Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Dividing Polynomials

1. Use long division to divide $3x^4 - 5x^2 + 3$ by $x + 2$.

Step 1
Let’s first perform the long division. Just remember that we keep going until the remainder has degree that is strictly less that the degree of the polynomial we’re dividing by, $x + 2$ in this case. The polynomial we’re dividing by has degree one and so, in this case, we’ll stop when the remainder is degree zero, i.e. a constant.

Here is the long division work for this problem.

$$
\begin{align*}
\frac{3x^3 - 6x^2 + 7x - 14}{x + 2} &= \frac{3x^4 - 5x^2 + 3}{x + 2} \\
&= \frac{3x^4 + 6x^3}{x + 2} - \frac{6x^3 - 5x^2 + 3}{x + 2} \\
&= \frac{-6x^3 - 6x^2 + 3}{x + 2} - \frac{-6x^3 - 12x^2}{x + 2} \\
&= \frac{7x^2 + 3}{x + 2} - \frac{7x^2 + 14x}{x + 2} \\
&= \frac{-14x + 3}{x + 2} - \frac{-14x - 28}{x + 2} \\
&= \frac{31}{x + 2}
\end{align*}
$$

Step 2
We can put the answer in either of the two following forms.

$$
\frac{3x^4 - 5x^2 + 3}{x + 2} = 3x^3 - 6x^2 + 7x - 14 + \frac{31}{x + 2}
$$

$$
3x^4 - 5x^2 + 3 = (x + 2)(3x^3 - 6x^2 + 7x - 14) + 31
$$

Either answer is acceptable here although one may be more useful than the other depending on the application that is being done when you need to actually do the long division.

2. Use long division to divide $x^3 + 2x^2 - 3x + 4$ by $x - 7$.

Step 1
Let’s first perform the long division. Just remember that we keep going until the remainder has degree that is strictly less that the degree of the polynomial we’re dividing by, $x - 7$ in this case. The polynomial we’re dividing by has degree one and so, in this case, we’ll stop when the remainder is degree zero, i.e. a constant.

Here is the long division work for this problem.

\[
\begin{array}{c|ccccc}
 & x^2 + 9x + 60 \\
\hline 
 x - 7 & x^3 + 2x^2 - 3x + 4 \\
 & -(x^3 - 7x^2) \\
 & 9x^2 - 3x + 4 \\
 & -(9x^2 - 63x) \\
 & 60x + 4 \\
 & -(60x - 420) \\
 & 424 \\
\end{array}
\]

Step 2
We can put the answer in either of the two following forms.

\[
\frac{x^3 + 2x^2 - 3x + 4}{x - 7} = x^2 + 9x + 60 + \frac{424}{x - 7}
\]

\[
x^3 + 2x^2 - 3x + 4 = (x - 7)(x^2 + 9x + 60) + 424
\]

Either answer is acceptable here although one may be more useful than the other depending on the application that is being done when you need to actually do the long division.

3. Use long division to divide $2x^5 + x^4 - 6x + 9$ by $x^2 - 3x + 1$.

Step 1
Let’s first perform the long division. Just remember that we keep going until the remainder has degree that is strictly less that the degree of the polynomial we’re dividing by, $x^2 - 3x + 1$ in this case. The polynomial we’re dividing by has degree two and so, in this case, we’ll stop when the remainder is degree one or zero.

Here is the long division work for this problem.
\[
\begin{array}{c}
(2x^3 + 7x^2 + 19x + 50) \\
(x^2 - 3x + 1) \\
- (2x^5 + x^4 - 6x + 9) \\
\end{array}
\]
\[
\begin{array}{c}
(2x^5 - 6x^4 + 2x^3) \\
7x^4 - 2x^3 - 6x + 9 \\
- (7x^4 - 21x^3 + 7x^2) \\
19x^3 - 7x^2 - 6x + 9 \\
- (19x^3 - 57x^2 + 19x) \\
50x^2 - 25x + 9 \\
- (50x^2 - 150x + 50) \\
125x - 41 \\
\end{array}
\]

Step 2
We can put the answer in either of the two following forms.

\[
\frac{2x^5 + x^4 - 6x + 9}{x^2 - 3x + 1} = 2x^3 + 7x^2 + 19x + 50 + \frac{125x - 41}{x^2 - 3x + 1}
\]

\[
2x^5 + x^4 - 6x + 9 = (x^2 - 3x + 1)(2x^3 + 7x^2 + 19x + 50) + 125x - 41
\]

Either answer is acceptable here although one may be more useful than the other depending on the application that is being done when you need to actually do the long division.

4. Use synthetic division to divide \(x^3 + x^2 + x + 1\) by \(x + 9\).

Step 1
Here is the synthetic division. We’ll leave it to you to check all the numbers.

\[
\begin{array}{c|cccc}
-9 & 1 & 1 & 1 & 1 \\
& & -9 & 72 & -657 \\
& & 1 & -8 & 73 & -656 \\
\end{array}
\]

Step 2
The answer is then,

\[
x^3 + x^2 + x + 1 = (x + 9)(x^2 - 8x + 73) - 656
\]

Note that we only gave one form of the answer (unlike the first couple of problems) since this is often the form we need when using synthetic division and it is also the form that method is set up to give.
5. Use synthetic division to divide \( 7x^3 - 1 \) by \( x + 2 \).

Step 1
Here is the synthetic division. We’ll leave it to you to check all the numbers.

\[
\begin{array}{c|cccc}
-2 & 7 & 0 & 0 & -1 \\
& -14 & 28 & -56 \\
\hline
& 7 & -14 & 28 & -57 \\
\end{array}
\]

Step 2
The answer is then,

\[
7x^3 - 1 = (x + 2) \left( 7x^2 - 14x + 28 \right) - 57
\]

Note that we only gave one form of the answer (unlike the first couple of problems) since this is often the form we need when using synthetic division and it is also the form that method is set up to give.

6. Use synthetic division to divide \( 5x^4 + x^2 - 8x + 2 \) by \( x - 4 \).

Step 1
Here is the synthetic division. We’ll leave it to you to check all the numbers.

\[
\begin{array}{c|ccccc}
4 & 5 & 0 & 1 & -8 & 2 \\
& 20 & 80 & 324 & 1264 \\
\hline
& 5 & 20 & 81 & 316 & 1266 \\
\end{array}
\]

Step 2
The answer is then,

\[
5x^4 + x^2 - 8x + 2 = (x - 4) \left( 5x^3 + 20x^2 + 81x + 316 \right) + 1266
\]

Note that we only gave one form of the answer (unlike the first couple of problems) since this is often the form we need when using synthetic division and it is also the form that method is set up to give.