Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Quadratic Equations – Part I

1. Solve the following quadratic equation by factoring.

\[ u^2 - 5u - 14 = 0 \]

Step 1
Not much to this problem. We already have zero on one side of the equation, which we need to proceed with this problem. Therefore, all we need to do is actually factor the quadratic.

\[(u + 2)(u - 7) = 0\]

Step 2
Now all we need to do is use the zero factor property to get,

\[ u + 2 = 0 \quad \text{OR} \quad u - 7 = 0 \]

\[ u = -2 \quad u = 7 \]

Therefore the two solutions are: \[ u = -2 \text{ and } u = 7 \]

We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.

2. Solve the following quadratic equation by factoring.

\[ x^2 + 15x = -50 \]

Step 1
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.

\[ x^2 + 15x + 50 = 0 \]

\[ (x + 5)(x + 10) = 0 \]

Step 2
Now all we need to do is use the zero factor property to get,

\[ x + 5 = 0 \quad \text{OR} \quad x + 10 = 0 \]

\[ x = -5 \quad x = -10 \]

Therefore the two solutions are: \[ x = -5 \text{ and } x = -10 \]

We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.
3. Solve the following quadratic equation by factoring.

\[ y^2 = 11y - 28 \]

**Step 1**
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.

\[ y^2 - 11y + 28 = 0 \]

\[ (y - 4)(y - 7) = 0 \]

**Step 2**
Now all we need to do is use the zero factor property to get,

\[ y - 4 = 0 \quad \text{OR} \quad y - 7 = 0 \]

\[ y = 4 \quad \text{OR} \quad y = 7 \]

Therefore the two solutions are: \( y = 4 \) and \( y = 7 \)

We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.

4. Solve the following quadratic equation by factoring.

\[ 19x = 7 - 6x^2 \]

**Step 1**
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.

\[ 6x^2 + 19x - 7 = 0 \]

\[ (3x - 1)(2x + 7) = 0 \]

**Step 2**
Now all we need to do is use the zero factor property to get,

\[ 3x - 1 = 0 \quad \text{OR} \quad 2x + 7 = 0 \]

\[ x = \frac{1}{3} \quad \text{OR} \quad x = -\frac{7}{2} \]

Therefore the two solutions are: \( x = \frac{1}{3} \) and \( x = -\frac{7}{2} \)

We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.
College Algebra

5. Solve the following quadratic equation by factoring.
   \[ 6w^2 - w = 5 \]

Step 1
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.
\[ 6w^2 - w - 5 = 0 \]
\[ (6w + 5)(w - 1) = 0 \]

Step 2
Now all we need to do is use the zero factor property to get,
\[ 6w + 5 = 0 \quad \text{OR} \quad w - 1 = 0 \]
\[ w = -\frac{5}{6} \quad \text{OR} \quad w = 1 \]

Therefore the two solutions are: \[ w = -\frac{5}{6} \text{ and } w = 1 \]

We’ll leave it to you to verify that they really are solutions if you’d like to by plugging them back into the equation.

6. Solve the following quadratic equation by factoring.
   \[ z^2 - 16z + 61 = 2z - 20 \]

Step 1
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.
\[ z^2 - 18z + 81 = 0 \]
\[ (z - 9)^2 = 0 \]

Step 2
From the factored form we can quickly see that the solution is: \[ z = 9 \]

7. Solve the following quadratic equation by factoring.
   \[ 12x^2 = 25x \]

Step 1
The first thing we need to do is get everything on one side of the equation and then factor the quadratic.
\[ 12x^2 - 25x = 0 \]
\[ x(12x - 25) = 0 \]

Make sure that you do not just cancel an \( x \) from both sides of the equation!
Step 2
Now all we need to do is use the zero factor property to get,

\[ x = 0 \quad \text{OR} \quad 12x - 25 = 0 \]
\[ x = \frac{25}{12} \]

Therefore the two solutions are: \( x = 0 \) and \( x = \frac{25}{12} \).

Note that if we’d canceled an \( x \) from both sides of the equation in the first step we would have missed the solution \( x = 0 \)!

