Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Radicals

1. Write the following expression in exponential form.

\[ \sqrt[7]{y} \]

Solution
All this problem is asking us to do is basically use the definition of the radical notation and write this in exponential form instead of radical form.

\[ \frac{1}{y^{\frac{1}{7}}} \]

2. Write the following expression in exponential form.

\[ \sqrt{x^2} \]

Solution
All this problem is asking us to do is basically use the definition of the radical notation and write this in exponential form instead of radical form.

\[ (x^2)^{\frac{1}{3}} \]

3. Write the following expression in exponential form.

\[ \sqrt[5]{ab} \]

Solution
All this problem is asking us to do is basically use the definition of the radical notation and write this in exponential form instead of radical form.

\[ (ab)^{\frac{1}{5}} \]

Be careful with parenthesis here! Recall that the only thing that gets the exponent is the term immediately to the left of the exponent. So, if we’d dropped parenthesis we’d get,
\[ab^{\frac{1}{6}} = a \left( b^{\frac{1}{6}} \right) = a \sqrt[6]{b}\]

which is most definitely not what we started with. The only way to make sure that we understand that both the \(a\) and the \(b\) were under the radical is to use parenthesis as we did above.

4. Write the following expression in exponential form.

\[\sqrt{w^2v^3}\]

Solution
All this problem is asking us to do is basically use the definition of the radical notation and write this in exponential form instead of radical form.

\[\left( w^2v^3 \right)^{\frac{1}{2}}\]

Recall that when no index is written on the radical it is assumed to be 2.

Also, be careful with parenthesis here! Recall that the only thing that gets the exponent is the term immediately to the left of the exponent and so we need parenthesis on the whole thing to make sure that we understand that both terms were under the root.

5. Evaluate : \(\sqrt[4]{81}\)

Hint : Recall that the easiest way to evaluate radicals is to convert to exponential form and then also recall that we evaluated exponential forms in the Rational Exponent section.

Solution
All we need to do here is to convert this to exponential form and then recall that we learned how to evaluate the exponential form in the Rational Exponent section.

\[\sqrt[4]{81} = 81^{\frac{1}{4}} = 3\] because \(3^4 = 81\)

6. Evaluate : \(\sqrt[3]{-512}\)

Hint : Recall that the easiest way to evaluate radicals is to convert to exponential form and then also recall that we evaluated exponential forms in the Rational Exponent section.

Solution
All we need to do here is to convert this to exponential form and then recall that we learned how to evaluate the exponential form in the Rational Exponent section.

\[ \sqrt[3]{-512} = (-512)^{\frac{1}{3}} = -8 \quad \text{because} \quad (-8)^3 = -512 \]

7. Evaluate : \( \sqrt[3]{1000} \)

Hint : Recall that the easiest way to evaluate radicals is to convert to exponential form and then also recall that we evaluated exponential forms in the Rational Exponent section.

Solution

All we need to do here is to convert this to exponential form and then recall that we learned how to evaluate the exponential form in the Rational Exponent section.

\[ \sqrt[3]{1000} = 1000^{\frac{1}{3}} = 10 \quad \text{because} \quad 10^3 = 1000 \]

8. Simplify the following expression. Assume that \( x \) is positive.

\[ \sqrt[3]{x^8} \]

Step 1

Recall that by simplify we mean we want to put the expression in simplified radical form (which we defined in the notes for this section).

To do this for this expression we’ll need to write the radicand as,

\[ x^8 = x^6 \cdot x^2 = (x^2)^3 \cdot x^2 \]

Step 2

Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.

\[ \sqrt[3]{x^8} = \sqrt[3]{(x^2)^3 \cdot x^2} = \sqrt[3]{x^2} \cdot \sqrt[3]{x^2} = x^{\frac{2}{3}} \cdot x^{\frac{2}{3}} \]

9. Simplify the following expression. Assume that \( y \) is positive.

\[ \sqrt[3]{8y^3} \]
Step 1
Recall that by simplify we mean we want to put the expression in simplified radical form (which we defined in the notes for this section).

