Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Symmetry

1. Determine the symmetry of each of the following equation.

\[ x = 4y^6 - y^2 \]

Step 1
Let’s first check for symmetry about the x-axis. To do this we need to replace all the y’s with \(-y\).

\[ x = 4(-y)^6 - (-y)^2 \quad \rightarrow \quad x = 4y^6 - y^2 \]

The resulting equation is identical to the original equation and so is equivalent to the original equation. Therefore the equation is has symmetry about the x-axis.

Step 2
Next we’ll check for symmetry about the y-axis. To do this we need to replace all the x’s with \(-x\).

\[ -x = 4y^6 - y^2 \]

The resulting equation is not equivalent to the original equation (i.e. it is not same nor is it the same equation except with opposite signs on every term). Therefore the equation is does not have symmetry about the y-axis.

Step 3
Finally a check for symmetry about the origin. For this check we need to replace all the y’s with \(-y\) and to replace all the x’s with \(-x\).

\[ -x = 4(-y)^6 - (-y)^2 \quad \rightarrow \quad -x = 4y^6 - y^2 \]

The resulting equation is not equivalent to the original equation (i.e. it is not same nor is it the same equation except with opposite signs on every term). Therefore the equation does not have symmetry about the origin.

2. Determine the symmetry of each of the following equation.

\[ \frac{y^2}{4} = 1 + \frac{x^2}{9} \]

Step 1
Let’s first check for symmetry about the x-axis. To do this we need to replace all the y’s with \(-y\).
\[
\frac{(-y)^2}{4} = 1 + \frac{x^2}{9} \quad \rightarrow \quad \frac{y^2}{4} = 1 + \frac{x^2}{9}
\]

The resulting equation is identical to the original equation and so is equivalent to the original equation. Therefore the equation is \textbf{has symmetry about the x-axis}.

Step 2
Next we’ll check for symmetry about the y-axis. To do this we need to replace all the x’s with \(-x\).

\[
\frac{y^2}{4} = 1 + \frac{(-x)^2}{9} \quad \rightarrow \quad \frac{y^2}{4} = 1 + \frac{x^2}{9}
\]

The resulting equation is identical to the original equation and so is equivalent to the original equation. Therefore the equation is \textbf{has symmetry about the y-axis}.

Step 3
Finally a check for symmetry about the origin. For this check we need to replace all the y’s with \(-y\) and to replace all the x’s with \(-x\).

\[
\frac{(-y)^2}{4} = 1 + \frac{(-x)^2}{9} \quad \rightarrow \quad \frac{y^2}{4} = 1 + \frac{x^2}{9}
\]

The resulting equation is identical to the original equation and so is equivalent to the original equation. Therefore the equation is \textbf{has symmetry about the origin}.

3. Determine the symmetry of each of the following equation.

\[x^2 = 7y - x^3 + 2\]

Step 1
Let’s first check for symmetry about the x-axis. To do this we need to replace all the y’s with \(-y\).

\[x^2 = 7(-y) - x^3 + 2 \quad \rightarrow \quad x^2 = -7y - x^3 + 2\]

The resulting equation is not equivalent to the original equation (\textit{i.e.} it is not same nor is it the same equation except with opposite signs on every term). Therefore the equation is \textbf{does not have symmetry about the x-axis}.

Step 2
Next we’ll check for symmetry about the y-axis. To do this we need to replace all the x’s with \(-x\).

\[(-x)^2 = 7y - (-x)^3 + 2 \quad \rightarrow \quad x^2 = 7y + x^3 + 2\]
The resulting equation is not equivalent to the original equation (i.e. it is not same nor is it the same equation except with opposite signs on every term). Therefore the equation is does not have symmetry about the y-axis.

Step 3
Finally a check for symmetry about the origin. For this check we need to replace all the y’s with −y and to replace all the x’s with −x.

\[
(−x)^2 = 7(−y) − (−x)^3 + 2 \quad \rightarrow \quad x^2 = −7y + x^3 + 2
\]

The resulting equation is not equivalent to the original equation (i.e. it is not same nor is it the same equation except with opposite signs on every term). Therefore the equation does not have symmetry about the origin.

4. Determine the symmetry of each of the following equation.

\[y = 4x^2 + x^6 − x^8\]

Step 1
Let’s first check for symmetry about the x-axis. To do this we need to replace all the y’s with −y.

\[−y = 4x^2 + x^6 − x^8\]

The resulting equation is not equivalent to the original equation (i.e. it is not same nor is it the same equation except with opposite signs on every term). Therefore the equation is does not have symmetry about the x-axis.

Step 2
Next we’ll check for symmetry about the y-axis. To do this we need to replace all the x’s with −x.

\[y = 4(−x)^2 + (−x)^6 − (−x)^8 \quad \rightarrow \quad y = 4x^2 + x^6 − x^8\]

The resulting equation is identical to the original equation and so is equivalent to the original equation. Therefore the equation is has symmetry about the y-axis.

Step 3
Finally a check for symmetry about the origin. For this check we need to replace all the y’s with −y and to replace all the x’s with −x.

\[−y = 4(−x)^2 + (−x)^6 − (−x)^8 \quad \rightarrow \quad −y = 4x^2 + x^6 − x^8\]

The resulting equation is not equivalent to the original equation (i.e. it is not same nor is it the same equation except with opposite signs on every term). Therefore the equation does not have symmetry about the origin.
5. Determine the symmetry of each of the following equation.

\[ y = 7x + 4x^5 \]

Step 1
Let’s first check for symmetry about the \(x\)-axis. To do this we need to replace all the \(y\)’s with \(-y\).

\[-y = 7x + 4x^5\]

The resulting equation is not equivalent to the original equation (i.e. it is not same nor is it the same equation except with opposite signs on every term). Therefore the equation is does not have symmetry about the \(x\)-axis.

Step 2
Next we’ll check for symmetry about the \(y\)-axis. To do this we need to replace all the \(x\)’s with \(-x\).

\[ y = 7(-x) + 4(-x)^5 \quad \rightarrow \quad y = -7x - 4x^5 \]

The resulting equation is not equivalent to the original equation (i.e. it is not same nor is it the same equation except with opposite signs on every term). Therefore the equation is does not have symmetry about the \(y\)-axis.

Step 3
Finally a check for symmetry about the origin. For this check we need to replace all the \(y\)’s with \(-y\) and to replace all the \(x\)’s with \(-x\).

\[-y = 7(-x) + 4(-x)^5 \quad \rightarrow \quad -y = -7x - 4x^5\]

The resulting equation is the same as the original equation except all the signs are the opposite and so is equivalent to the original equation. Therefore the equation has symmetry about the origin.