Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
1. Solve the following inequality.

\[ u^2 + 4u \geq 21 \]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the polynomial.

\[ (u+7)(u-3) \geq 0 \]

Hint: Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored form we can quickly see that the polynomial will be zero at,

\[ u = -7 \quad u = 3 \]

Remember that these points are important because they are the only places where the polynomial on the left side of the inequality might change sign. Given that we want to know where the polynomial is zero (which we now know) or positive knowing where the polynomial might change sign will help considerably with determining the answer we’re looking for.

Hint: Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[ (-1)(-11) > 0 \quad (7)(-3) < 0 \quad (11)(1) > 0 \]

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Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[
\begin{align*}
 &u \leq -7 \quad \text{and} \quad u \geq 3 \\
 &(-\infty, -7] \quad \text{and} \quad [3, \infty)
\end{align*}
\]

2. Solve the following inequality.

\[x^2 + 8x + 12 < 0\]

Step 1
The first thing we need to do is get a zero on one side of the inequality (which is already done for this problem) and then, if possible, factor the polynomial.

\[(x + 6)(x + 2) < 0\]

Hint: Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored from we can quickly see that the polynomial will be zero at,

\[x = -6 \quad x = -2\]

Remember that these points are important because they are the only places where the polynomial on the left side of the inequality might change sign. Given that we want to know where the polynomial is negative knowing where the polynomial might change sign will help considerably with determining the answer we’re looking for.

Hint: Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.
Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[-6 < x < -2\]
\[(-6, -2)\]

3. Solve the following inequality.

\[4t^2 \leq 15 - 17t\]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the polynomial.

\[4t^2 + 17t - 15 \leq 0\]
\[(t + 5)(4t - 3) \leq 0\]

Hint: Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored from we can quickly see that the polynomial will be zero at,

\[t = -5\]  \[t = \frac{3}{4}\]

Remember that these points are important because they are the only places where the polynomial on the left side of the inequality might change sign. Given that we want to know where the polynomial is zero (which we now know) or negative knowing where the polynomial might change sign will help considerably with determining the answer we’re looking for.

Hint: Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[
\begin{align*}
  t &= -6 & (10)(-27) > 0 \\
  t &= 0 & (5)(-3) < 0 \\
  t &= 1 & (6)(1) > 0 \\
\end{align*}
\]

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[
\begin{cases}
  -5 \leq t \leq \frac{3}{4} \\
  [-5, \frac{3}{4}] 
\end{cases}
\]

4. Solve the following inequality.

\[ z^2 + 34 > 12z \]

Step 1
The first thing we need to do is get a zero on one side of the inequality and then, if possible, factor the polynomial.

\[ z^2 - 12z + 34 > 0 \]

In this case the polynomial doesn’t factor.

Hint : Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. Because the polynomial didn’t factor we’ll need to use the quadratic formula to determine where it’s zero.
We’ll need these points in decimal form to make the rest of the problem easier.

Remember that these points are important because they are the only places where the polynomial on the left side of the inequality might change sign. Given that we want to know where the polynomial is positive knowing where the polynomial might change sign will help considerably with determining the answer we’re looking for.

Hint : Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[
\begin{align*}
z \to 4 & \quad | & z \to 6 & \quad | & z \to 8 \\
4^2 - 12(4) + 34 > 0 & \quad | & 6^2 - 12(6) + 34 < 0 & \quad | & 8^2 - 12(8) + 34 > 0
\end{align*}
\]

5. Solve the following inequality.

\[
y^2 - 2y + 1 \leq 0
\]

Step 1
The first thing we need to do is get a zero on one side of the inequality (which is already done for this problem) and then, if possible, factor the polynomial.
\[(y-1)^2 \leq 0\]

Hint: Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored form we can quickly see that the polynomial will be zero at,

\[y = 1\]

Hint: Is it possible for the polynomial to ever be negative?

Step 3
This problem works a little differently than the others in this section. Because the polynomial is a perfect square we know that it can never be negative! It is only possible for it to be zero or positive.

We are being asked to determine where the polynomial is negative or zero. As noted however it isn’t possible for it to be negative. Therefore the only solution we can get for this inequality is where it is zero and we found that in the previous step.

The answer is then,

\[y = 1\]

In this case the answer is a single number and not an inequality. This happens on occasion and we shouldn’t worry about these kinds of “unusual” answers.

6. Solve the following inequality.

\[t^4 + t^3 - 12t^2 < 0\]

Step 1
The first thing we need to do is get a zero on one side of the inequality (which is already done for this problem) and then, if possible, factor the polynomial.

\[t^2(t^2 + t - 12) < 0\]
\[t^2(t^2 + 4)(t - 3) < 0\]

Hint: Where are the only places where the polynomial might change signs?

Step 2
Despite the fact that this is an inequality we first need to know where the polynomial is zero. From the factored form we can quickly see that the polynomial will be zero at,

\[t = -4 \quad t = 0 \quad t = 3\]
Remember that these points are important because they are the only places where the polynomial on the left side of the inequality might change sign. Given that we want to know where the polynomial is negative knowing where the polynomial might change sign will help considerably with determining the answer we’re looking for.

Hint : Knowing that the polynomial can only change sign at the points above how can we quickly determine if the polynomial is positive or negative in the ranges between those points?

Step 3
Recall from the discussion in the notes for this section that because the points from the previous step are the only places where the polynomial might change sign we can quickly determine if the polynomial is positive/negative in the ranges between each of these points simply by plugging in “test points” from each region into the polynomial to check the sign.

So, let’s sketch a quick number line with the points where the polynomial is zero graphed on it. We’ll also show the test point computations on the number line as well. Here is the number line.

\[
\begin{array}{cccccc}
\text{Be careful with the first term in the factored form when plugging in the test points! It is squared and so will always be positive regardless of the sign of the test points. One of the bigger mistakes that students make with this kind of problem is to miss the square and treat that term as negative when plugging in a negative test point.}
\end{array}
\]

Step 4
All we need to do now is get the solution from the number line in the previous step. Here is both the inequality and interval notation from of the answer.

\[
\begin{array}{cccc}
-4 & < t & < 0 & \text{and} \quad 0 & < t & < 3 \\
(-4,0) & \text{and} & (0,3) \\
\end{array}
\]

Be careful with your answer here and don’t include \( t = 0 \) ! It might be tempting to do that to “simplify” the answer into a single inequality/interval but the polynomial is zero at \( t = 0 \) and we only want to know where the polynomial is negative! Therefore we cannot include \( t = 0 \) in our answer and we’ll need to write it as two inequalities/intervals.