4}\).

Here is a quick sketch of the hyperbola.