To do this for this expression we’ll need to write the radicand as,

$$8y^3 = (4y^2)(2y)$$

Step 2
Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.

$$\sqrt{8y^3} = \sqrt{(4y^2)(2y)} = \sqrt{4y^2} \sqrt{2y} = 2y\sqrt{2y}$$

10. Simplify the following expression. Assume that $x, y$ and $z$ are positive.

$$\sqrt{x^7y^{20}z^{11}}$$

Step 1
Recall that by simplify we mean we want to put the expression in simplified radical form (which we defined in the notes for this section).

To do this for this expression we’ll need to write the radicand as,

$$x^7y^{20}z^{11} = x^4 y^{20} z^8 x^3 z^3 = x^4 \left(y^5\right)^4 \left(z^2\right)^4 x^3 z^3$$

Step 2
Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.

$$\sqrt[4]{x^7y^{20}z^{11}} = \sqrt[4]{x^4 \left(y^5\right)^4 \left(z^2\right)^4 x^3 z^3} = x y^5 z^2 \sqrt[4]{x^3 z^3}$$

11. Simplify the following expression. Assume that $x, y$ and $z$ are positive.

$$\sqrt[3]{54x^6y^7z^2}$$

Step 1
Recall that by simplify we mean we want to put the expression in simplified radical form (which we defined in the notes for this section).

To do this for this expression we’ll need to write the radicand as,
\[ 54x^6y^7z^2 = (27x^6y^6)(2y^4z^2) = 3^3 \left( x^2 \right) \left( y^2 \right)^3 \left( 2yz^2 \right) \]

Step 2
Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.

\[ \sqrt[3]{54x^6y^7z^2} = \sqrt[3]{3^3 \left( x^2 \right) \left( y^2 \right)^3 \left( 2yz^2 \right)} = 3 \left( x^2 \right)^{\frac{1}{3}} \left( y^2 \right)^{\frac{3}{3}} \sqrt[3]{2yz^2} = 3x^2y^2 \sqrt[3]{2yz^2} \]

12. Simplify the following expression. Assume that \( x, y \) and \( z \) are positive.

\[ \sqrt[4]{4x^3y} \sqrt[4]{8x^2y^3z^5} \]

Step 1
Remember that when we have a product of two radicals with the same index in an expression we first need to combine them into one root before we start the simplification process.

\[ \sqrt[4]{4x^3y} \sqrt[4]{8x^2y^3z^5} = \sqrt[4]{(4x^3y)(8x^2y^3z^5)} = \sqrt[4]{32x^5y^4z^5} \]

Step 2
Now that the expression has been written as a single radical we can proceed as we did in the earlier problems.

The radicand can be written as,

\[ 32x^5y^4z^5 = (2^4x^4y^4z^4)(2xz) \]

Step 3
Now that we’ve gotten the radicand rewritten it’s easy to deal with the radical and get the expression in simplified radical form.

\[ \sqrt[4]{32x^5y^4z^5} = \sqrt[4]{2^4x^4y^4z^4} = 2xz \sqrt[4]{2xz} \]

13. Multiply the following expression. Assume that \( x \) is positive.

\[ \sqrt{x} \left( 4 - 3\sqrt{x} \right) \]

Solution
All we need to do here is do the multiplication so here is that.
\[
\sqrt{x} \left(4 - 3\sqrt{x}\right) = 4\sqrt{x} - 3\sqrt{x} \left(\sqrt{x}\right) = 4\sqrt{x} - 3\sqrt{x^2} = 4\sqrt{x} - 3x
\]

Don’t forget to simplify any resulting roots that can be. That is an often missed part of these problems.

14. Multiply the following expression. Assume that \(x\) is positive.

\[
(2\sqrt{x} + 1) \left(3 - 4\sqrt{x}\right)
\]

Solution

All we need to do here is do the multiplication so here is that.

\[
(2\sqrt{x} + 1) \left(3 - 4\sqrt{x}\right) = 6\sqrt{x} - 8\sqrt{x} \left(\sqrt{x}\right) + 3 - 4\sqrt{x} = 3 + 2\sqrt{x} - 8\sqrt{x^2} = 3 + 2\sqrt{x} - 8x
\]

Don’t forget to simplify any resulting roots that can be. That is an often missed part of these problems.