8. Use factoring to solve the following equation.

\[ x^4 - 2x^3 - 3x^2 = 0 \]

Step 1
Do not let the fact that this equation is not a quadratic equation convince you that you can’t do it! Note that we can factor an \( x^2 \) out of the equation. Doing that gives,

\[ x^2 \left( x^2 - 2x - 3 \right) = 0 \]

The quantity in the parenthesis is a quadratic and we can factor it. The full factoring of the equation is then,

\[ x^2 \left( x - 3 \right) \left( x + 1 \right) = 0 \]

Step 2
Now all we need to do is use the zero factor property to get,

\[ x^2 = 0 \quad \text{OR} \quad x - 3 = 0 \quad \text{OR} \quad x + 1 = 0 \]
\[ x = 0 \quad \quad x = 3 \quad \quad x = -1 \]

Therefore the three solutions are: \( x = 0, x = 3 \) and \( x = -1 \).

9. Use factoring to solve the following equation.

\[ t^4 = 9t^3 \]

Step 1
Do not let the fact that this equation is not a quadratic equation convince you that you can’t do it! Note that we move both terms to one side we can factor a $t^3$ out of the equation. Doing that gives,

\[ t^5 - 9t^3 = 0 \]
\[ t^3(t^2 - 9) = 0 \]

The quantity in the parenthesis is a quadratic and we can factor it. The full factoring of the equation is then,

\[ t^3(t - 3)(t + 3) = 0 \]

Step 2
Now all we need to do is use the zero factor property to get,

\[ t^3 = 0 \quad \text{OR} \quad t - 3 = 0 \quad \text{OR} \quad t + 3 = 0 \]
\[ t = 0 \quad \text{OR} \quad t = 3 \quad \text{OR} \quad t = -3 \]

Therefore the three solutions are: $t = 0, t = 3$ and $t = -3$

10. Use factoring to solve the following equation.

\[ \frac{w^2 - 10}{w + 2} + w - 4 = w - 3 \]

Step 1
This is an equation containing rational expressions so we know that the first step is to clear out the denominator by multiplying by the LCD, which is $w + 2$ in this case. Also, note that we now know that we must avoid $w = -2$ so we do not get division by zero.

Multiplying by the LCD and doing some basic simplification gives,

\[ (w + 2)\left(\frac{w^2 - 10}{w + 2} + w - 4\right) = (w - 3)(w + 2) \]
\[ w^2 - 10 + (w - 4)(w + 2) = (w - 3)(w + 2) \]
\[ w^2 - 10 + w^2 - 2w - 8 = w^2 - w - 6 \]
\[ w^2 - w - 12 = 0 \]

Step 2
We can now factor the quadratic to get,

\[ (w - 4)(w + 3) = 0 \]
The zero factor property now tells us,

\[ w - 4 = 0 \quad \text{OR} \quad w + 3 = 0 \]
\[ w = 4 \quad \text{OR} \quad w = -3 \]

Therefore the two solutions are: \( w = 4 \) and \( w = -3 \).

Note as well that because neither of these are \( w = -2 \) we know that we won’t get division by zero. Do not forget this important part of the solution process for equations involving rational expressions!

11. Use factoring to solve the following equation.

\[ \frac{4z}{z + 1} + \frac{5}{z} = \frac{6z + 5}{z^2 + z} \]

Step 1
This is an equation containing rational expressions so we know that the first step is to clear out the denominator by multiplying by the LCD, which is \( z(z + 1) \) in this case. Also, note that we now know that we must avoid \( z = 0 \) and \( z = -1 \) so we do not get division by zero.

Multiplying by the LCD and doing some basic simplification gives,

\[ z(z + 1) \left( \frac{4z}{z + 1} + \frac{5}{z} \right) = z(z + 1) \left( \frac{6z + 5}{z^2 + z} \right) \]
\[ (z)(4z) + 5(z + 1) = 6z + 5 \]
\[ 4z^2 + 5z + 5 = 6z + 5 \]
\[ 4z^2 - z = 0 \]

Step 2
We can now factor the quadratic to get,

\[ z(4z - 1) = 0 \]

The zero factor property now tells us,

\[ z = 0 \quad \text{OR} \quad 4z - 1 = 0 \]
\[ z = \frac{1}{4} \]

Note that we cannot use the first potential solution since that would give us division by zero! Therefore the only solution is: \( z = \frac{1}{4} \).
When dealing with equations that have rational expressions do not forget to verify that you do not get
division by zero with any of the potential solutions! As we saw in this case if we had not checked we
would have gotten a value of  \( x \) that seemed to be a solution but in fact was not because of the division by
zero issue.

12. Use factoring to solve the following equation.

\[
\frac{2x - 7}{x + 5} - \frac{5x + 8}{x + 5} = x + 1
\]

Step 1
This is an equation containing rational expressions so we know that the first step is to clear out the
denominator by multiplying by the LCD, which is \( x + 5 \) in this case. Also, note that we now know that
we must avoid \( x = -5 \) so we do not get division by zero.

