Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Application of Quadratic Equations

1. The width of a rectangle is 1 m less than twice the length. If the area of the rectangle is 100 m² what are the dimensions of the rectangle?

Step 1
We’ll start by letting \( L \) be the length of the rectangle. From the problem statement we now know that the width of the rectangle is 1 m less than twice the length and so must be \( 2L - 1 \).

Step 2
We also know that the area of any rectangle is length times width and we are given that the area of this particular rectangle is 100. Therefore the equation for this problem is,

\[
100 = (L)(2L - 1) = 2L^2 - L
\]

Step 3
This is a quadratic equation and we know how to solve that so let’s do that. First, we need to get the quadratic equation in standard form.

\[
2L^2 - L - 100 = 0
\]

We can now use the quadratic formula on this to get,

\[
L = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-100)}}{2(2)} = \frac{1 \pm \sqrt{801}}{4}
\]

Step 4
Reducing the two values we got in the previous steps to decimals we arrive at the following two solutions to the quadratic equation from Step 2.

\[
L = \frac{1 - \sqrt{801}}{4} = -6.8255 \quad \text{and} \quad L = \frac{1 + \sqrt{801}}{4} = 7.3255
\]

We are dealing with a rectangle and so having a negative length doesn’t make much sense. Therefore the first solution to the quadratic equation can’t be the length of the rectangle.

This means that the length of the rectangle must be 7.3255 m and the width of the rectangle is then

\[
2(7.3255) - 1 = 13.651 \text{ m}
\]
2. Two cars start out at the same spot. One car starts to drive north at 40 mph and 3 hours later the second car starts driving to the east at 60 mph. How long after the first car starts driving does it take for the two cars to be 500 miles apart?

Step 1
Let’s start out this problem by defining Car A to be the car that drives 40 mph and Car B to be the car that drives 60 mph. Let’s also let \( t \) be the time that Car A is driving. From the problem statement we know that Car B starts 3 hours after Car A and so drives for 3 hours less than Car A. This means that \( t - 3 \) is the time that Car B is driving.

Step 2
Next let’s set up a sketch for this situation.

Step 3
Okay. Now we need to get an equation for this situation. The first thing to notice about our sketch is that we have a right triangle! This means we can relate all three lengths using the Pythagorean Theorem (this is one of the reasons to have a sketch – to see these kinds of things).

The Pythagorean Theorem tells us that,

\[
\left( \frac{\text{Distance}}{\text{Car A drives}} \right)^2 + \left( \frac{\text{Distance}}{\text{Car B drives}} \right)^2 = (500)^2 = 250,000
\]

Step 4
Next, we know that we can find the distance of each car using the formula,

\[
\text{Distance} = (\text{Speed of Car})(\text{Time driving})
\]

So, for each car we have,
Distance of Car A = \((40)(t) = 40t\)
Distance of Car B = \((60)(t-3) = 60(t-3)\)

Putting all of this into the “word equation” we wrote down in Step 3 we get the following equation.

\[
\begin{align*}
(40t)^2 + (60(t-3))^2 &= 250,000 \\
40^2t^2 + 60^2(t-3)^2 &= 250,000 \\
1600t^2 + 3600(t^2 - 6t + 9) &= 250,000 \\
1600t^2 + 3600t^2 - 21,600t + 32,400 &= 250,000 \\
5200t^2 - 21,600t - 217,600 &= 0 \\
\end{align*}
\]

Note as well that we did quite a bit of simplification to get the equation into a standard form. Also, do not get excited about the “large” numbers here! They happen on occasion so they are nothing to worry about. This is still just a quadratic and we know how to solve quadratic equations. It doesn’t matter if the numbers are single digit numbers of significantly larger numbers as they are here.

