Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Linear Inequalities

1. Solve the following inequality and give the solution in both inequality and interval notation.

\[4(z + 2) - 1 > 5 - 7(4 - z)\]

Hint: Remember that solving linear inequalities is pretty much the same as solving a linear equation. Just remember to be careful when multiplying/dividing by a negative number.

Step 1
We know that the process of solving a linear inequality is pretty much the same process as solving a linear equation. We do basic algebraic manipulations to get all the \(z\)'s on one side of the inequality and the numbers on the other side. Just remember that what you do to one side of the inequality you have to do to the other side as well. So, let's go through the solution process for this linear inequality.

First, we should clear out the parenthesis on both sides and do any simplification that we can. Doing this gives,

\[4z + 8 - 1 > 5 - 28 + 7z\]
\[4z + 7 > -23 + 7z\]

Step 2
We can now subtract \(7z\) from both sides and subtract 7 to both sides to get,

\[-3z > -30\]

Note that we could just have easily subtracted \(4z\) from both sides and added 23 to both sides. Each will get the same result in the end.

Step 3
For the final step we need to divide both sides by -3. Recall however that because we are dividing by a negative number we need to switch the direction of the inequality to get,

\[z < 10\]

So, the inequality form of the solution is \(z < 10\) and the interval notation form of the solution is \((-\infty, 10]\).

Remember that we use a parenthesis, i.e. “)“, for the right side of the interval notation because we are not including 10 in the solution. Also recall that infinities always get parenthesis!

2. Solve the following inequality and give the solution in both inequality and interval notation.

\[\frac{1}{2}(3 + 4t) \leq 6\left(\frac{1}{3} - \frac{1}{2}t\right) - \frac{1}{4}(2 + 10t)\]
Hint: Remember that solving linear inequalities is pretty much the same as solving a linear equation. Just remember to be careful when multiplying/dividing by a negative number.

Step 1
We know that the process of solving a linear inequality is pretty much the same process as solving a linear equation. We do basic algebraic manipulations to get all the $t$'s on one side of the inequality and the numbers on the other side. Just remember that what you do to one side of the inequality you have to do to the other side as well. So, let's go through the solution process for this linear inequality.

First, we should clear out the parenthesis on both sides and do any simplification that we can. Doing this gives,

$$\frac{3}{2} + 2t \leq 2 - 3t - \frac{1}{2} - \frac{5}{2}t$$

$$\frac{3}{2} + 2t \leq \frac{3}{2} - \frac{11}{2}t$$

Step 2
We can now add $\frac{1}{2}t$ to both sides and subtract $\frac{3}{2}$ from both sides to get,

$$\frac{15}{2} t \leq 0$$

Step 3
For the final step we need to multiply both sides by $\frac{2}{15}$ to get,

$$t \leq 0$$

So, the inequality form of the solution is $[t \leq 0]$ and the interval notation form of the solution is $[\infty, 0]$.

Remember that we use a square bracket, i.e. “[“, for the left portion of the interval because we are including zero in the solution. Also recall that infinities never get square brackets!

3. Solve the following inequality and give the solution in both inequality and interval notation.

$$-1 < 4x + 2 < 10$$

Hint: Solving double inequalities uses the same basic process as solving single inequalities. Just remember that what you do to one part you have to do to all parts of the inequality.

Step 1
Just like with single inequalities solving these follow pretty much the same process as solving a linear equation. The only difference between this and a single inequality is that we now have three parts of the inequality and so we just need to remember that what we do to one part we need to do to all parts.
Also, recall that the main goal is to get the variable all by itself in the middle and all the numbers on the two outer parts of the inequality.

So, let’s start by subtracting 2 from all the parts. This gives,

\[-3 < 4x < 8\]

Step 2
Finally, all we need to do is divide all three parts by 4 to get,

\[-\frac{3}{4} < x < 2\]

So, the inequality form of the solution is \(-\frac{3}{4} < x < 2\) and the interval notation form of the solution is \((-\frac{3}{4}, 2)\).

4. Solve the following inequality and give the solution in both inequality and interval notation.

\[8 \leq 3 - 5z < 12\]

Hint: Solving double inequalities uses the same basic process as solving single inequalities. Just remember that what you do to one part you have to do to all parts of the inequality.

Step 1
Just like with single inequalities solving these follow pretty much the same process as solving a linear equation. The only difference between this and a single inequality is that we now have three parts of the inequality and so we just need to remember that what we do to one part we need to do to all parts.

Also, recall that the main goal is to get the variable all by itself in the middle and all the numbers on the two outer parts of the inequality.