Multiplying by the LCD and doing some basic simplification gives,

\[
(x + 5)(x + 1) = \left( \frac{2x - 7}{x + 5} - \frac{5x + 8}{x + 5} \right)(x + 5)
\]

\[
(x + 5)(x + 1) = \left( \frac{2x - 7}{x + 5} \right)(x + 5) - \left( \frac{5x + 8}{x + 5} \right)(x + 5)
\]

\[
(x + 5)(x + 1) = 2x - 7 - (5x + 8)
\]

\[
x^2 + 6x + 5 = 2x - 7 - 5x - 8
\]

\[
x^2 + 9x + 20 = 0
\]

Step 2
We can now factor the quadratic to get,

\[
(x + 4)(x + 5) = 0
\]

The zero factor property now tells us,

\[
x + 4 = 0 \quad \text{OR} \quad x + 5 = 0
\]

\[
x = -4 \quad \text{OR} \quad x = -5
\]

Note that we cannot use the second potential solution since that would give us division by zero!
Therefore the only solution is : \( x = -4 \).

When dealing with equations that have rational expressions do not forget to verify that you do not get
division by zero with any of the potential solutions! As we saw in this case if we had not checked we
would have gotten a value of \( x \) that seemed to be a solution but in fact was not because of the division by
zero issue.
13. Use the Square Root Property to solve the equation.

\[ 9u^2 - 16 = 0 \]

Step 1
There really isn’t too much to this problem. Just recall that we need to get the variable on one side of the equation by itself with a coefficient of one. For this problem that gives,

\[ 9u^2 = 16 \]
\[ u^2 = \frac{16}{9} \]

Step 2
Now all we need to do is use the Square Root Property to get,

\[ u = \pm \sqrt{\frac{16}{9}} = \pm \frac{\sqrt{16}}{\sqrt{9}} = \pm \frac{4}{3} \]

So we have the following two solutions:

\[ u = -\frac{4}{3} \quad \text{and} \quad u = \frac{4}{3} \]

14. Use the Square Root Property to solve the equation.

\[ x^2 + 15 = 0 \]

Step 1
There really isn’t too much to this problem. Just recall that we need to get the variable on one side of the equation by itself with a coefficient of one. For this problem that gives,

\[ x^2 = -15 \]

Step 2
Now all we need to do is use the Square Root Property to get,

\[ x = \pm \sqrt{-15} = \pm \sqrt{15} i \]

So we have the following two solutions:

\[ x = -\sqrt{15} i \quad \text{and} \quad x = \sqrt{15} i \]

Do not get excited about complex solutions. They will happen fairly regularly when solving quadratic equations so we need to be able to deal with them.

15. Use the Square Root Property to solve the equation.

\[ (z - 2)^2 - 36 = 0 \]
Step 1
There really isn’t too much to this problem. Just recall that we need to get the squared term on one side of the equation by itself with a coefficient of one. For this problem that gives,

\[(z - 2)^2 = 36\]

Step 2
Using the Square Root Property gives,

\[z - 2 = \pm \sqrt{36} = \pm 6\]

To finish this off all we need to do then is solve for \(z\) by adding 2 to both sides. This gives,

\[z = 2 \pm 6 \Rightarrow z = 2 - 6 = -4, \quad z = 2 + 6 = 8\]

So, after we did a little arithmetic, have the following two solutions: \(z = -4\) and \(z = 8\).

16. Use the Square Root Property to solve the equation.

\[(6t + 1)^2 + 3 = 0\]

Step 1
There really isn’t too much to this problem. Just recall that we need to get the squared term on one side of the equation by itself with a coefficient of one. For this problem that gives,

\[(6t + 1)^2 = -3\]

Step 2
Using the Square Root Property gives,

\[6t + 1 = \pm \sqrt{-3} = \pm \sqrt{3}i\]

To finish this off all we need to do then is solve for \(t\) by subtracting 1 from both sides and then dividing by the 6. This gives,

\[6t = -1 \pm \sqrt{3}i \Rightarrow t = \frac{-1 \pm \sqrt{3}i}{6} = \frac{-1}{6} \pm \frac{\sqrt{3}}{6}i\]

Note that we did a little rewrite after dividing by the 6 to put the answer in a more standard form for complex numbers.
We then have the following two solutions: $t = -\frac{1}{6} - \frac{\sqrt{6}}{6}i$ and $t = -\frac{1}{6} + \frac{\sqrt{6}}{6}i$. 