Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Applications

1. We have $10,000 to invest for 44 months. How much money will we have if we put the money into an account that has an annual interest rate of 5.5% and interest is compounded,
   (a) quarterly    (b) monthly   (c) continuously

(a) quarterly
From the problem statement we can see that,
\[ P = 10000 \quad r = \frac{5.5}{100} = 0.055 \quad t = \frac{44}{12} = \frac{11}{3} \]

Remember that the value of \( r \) must be given as a decimal, \( i.e. \) the percentage divided by 100. Also remember that \( t \) must be in years and so we’ll need to convert the months we are given to years.

For this part we are compounding interest rate quarterly and that means it will compound 4 times per year and so we also then know that,
\[ m = 4 \]

At this point all that we need to do is plug into the equation and run the numbers through a calculator to compute the amount of money that we’ll have.
\[ A = 10000 \left( 1 + \frac{0.055}{4} \right)^{4\left(\frac{11}{3}\right)} = 10000 \left( 1.01375 \right)^{44} = 10000 \left( 1.221760422 \right) = 12217.60 \]

So, we’ll have $12,217.60 in the account after 44 months.

(b) monthly
From the problem statement we can see that,
\[ P = 10000 \quad r = \frac{5.5}{100} = 0.055 \quad t = \frac{44}{12} = \frac{11}{3} \]

Remember that the value of \( r \) must be given as a decimal, \( i.e. \) the percentage divided by 100. Also remember that \( t \) must be in years and so we’ll need to convert the months we are given to years.

For this part we are compounding interest rate monthly and that means it will compound 12 times per year and so we also then know that,
\[ m = 12 \]

At this point all that we need to do is plug into the equation and run the numbers through a calculator to compute the amount of money that we’ll have.
\[ A = 10000 \left( 1 + \frac{0.055}{12} \right)^{12\left(\frac{11}{3}\right)} = 10000 \left( 1.00453333 \right)^{44} = 10000 \left( 1.222876562 \right) = 12228.77 \]

So, we’ll have $12,228.77 in the account after 44 months.
(c) **continuously**
From the problem statement we can see that,

\[ P = 10000 \quad r = \frac{5.5}{100} = 0.055 \quad t = \frac{44}{12} = \frac{11}{3} \]

Remember that the value of \( r \) must be given as a decimal, *i.e.* the percentage divided by 100. Also remember that \( t \) must be in years and so we’ll need to convert the months we are given to years.

For this part we are compounding continuously and so we won’t have an \( m \) and will be using the other equation and all we have all we need to do the computation so,

\[ A = 10000e^{(0.055)(\frac{11}{3})} = 10000e^{0.201666667} = 10000(1.223440127) = 12234.40 \]

So, we’ll have $12,234.40 in the account after 44 months.

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2. We are starting with $5000 and we’re going to put it into an account that earns an annual interest rate of 12%. How long should we leave the money in the account in order to double our money if interest is compounded,

(a) quarterly  (b) monthly  (c) continuously

Hint : Identify the given quantities, plug into the appropriate equation and use the techniques from an earlier section to solve for \( t \).

(a) **quarterly**
From the problem statement we can see that,

\[ A = 10000 \quad P = 5000 \quad r = \frac{12}{100} = 0.12 \]

Remember that the value of \( r \) must be given as a decimal, *i.e.* the percentage divided by 100. Also, for this part we are compounding interest rate quarterly and that means it will compound 4 times per year and so we also then know that,

\[ m = 4 \]

Plugging into the equation gives us,

\[ 10000 = 5000\left(1+\frac{0.12}{4}\right)^{4t} = 5000\left(1.03\right)^{4t} \]

Using the techniques from the *Solve Exponential Equations* section we can solve for \( t \).

\[ 2 = 1.03^{4t} \]

\[ \ln(2) = \ln(1.03^{4t}) \]

\[ \ln(2) = 4t \ln(1.03) \]

\[ t = \frac{\ln(2)}{4\ln(1.03)} = 5.8624 \]
So, we’ll double our money in approximately 5.8624 years.

(b) **monthly**

From the problem statement we can see that,

\[
A = 10000 \quad P = 5000 \quad r = \frac{12}{100} = 0.12
\]

Remember that the value of \( r \) must be given as a decimal, i.e. the percentage divided by 100. Also, for this part we are compounding interest rate monthly and that means it will compound 12 times per year and so we also then know that,

\[ m = 12 \]

Plugging into the equation gives us,

\[
10000 = 5000 \left(1 + \frac{0.12}{12}\right)^{12t}
\]

Using the techniques from the Solve Exponential Equations section we can solve for \( t \).

\[
\frac{2}{1.01^{12t}} = \frac{\ln(2)}{12 \ln(1.01)} = 5.8051
\]

So, we’ll double our money in approximately 5.8051 years.

(c) **continuously**

From the problem statement we can see that,

\[
A = 10000 \quad P = 5000 \quad r = \frac{12}{100} = 0.12
\]

Remember that the value of \( r \) must be given as a decimal, i.e. the percentage divided by 100. For this part we are compounding continuously and so we won’t have an \( m \) and will be using the other equation.

Plugging into the continuously compounding interest equation gives,

\[
10000 = 5000e^{0.12t}
\]

Now, solving this using the techniques from the Solve Exponential Equations section gives,
\[ 2 = e^{0.12t} \]
\[ \ln(2) = \ln(e^{0.12t}) \]
\[ \ln(2) = 0.12t \]
\[ t = \frac{\ln(2)}{0.12} = 5.7762 \]

So, we’ll double our money in approximately 5.7762 years.