So, let’s start by subtracting 3 from all the parts. This gives,

\[5 \leq -5z < 9\]

Step 2
Finally, all we need to do is divide all three parts by -5 to get,

\[-1 \geq z > -\frac{9}{5}\]

Don’t forget that because we were dividing everything by a negative number we needed to switch the direction of the inequalities.
So, the inequality form of the solution is \(-\frac{9}{5} < z \leq -1\) (we flipped the inequality around to get the smaller number on the left as that is a more “standard” form). The interval notation form of the solution is \([-\frac{9}{5}, -1]\).

For the interval notation form remember that the smaller number is always on the left (hence the reason for flipping the inequality form above!) and be careful with parenthesis and square brackets. We use parenthesis if we don’t include the number and square brackets if we do include the number.

5. Solve the following inequality and give the solution in both inequality and interval notation.

\[0 \leq 10w - 15 \leq 23\]

Hint : Solving double inequalities uses the same basic process as solving single inequalities. Just remember that what you do to one part you have to do to all parts of the inequality.

Step 1
Just like with single inequalities solving these follow pretty much the same process as solving a linear equation. The only difference between this and a single inequality is that we now have three parts of the inequality and so we just need to remember that what we do to one part we need to do to all parts.

Also, recall that the main goal is to get the variable all by itself in the middle and all the numbers on the two outer parts of the inequality.

So, let’s start by add 15 to all the parts. This gives,

\[15 \leq 10w \leq 38\]

Step 2
Finally, all we need to do is divide all three parts by 10 to get,

\[\frac{3}{2} \leq w \leq \frac{19}{5}\]

So, the inequality form of the solution is \(\frac{3}{2} \leq w \leq \frac{19}{5}\) and the interval notation form of the solution is \(\left[\frac{3}{2}, \frac{19}{5}\right]\).

6. Solve the following inequality and give the solution in both inequality and interval notation.

\[2 < \frac{1}{6} - \frac{1}{2}x \leq 4\]

Hint : Solving double inequalities uses the same basic process as solving single inequalities. Just remember that what you do to one part you have to do to all parts of the inequality.
Step 1
Just like with single inequalities solving these follow pretty much the same process as solving a linear equation. The only difference between this and a single inequality is that we now have three parts of the inequality and so we just need to remember that what we do to one part we need to do to all parts.

Also, recall that the main goal is to get the variable all by itself in the middle and all the numbers on the two outer parts of the inequality.

So, let’s start by subtracting \( \frac{1}{6} \) from all the parts. This gives,

\[
\frac{11}{6} < -\frac{1}{2}x \leq \frac{23}{6}
\]

Step 2
Finally, all we need to do is multiply all three parts by -2 to get,

\[
-\frac{11}{3} > x \geq -\frac{23}{3}
\]

Don’t forget that because we were multiplying everything by a negative number we needed to switch the direction of the inequalities.

So, the inequality form of the solution is \( \left[ -\frac{23}{3}, -\frac{11}{3} \right] \) (we flipped the inequality around to get the smaller number on the left as that is a more “standard” form). The interval notation form of the solution is \( (-\infty, -\frac{23}{3}] \).  

For the interval notation form remember that the smaller number is always on the left (hence the reason for flipping the inequality form above!) and be careful with parenthesis and square brackets. We use parenthesis if we don’t include the number and square brackets if we do include the number.

7. If \( 0 \leq x < 3 \) determine \( a \) and \( b \) for the inequality : \( a \leq 4x + 1 < b \)

Hint : Can you make the middle part of the first inequality look like the middle part of the second inequality?

Step 1
This problem is really the reverse of the previous problems in this section. In the previous problems we started with something like the second inequality (of course we also had numbers in the two outer portions instead of \( a \) and \( b \)) and we had to manipulate it to get the \( x \) by itself in the middle.

The process here is basically the same just in reverse. We need to do algebraic manipulations to make the middle part of the first inequality look like the middle part of the second manipulation. The only real
difference is that with the solving problems we added/subtracted the number before we dealt with the coefficient of the $x$. Here we need to get the coefficient on the $x$ before we get the number.

So, the first thing we’ll do is multiply all three parts of the first inequality by 4. This gives,

$$0(4) \leq 4x < 3(4) \quad \Rightarrow \quad 0 \leq 4x < 12$$

Step 2
Now all we need to do is add one to all three parts.

$$1 \leq 4x + 1 < 13$$

Step 3
Comparing this inequality in the second step to the second inequality in the problem statement we can see that we must have $a = 1$ and $b = 13$. 
