Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
Linear Systems with Two Variables

1. Use the Method of Substitution to find the solution to the following system or to determine if the system is inconsistent or dependent.

\[ x - 7y = -11 \]
\[ 5x + 2y = -18 \]

Step 1
Before we get started with the solution process for this system we need to make it clear that there is no “one correct solution path”. There are lots of solution paths that we can take to find the solution to this system. All are correct and all will end up with the same solution to the system (provided the work has been done correctly of course…).

Okay, let’s get started on the solution to this system.

The Method of Substitution tells us that we first need to solve one of the equations for one of the variables. The equation we solve and the variable we solve for technically doesn’t matter as noted above.

However, there is often one equation/variable combination that is “easier” than the others. In this case we can quickly solve the first equation for \( x \) without a lot of extra work so let’s do that.

\[ x = 7y - 11 \]

Step 2
We now take the equation for \( x \) we found above and substitute this into the other equation (the second equation in this case). Doing this gives,

\[ 5(7y - 11) + 2y = -18 \]

Step 3
We can now solve the equation we found in the previous step for \( y \). Doing this gives,

\[ 5y = 37 \quad \rightarrow \quad y = 1 \]

Step 4
Finally, we can plug the value of \( y \) we found in the previous step into the equation for \( x \) we found in the first step. This gives,

\[ x = 7(1) - 11 = -4 \]
The solution to the system is then: \[ x = -4, \ y = 1. \]

2. Use the Method of Substitution to find the solution to the following system or to determine if the system is inconsistent or dependent.

\[
\begin{align*}
7x - 8y &= -12 \\
-4x + 2y &= 3
\end{align*}
\]

Step 1
Before we get started with the solution process for this system we need to make it clear that there is no “one correct solution path”. There are lots of solution paths that we can take to find the solution to this system. All are correct and all will end up with the same solution to the system (provided the work has been done correctly of course…).

Okay, let’s get started on the solution to this system.

The Method of Substitution tells us that we first need to solve one of the equations for one of the variables. The equation we solve and the variable we solve for technically doesn’t matter as noted above.

However, there is often one equation/variable combination that is “easier” than the others. In this case we can solve the second equation for \( y \) without a lot of extra work so let’s do that.

\[
\begin{align*}
-4x + 2y &= 3 \\
2y &= 4x + 3 \\
\Rightarrow \quad y &= 2x + \frac{3}{2}
\end{align*}
\]

Note that you will often get fractions showing up at this step and there isn’t going to be a whole lot that you can do about it so don’t worry when they show up!

Step 2
We now take the equation for \( y \) we found above and substitute this into the other equation (the first equation in this case). Doing this gives,

\[
\begin{align*}
7x - 8y &= -12 \\
7x - 8\left(2x + \frac{3}{2}\right) &= -12
\end{align*}
\]

Step 3
We can now solve the equation we found in the previous step for \( x \). Doing this gives,

\[
\begin{align*}
7x - 8\left(2x + \frac{3}{2}\right) &= -12 \\
7x - 16x - 12 &= -12 \\
-8x &= 0 \\
\Rightarrow \quad x &= 0
\end{align*}
\]
Do not get excited about the zero here! They will be answers occasionally.

Step 4
Finally, we can plug the value of \( x \) we found in the previous step into the equation for \( y \) we found in the first step. This gives,

\[
y = 2(0) + \frac{3}{2} = \frac{3}{2}
\]

The solution to the system is then: \( x = 0, \ y = \frac{3}{2} \).

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3. Use the Method of Substitution to find the solution to the following system or to determine if the system is inconsistent or dependent.

\[
\begin{align*}
3x + 9y & = -6 \\
-4x - 12y & = 8
\end{align*}
\]

Step 1
Before we get started with the solution process for this system we need to make it clear that there is no “one correct solution path”. There are lots of solution paths that we can take to find the solution to this system. All are correct and all will end up with the same solution to the system (provided the work has been done correctly of course…).

Okay, let’s get started on the solution to this system.

The Method of Substitution tells us that we first need to solve one of the equations for one of the variables. The equation we solve and the variable we solve for technically doesn’t matter as noted above.