3. A population of bacteria initially has 250 present and in 5 days there will be 1600 bacteria present.
   (a) Determine the exponential growth equation for this population.
   (b) How long will it take for the population to grow from its initial population of 250 to a population of 2000?

(a) **Determine the exponential growth equation for this population.**

We can start off here by acknowledging that we know the initial number of bacteria is 250 and so \( Q_0 = 250 \). Therefore the equation is then,

\[ Q(t) = 250e^{kt} \]

Now, we also know that \( Q(5) = 1600 \) and plugging this into the equation above gives,

\[ 1600 = Q(5) = 250e^{5k} \]

We can use techniques from the Solve Logarithm Equations section to determine the value of \( k \).

\[ 1600 = 250e^{5k} \]
\[ \frac{1600}{250} = e^{5k} \]
\[ \ln \left( \frac{32}{5} \right) = 5k \]
\[ k = \frac{1}{5} \ln \left( \frac{32}{5} \right) = 0.3712596 \]

Depending upon your preferences we can use either the exact value or the decimal value. Note however that because \( k \) is in the exponent of an exponential function we’ll need to use quite a few decimal places to avoid potentially large differences in the value that we’d get if we rounded off too much.

Putting all of this together the exponential growth equation for this population is,

\[ Q = 250e^{\frac{1}{5} \ln \left( \frac{32}{5} \right) t} \]
Hint: Identify the given quantities, plug into the appropriate equation and use the techniques from an earlier section to solve for $t$.

(b) How long will it take for the population to grow from its initial population of 250 to a population of 2000?

What we’re really being asked to do here is to solve the equation,

$$2000 = Q(t) = 250e^{\frac{1}{32} \ln\left(\frac{32}{5}\right)t}$$

and we know from the Solve Logarithm Equations section how to do that. Here is the solution work for this part.

$$\frac{2000}{250} = e^{\frac{1}{32} \ln\left(\frac{32}{5}\right)t}$$

$$\ln(8) = \frac{1}{5} \ln\left(\frac{32}{5}\right)t$$

$$t = \frac{5 \ln(8)}{\ln\left(\frac{32}{5}\right)} = 5.6010$$

It will take 5.601 days for the population to reach 2000.

4. We initially have 100 grams of a radioactive element and in 1250 years there will be 80 grams left.
   (a) Determine the exponential decay equation for this element.
   (b) How long will it take for half of the element to decay?
   (c) How long will it take until there is only 1 gram of the element left?

(a) Determine the exponential decay equation for this element.

We can start off here by acknowledging that we know the initial amount of the radioactive element is 100 and so $Q_0 = 100$. Therefore the equation is then,

$$Q(t) = 100e^{kt}$$

Now, we also know that $Q(1250) = 80$ and plugging this into the equation above gives,

$$80 = Q(1250) = 100e^{1250k}$$

We can use techniques from the Solve Logarithm Equations section to determine the value of $k$. 

\[
80 = 100e^{1250k}
\]
\[
\frac{80}{100} = e^{1250k}
\]
\[
\ln\left(\frac{4}{5}\right) = 1250k
\]
\[
k = \frac{1}{1250} \ln\left(\frac{4}{5}\right) = -0.000178515
\]

Depending upon your preferences we can use either the exact value or the decimal value. Note however that because \(k\) is in the exponent of an exponential function we’ll need to use quite a few decimal places to avoid potentially large differences in the value that we’d get if we rounded off too much.

Putting all of this together the exponential decay equation for this population is,
\[
Q = 100e^{\frac{1}{1250} \ln\left(\frac{4}{5}\right)t}
\]

Hint: Identify the given quantities, plug into the appropriate equation and use the techniques from an earlier section to solve for \(t\).

(b) How long will it take for half of the element to decay?

What we’re really being asked to do here is to solve the equation,
\[
50 = Q(t) = 100e^{\frac{1}{1250} \ln\left(\frac{4}{5}\right)t}
\]
and we know from the Solve Logarithm Equations section how to do that. Here is the solution work for this part.

\[
\frac{50}{100} = e^{\frac{1}{1250} \ln\left(\frac{4}{5}\right)t}
\]
\[
\ln\left(\frac{1}{2}\right) = \frac{1}{1250} \ln\left(\frac{4}{5}\right)t
\]
\[
t = \frac{1250 \ln\left(\frac{1}{2}\right)}{\ln\left(\frac{4}{5}\right)} = 3882.8546
\]

It will take 3882.8546 years for half of the element to decay. On a side note this time is called the half-life of the element.

Hint: This part is pretty much the same as the previous part.

(c) How long will it take until there is only 1 gram of the element left?

In this part we’re being asked to solve the equation,
The solution process for this part is the same as that for the previous part. Here is the solution work for this part.

\[
\frac{1}{100} = e^{\frac{1}{1250} \ln\left(\frac{4}{5}\right) t}
\]

\[
\ln\left(\frac{1}{100}\right) = \frac{1}{1250} \ln\left(\frac{4}{5}\right) t
\]

\[
t = \frac{1250 \ln\left(\frac{1}{100}\right)}{\ln\left(\frac{4}{5}\right)} = 25797.1279
\]

There will only be 1 gram of the element left after 25,797.1279 years.