Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.
1. For the following system of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

\[ \begin{align*}
  x - 7y & = -11 \\
  5x + 2y & = -18
\end{align*} \]

Step 1
The first step is to write down the augmented matrix for the system of equations.

\[
\begin{bmatrix}
  1 & -7 & | & -11 \\
  5 & 2 & | & -18
\end{bmatrix}
\]

Step 2
We need to make the number in the upper left corner a one. In this case it already is and so there really isn’t anything to do in this step for this particular problem.

Step 3
Next, we need to convert the 5 below the 1 into a zero and we can do that with the following elementary row operation.

\[
\begin{bmatrix}
  1 & -7 & | & -11 \\
  5 & 2 & | & -18
\end{bmatrix} \rightarrow \begin{bmatrix}
  1 & -7 & | & -11 \\
  0 & 37 & | & 37
\end{bmatrix}
\]

Step 4
The next step is to turn the number at the bottom of the second column (37 in this case) into a one. The following elementary row operation will do that for us.

\[
\begin{bmatrix}
  1 & -7 & | & -11 \\
  0 & 37 & | & 37
\end{bmatrix} \rightarrow \begin{bmatrix}
  1 & -7 & | & -11 \\
  0 & 1 & | & 1
\end{bmatrix}
\]

Step 5
Finally we need to convert the number above the one we got in Step 4 into a zero. To do that we can use the following elementary row operation.

\[
\begin{bmatrix}
  1 & -7 & | & -11 \\
  0 & 1 & | & 1
\end{bmatrix} \rightarrow \begin{bmatrix}
  1 & 0 & | & -4 \\
  0 & 1 & | & 1
\end{bmatrix}
\]

Step 6
From the final augmented matrix we found in Step 5 we get the solution to the system is:

\[
\begin{align*}
  x & = -4 \\
  y & = 1
\end{align*}
\]
2. For the following system of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

\[
\begin{align*}
7x - 8y &= -12 \\
-4x + 2y &= 3
\end{align*}
\]

Step 1
The first step is to write down the augmented matrix for the system of equations.

\[
\begin{bmatrix}
7 & -8 & | & -12 \\
-4 & 2 & | & 3
\end{bmatrix}
\]

Step 2
We need to make the number in the upper left corner a one. There are several ways to do this. One way would be to use the elementary row operation \( \frac{1}{7} R_1 \). However, this would put fractions into the other two entries in the first row and it might be nice to avoid them.

So, instead let’s do the following elementary row operation.

\[
\begin{bmatrix}
7 & -8 & | & -12 \\
-4 & 2 & | & 3
\end{bmatrix} \rightarrow \begin{bmatrix}
1 & -4 & | & -6 \\
-4 & 2 & | & 3
\end{bmatrix}
\]

Now, this isn’t quite what we want since the number in the upper left is a minus one and not a positive one. However, we can easily fix that by multiplying the first row by -1.

\[
\begin{bmatrix}
-1 & -4 & | & -6 \\
-4 & 2 & | & 3
\end{bmatrix} \rightarrow \begin{bmatrix}
1 & 4 & | & 6 \\
-4 & 2 & | & 3
\end{bmatrix}
\]

Note that as this step has shown there are several different paths to do these problems. Some will result in “messier” intermediate steps, but the solution we get in the end will be the same regardless of the path we chose to follow in the solution process.

Step 3
Next, we need to convert the -4 below the 1 into a zero and we can do that with the following elementary row operation.

\[
\begin{bmatrix}
1 & 4 & | & 6 \\
-4 & 2 & | & 3
\end{bmatrix} \rightarrow \begin{bmatrix}
1 & 4 & | & 6 \\
0 & 18 & | & 27
\end{bmatrix}
\]

Step 4
The next step is to turn the number at the bottom of the second column (18 in this case) into a one. The following elementary row operation will do that for us.
In the first step we chose to avoid the step that put fractions into the augmented matrix, but sometimes, as in this step, they can’t be avoided.