In this case both equations seem equally “easy” to deal with and so let’s solve the second equation for \( x \) since that is a combination we didn’t use in the first couple of problems.

\[
\begin{align*}
-4x - 12y & = 8 \\
-4x & = 12y + 8 \\
\Rightarrow \quad x & = -3y - 2
\end{align*}
\]

Step 2
We now take the equation for \( x \) we found above and substitute this into the other equation (the first equation in this case). Doing this gives,

\[
3x + 9y = -6
\]

\[
3(-3y - 2) + 9y = -6
\]

Step 3
We can now solve the equation we found in the previous step for \( y \). Doing this gives,
Step 4
Now, the result from the previous step is true for any value of $y$ or $x$ and so we know that the system is dependent and there will be an infinite number of solutions to the system. We can write the “solution” to this system as follows,

$$x = -3t - 2$$
$$y = t$$

$t$ is any number

4. Use the Method of Elimination to find the solution to the following system or to determine if the system is inconsistent or dependent.

$$6x - 5y = 8$$
$$-12x + 2y = 0$$

Step 1
Before we get started with the solution process for this system we need to make it clear that there is no “one correct solution path”. There are lots of solution paths that we can take to find the solution to this system. All are correct and all will end up with the same solution to the system (provided the work has been done correctly of course…).

Okay, let’s get started on the solution to this system.

The Method of Elimination tells us that we first need to multiply one or both of the equations by constants so that one of the variables has the same coefficient but with opposite signs and then add the two equations.

For this system if we multiply the first equation by 2 then the first equation will have an $x$ coefficient of 12 while the second equation will have an $x$ coefficient of -12. This is exactly what we need so we’ll do that and then add the resulting equations.

$$6x - 5y = 8 \quad \times 2$$
$$-12x + 2y = 0 \quad \text{same}$$
$$12x - 10y = 16$$
$$-12x + 2y = 0$$
$$-8y = 16$$

Step 2
We can now easily solve the result from the above step to see that $y = -2$.

Step 3
Finally we can plug the value of \( y \) we found in the previous step in either of the original equations and solve for \( x \). We’ll use the first equation for this.

\[
6x - 5(-2) = 8 \\
6x + 10 = 8 \\
6x = -2 \quad \rightarrow \quad x = -\frac{1}{3}
\]

The solution to the system is then: \( x = -\frac{1}{3}, \ y = -2 \).

5. Use the Method of Elimination to find the solution to the following system or to determine if the system is inconsistent or dependent.

\[
\begin{align*}
-2x + 10y &= 2 \\
5x - 25y &= 3
\end{align*}
\]

Step 1
Before we get started with the solution process for this system we need to make it clear that there is no “one correct solution path”. There are lots of solution paths that we can take to find the solution to this system. All are correct and all will end up with the same solution to the system (provided the work has been done correctly of course…).

Okay, let’s get started on the solution to this system.

The Method of Elimination tells us that we first need to multiply one or both of the equations by constants so that one of the variables has the same coefficient but with opposite signs and then add the two equations.

For this system if we multiply the first equation by 5 and the second equation by 2 then the first equation will have an \( x \) coefficient of -10 while the second equation will have an \( x \) coefficient of 10. This is exactly what we need so we’ll do that and then add the resulting equations.

\[
\begin{align*}
-2x + 10y &= 2 \quad \times 5 \\
5x - 25y &= 3 \quad \times 2 \\
-10x + 50y &= 10 \\
10x - 50y &= 6
\end{align*}
\]

\[
0 = 16
\]

Step 2
The result above is clearly not true and so this system is \textbf{inconsistent} and has \textbf{no solution}. 

6. Use the Method of Elimination to find the solution to the following system or to determine if the system is inconsistent or dependent.

\[ \begin{align*} 2x + 3y &= 20 \\ 7x + 2y &= 53 \end{align*} \]

Step 1
Before we get started with the solution process for this system we need to make it clear that there is no “one correct solution path”. There are lots of solution paths that we can take to find the solution to this system. All are correct and all will end up with the same solution to the system (provided the work has been done correctly of course…).

Okay, let’s get started on the solution to this system.

The Method of Elimination tells us that we first need to multiply one or both of the equations by constants so that one of the variables has the same coefficient but with opposite signs and then add the two equations.

For this system if we multiply the first equation by 2 and the second equation by -3 then the first equation will have a \( y \) coefficient of 6 while the second equation will have a \( y \) coefficient of -6. This is exactly what we need so we’ll do that and then add the resulting equations.

\[ \begin{align*} 2x + 3y &= 20 \\ 7x + 2y &= 53 \end{align*} \]

\[ \begin{align*} \times 2 & \quad \times -3 \\ 4x + 6y &= 40 \\ -21x - 6y &= -159 \end{align*} \]

\[ -17x = -119 \]

Step 2
We can now easily solve the result from the above step to see that \( x = 7 \).

Step 3
Finally we can plug the value of \( y \) we found in the previous step in either of the original equations and solve for \( y \). We’ll use the first equation for this.

\[ 7(7) + 2y = 53 \]
\[ 49 + 2y = 53 \]
\[ 2y = 4 \quad \rightarrow \quad y = 2 \]

The solution to the system is then \( x = 7, \ y = 2 \).