Preface

Here are the solutions to the practice problems for my Calculus II notes. Some solutions will have more or less detail than other solutions. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Cross Product

1. If $\mathbf{w} = \langle 3, -1, 5 \rangle$ and $\mathbf{v} = \langle 0, 4, -2 \rangle$ compute $\mathbf{v} \times \mathbf{w}$.

Solution

Not really a whole lot to do here. We just need to run through one of the various methods for computing the cross product. We’ll be using the “trick” we used in the notes.

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -2 \\ 3 & -1 & 5 \end{vmatrix} = (4 \cdot 5 - (-2 \cdot -1)) \mathbf{i} - (0 \cdot 5 - (-2 \cdot 3)) \mathbf{j} + (0 \cdot -1 - 4 \cdot 3) \mathbf{k} = 18\mathbf{i} - 6\mathbf{j} - 12\mathbf{k}$$

2. If $\mathbf{w} = \langle 1, 6, -8 \rangle$ and $\mathbf{v} = \langle 4, -2, -1 \rangle$ compute $\mathbf{w} \times \mathbf{v}$.

Solution

Not really a whole lot to do here. We just need to run through one of the various methods for computing the cross product. We’ll be using the “trick” we used in the notes.
3. Find a vector that is orthogonal to the plane containing the points \( P = (3,0,1) \), \( Q = (4,-2,1) \) and \( R = (5,3,-1) \).

Step 1
We first need two vectors that are both parallel to the plane. Using the points that we are given (all in the plane) we can quickly get quite a few vectors that are parallel to the plane. We’ll use the following two vectors.

\[
\overrightarrow{PQ} = \langle 1,-2,0 \rangle \quad \overrightarrow{PR} = \langle 2,3,-2 \rangle
\]

Step 2
Now we know that the cross product of any two vectors will be orthogonal to the two original vectors. Since the two vectors from Step 1 are parallel to the plane (they actually lie in the plane in this case!) we know that the cross product must then also be orthogonal, or normal, to the plane.

So, using the “trick” we used in the notes the cross product is,

\[
\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 0 \\ 2 & 3 & -2 \end{vmatrix} = 4\hat{i} + 0\hat{j} + 3\hat{k} - (\hat{j}) - 0\hat{i} - (-4\hat{k}) = 4\hat{i} + 2\hat{j} + 7\hat{k}
\]

4. Are the vectors \( \vec{u} = \langle 1,2,-4 \rangle \), \( \vec{v} = \langle -5,3,-7 \rangle \) and \( \vec{w} = \langle -1,4,2 \rangle \) are in the same plane?

Solution
As discussed in the notes to answer this question all we need to do is compute the following quantity,
\[ \vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} 1 & 2 & -4 \\ -5 & 3 & -7 \\ -1 & 4 & 2 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ -5 & 3 \\ -1 & 4 \end{vmatrix} \\
= 6 + 14 + 80 - (-20) - (-28) - 12 = 136\]

Okay, since this is not zero we know that they are not in the same plane.