Step 5
As noted in the previous step this is just a quadratic equation and we know how to solve those! Using the quadratic formula gives,

\[
t = \frac{21,600 \pm \sqrt{(-21,600)^2 - 4(5200)(-217,600)}}{2(5200)} = \frac{21,600 \pm \sqrt{4,992,640,000}}{10,400}
\]

Step 6
Reducing the two values we got in the previous steps to decimals we arrive at the following two solutions to the quadratic equation from Step 4.

\[
t = \frac{21,600 - \sqrt{4,992,640,000}}{10,400} = -4.7172 \quad t = \frac{21,600 + \sqrt{4,992,640,000}}{10,400} = 8.8710
\]

The first solution to the equation doesn’t make any sense since it is negative (we are working with time and so it’s safe to assume we are starting at \(t = 0\) after all!) so that means the second is the answer we need.

This means that Car A (i.e. the one traveling at 40 mph) travels for 8.871 hours while Car B (i.e. the one traveling at 60 mph) travels for 5.871 hours (three hours less than Car A time!).

3. Two people can paint a house in 14 hours. Working individually one of the people takes 2 hours more than it takes the other person to paint the house. How long would it take each person working individually to paint the house?
Step 1
First, let Person A be the faster of the two painters and let \( t \) be the amount of time it takes to paint the house by himself. Next, let Person B be the slower of the two painters and so it will take this person \( t + 2 \) hours to paint the house by himself.

Step 2
Working together they can paint the house in 14 hours so we have the following word equation for them working together to paint the house.

\[
\left( \text{Portion of job done by Person A} \right) + \left( \text{Portion of job done by Person B} \right) = 1 \text{ Job}
\]

We know that Portion of Job = Work Rate \times Work Time so this gives the following word equation.

\[
\left( \text{Work Rate of Person A} \right) \left( \text{Work Time of Person A} \right) + \left( \text{Work Rate of Person B} \right) \left( \text{Work Time of Person B} \right) = 1
\]

\[
\left( \text{Work Rate of Person A} \right) (14) + \left( \text{Work Rate of Person B} \right) (14) = 1
\]

Step 3
Now we need the work rate of each person which we can get from their individual painting times as follows,

\[
\left( \text{Work Rate of Person A} \right) \left( \text{Work Time of Person A} \right) = \left( \text{Work Rate of Person A} \right) (t) = 1
\]

\[
\Rightarrow \text{ Work Rate of Person A} = \frac{1}{t}
\]

\[
\left( \text{Work Rate of Person B} \right) \left( \text{Work Time of Person B} \right) = \left( \text{Work Rate of Person B} \right) (t + 2) = 1
\]

\[
\Rightarrow \text{ Work Rate of Person B} = \frac{1}{t + 2}
\]

Step 4
Plugging these into the word equation from Step 2 we arrive at the following equation.

\[
\left( \frac{1}{t} \right) (14) + \left( \frac{1}{t + 2} \right) (14) = 1
\]

\[
\frac{14}{t} + \frac{14}{t + 2} = 1
\]
To solve this we know that we’ll need to multiply by the LCD, \( t(t + 2) \) in this case, to clear the denominators. Doing this gives,

\[
14(t + 2) + 14t = t(t + 2) \\
28t + 28 = t^2 + 2t \\
t^2 - 26t - 28 = 0
\]

After some simplification we arrive a fairly simple quadratic equation to solve. Using the quadratic formula gives,

\[
L = \frac{-(-26) \pm \sqrt{(-26)^2 - 4(1)(-28)}}{2(1)} = \frac{26 \pm \sqrt{788}}{2}
\]

Step 6
Reducing the two values we got in the previous steps to decimals we arrive at the following two solutions to the quadratic equation from Step 2.

\[
t = \frac{26 - \sqrt{788}}{2} = -1.0357 \quad \quad \quad t = \frac{26 + \sqrt{788}}{2} = 27.0357
\]

The first solution to the equation doesn’t make any sense since it is negative (we are working with time and so it’s safe to assume we are starting at \( t = 0 \) at all!) so that means the second is the answer we need.

This means that Person A can paint the house in 27.0357 hours while Person B can paint the house in 29.0357 hours (two hours more than Person A).