**Step 5**
Finally we need to convert the number above the one we got in Step 4 into a zero. To do that we can use the following elementary row operation.

\[
\begin{bmatrix} 1 & 4 & 6 \\ 0 & 1 & \frac{22}{18} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 6 \\ 0 & 1 & \frac{1}{2} \end{bmatrix}
\]

**Step 6**
From the final augmented matrix we found in Step 5 we get the solution to the system is \( x = 0, \ y = \frac{2}{3} \).

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3. For the following system of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

\[
\begin{align*}
3x + 9y &= -6 \\
-4x - 12y &= 8
\end{align*}
\]

**Step 1**
The first step is to write down the augmented matrix for the system of equations.

\[
\begin{bmatrix} 3 & 9 & -6 \\ -4 & -12 & 8 \end{bmatrix}
\]

**Step 2**
We need to make the number in the upper left corner a one. In this case we can quickly do that by dividing the top row by 3.

\[
\begin{bmatrix} 3 & 9 & -6 \\ -4 & -12 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -4 & -12 & 8 \end{bmatrix}
\]

**Step 3**
Next, we need to convert the -4 below the 1 into a zero and we can do that with the following elementary row operation.
The minute we see the bottom row of all zeroes we know that the system is dependent. We can convert the top row into an equation and solve for $x$ as follows,

$$x + 3y = -2$$

$$x = -3y - 2$$

From this we can write the solution as,

$$x = -3t - 2$$

$$y = t$$

$t$ is any number

4. For the following system of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

$$6x - 5y = 8$$

$$-12x + 2y = 0$$

Step 1
The first step is to write down the augmented matrix for the system of equations.

$$\begin{bmatrix}
6 & -5 & 8 \\
-12 & 2 & 0
\end{bmatrix}$$

Step 2
We need to make the number in the upper left corner a one. There are several ways to do this. One way would be to use the elementary row operation $\frac{1}{6} R_1$. However, this would put fractions into the other two entries in the first row.

We’re not going to be able to avoid fractions after this step and the above idea would do what we need but it would lead to two fractions. Note however that if we interchange the two rows we get,

$$\begin{bmatrix}
6 & -5 & 8 \\
-12 & 2 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
-12 & 2 & 0 \\
6 & -5 & 8
\end{bmatrix}$$

We could now do the elementary row operation $\frac{1}{12} R_1$ and we’ll only end up with one fraction in the first row instead of two so let’s do that.
Note that as this step has shown there are several different paths to do these problems. Some will result in “messier” intermediate steps, but the solution we get in the end will be the same regardless of the path we chose to follow in the solution process.

Step 3
Next, we need to convert the 6 below the 1 into a zero and we can do that with the following elementary row operation.

Step 4
The next step is to turn the number at the bottom of the second column (-4 in this case) into a one. The following elementary row operation will do that for us.

Step 5
Finally we need to convert the number above the one we got in Step 4 into a zero. To do that we can use the following elementary row operation.

Step 6
From the final augmented matrix we found in Step 5 we get the solution to the system is:

\[ x = -\frac{1}{3}, \ y = -2. \]

5. For the following system of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

\[ 5x - 25y = 3 \]
\[ -2x + 10y = 2 \]

Step 1
The first step is to write down the augmented matrix for the system of equations.
Step 2
We need to make the number in the upper left corner a one. In this case we can do this with the following elementary row operation.

\[
\begin{bmatrix}
  5 & -25 & 3 \\
  -2 & 10 & 2 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  0 & 0 & 8 \\
  -2 & 10 & 2 \\
\end{bmatrix}
\]

Step 3
Okay let’s step back for a second and convert the first row back to an equation. Doing this gives,

\[0 = 8\]

That is clearly not true and we’ve done all our work correctly and so this system is inconsistent and there is no solution to the system.

6. For the following system of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

\[
2x + 3y = 20 \\
7x + 2y = 53
\]

Step 1
The first step is to write down the augmented matrix for the system of equations.

\[
\begin{bmatrix}
  2 & 3 & 20 \\
  7 & 2 & 53 \\
\end{bmatrix}
\]

Step 2
We need to make the number in the upper left corner a one. There are several ways to do this. One way would be to use the elementary row operation \( \frac{1}{2} R_1 \). However, this would put fractions into the other two entries in the first row and it might be nice to avoid them.