15. Multiply the following expression. Assume that \(x\) is positive.

\[
\left(\sqrt[3]{x} + 2 \sqrt[3]{x^2}\right) \left(4 - \sqrt[3]{x^2}\right)
\]

Solution

All we need to do here is do the multiplication so here is that.

\[
\left(\sqrt[3]{x} + 2 \sqrt[3]{x^2}\right) \left(4 - \sqrt[3]{x^2}\right) = 4\sqrt[3]{x} - \sqrt[3]{x} \sqrt[3]{x^2} + 8 \sqrt[3]{x^2} - 2 \sqrt[3]{x^2} \sqrt[3]{x^2}
\]

\[
= 4\sqrt[3]{x} - \sqrt[3]{x^3} + 8 \sqrt[3]{x^2} - 2 \sqrt[3]{x^4}
\]

\[
= 4\sqrt[3]{x} - \sqrt[3]{x^3} + 8 \sqrt[3]{x^2} - 2 \sqrt[3]{x^3} \sqrt[3]{x}
\]

\[
= \frac{4\sqrt[3]{x} - 8 \sqrt[3]{x^2} - 2x \sqrt[3]{x}}{\sqrt[3]{x}}
\]

Don’t forget to simplify any resulting roots that can be. That is an often missed part of these problems and when dealing with roots other than square roots there can be quite a bit of work in the simplification process as we saw with this problem.

16. Rationalize the denominator. Assume that \(x\) is positive.

\[
\frac{6}{\sqrt{x}}
\]
Solution
For this problem we need to multiply the numerator and denominator by \( \sqrt{x} \) in order to rationalize the denominator.

\[
\frac{6}{\sqrt{x}} = \frac{6}{\sqrt{x} \sqrt{x}} = \frac{6\sqrt{x}}{x}
\]

17. Rationalize the denominator. Assume that \( x \) is positive.

\[
\frac{9}{\sqrt[3]{2x}}
\]

Solution
For this problem we need to multiply the numerator and denominator by \( \sqrt[3]{(2x)^2} \) in order to rationalize the denominator.

\[
\frac{9}{\sqrt[3]{2x}} = \frac{9\sqrt[3]{(2x)^2}}{\sqrt[3]{(2x)^2}} = \frac{9\sqrt[3]{(2x)^2}}{2x} = \frac{9\sqrt[3]{4x^2}}{2x}
\]

18. Rationalize the denominator. Assume that \( x \) and \( y \) are positive.

\[
\frac{4}{\sqrt{x} + 2\sqrt{y}}
\]

Solution
For this problem we need to multiply the numerator and denominator by \( \sqrt{x} - 2\sqrt{y} \) in order to rationalize the denominator.

\[
\frac{4}{\sqrt{x} + 2\sqrt{y}} = \frac{4\sqrt{x} - 2\sqrt{y}}{\sqrt{x} + 2\sqrt{y} \sqrt{x} - 2\sqrt{y}} = \frac{4(\sqrt{x} - 2\sqrt{y})}{(\sqrt{x} + 2\sqrt{y})(\sqrt{x} - 2\sqrt{y})} = \frac{4\sqrt{x} - 8\sqrt{y}}{x - 4y}
\]

19. Rationalize the denominator. Assume that \( x \) is positive.
\begin{equation*}
\frac{10}{3 - 5\sqrt{x}}
\end{equation*}

Solution

For this problem we need to multiply the numerator and denominator by \(3 + 5\sqrt{x}\) in order to rationalize the denominator.

\[
\frac{10}{3 - 5\sqrt{x}} = \frac{10}{3 - 5\sqrt{x}} \cdot \frac{3 + 5\sqrt{x}}{3 + 5\sqrt{x}} = \frac{10(3 + 5\sqrt{x})}{(3 - 5\sqrt{x})(3 + 5\sqrt{x})} = \frac{30 + 50\sqrt{x}}{9 - 25x}
\]