While this may seem to be of any use let’s take a look at the following elementary row operation.

\[
\begin{bmatrix}
  2 & 3 & 20 \\
  7 & 2 & 53 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  2 & 3 & 20 \\
  1 & -7 & -7 \\
\end{bmatrix}
\]

This operation worked on the second row instead of the first row that we need to work on. Note however, that we did put a 1 in the lower number of the first column. We need a 1 in the upper number of the first column and we can do that now simply by switching rows as follows,
Step 3
Next, we need to convert the 2 below the 1 into a zero and we can do that with the following elementary row operation.

\[
\begin{bmatrix}
1 & -7 & -7 \\
2 & 3 & 20
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -7 & -7 \\
0 & 17 & 34
\end{bmatrix}
\]

Step 4
The next step is to turn the number at the bottom of the second column (17 in this case) into a one. The following elementary row operation will do that for us.

\[
\begin{bmatrix}
1 & -7 & -7 \\
0 & 17 & 34
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -7 & -7 \\
0 & 1 & 2
\end{bmatrix}
\]

Step 5
Finally we need to convert the number above the one we got in Step 4 into a zero. To do that we can use the following elementary row operation.

\[
\begin{bmatrix}
1 & -7 & -7 \\
0 & 1 & 2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 7 \\
0 & 1 & 2
\end{bmatrix}
\]

Step 6
From the final augmented matrix we found in Step 5 we get the solution to the system is: \(x = 7, y = 2\).

7. For the following system of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

\[
\begin{align*}
2x + 5y + 2z &= -38 \\
3x - 2y + 4z &= 17 \\
-6x + y - 7z &= -12
\end{align*}
\]

Step 1
The first step is to write down the augmented matrix for the system of equations.
Step 2
We need to make the number in the upper left corner a one. Much like with the previous problems (i.e. solving systems with two variables) we can quickly do it with the elementary row operation \( \frac{1}{2} R_1 \) but that will put fractions into the augmented matrix and they would probably be around for quite a few steps and it would be really nice to avoid them for as long as possible when the augmented matrix starts getting this size.

So, let’s start with the following elementary row operation.

\[
\begin{bmatrix}
2 & 5 & 2 & -38 \\
3 & -2 & 4 & 17 \\
-6 & 1 & -7 & -12
\end{bmatrix}
\]

Step 3
Next, we need to convert the 3 and the -6 below the 1 in the first column into zeroes and we can do that with the following elementary row operations.

\[
\begin{bmatrix}
1 & -7 & 2 & 55 \\
3 & -2 & 4 & 17 \\
-6 & 1 & -7 & -12
\end{bmatrix}
\]

Step 4
We now need to turn the 19 in the second row into a one and it seems like the only easy way to do that is the following elementary row operation.
In the first step we chose to avoid the step that put fractions into the augmented matrix, but sometimes, as in this step, they can’t be avoided. With augmented matrices for systems with three variables fractions will almost inevitably show up and they will often be “messy” when they do.

This is just something we’ll need to deal with when solving these systems. We try to avoid them for as long as possible but except it when they show up and continue with the solution process.

Step 5
Next we need to turn the -41 in the third row into a zero. The following elementary row operation will do that for us.

\[
\begin{bmatrix}
1 & -7 & 2 & | 55 \\
0 & 1 & -\frac{2}{19} & | -\frac{148}{19} \\
0 & -41 & 5 & | 318
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -7 & 2 & | 55 \\
0 & 1 & -\frac{2}{19} & | -\frac{148}{19} \\
0 & 0 & 1 & | 13
\end{bmatrix}
\]

Again, we had to put more fraction into the augmented matrix. This is just a fact of life with these types of problems. However, as we’ll see in the next step they do often disappear as well.

Step 6
Okay, we need to turn the \( \frac{13}{19} \) in the third row into a one and we can do that as follows,

\[
\begin{bmatrix}
1 & -7 & 2 & | 55 \\
0 & 1 & -\frac{2}{19} & | -\frac{148}{19} \\
0 & 0 & \frac{13}{19} & | -\frac{26}{19}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -7 & 2 & | 55 \\
0 & 1 & -\frac{2}{19} & | -\frac{148}{19} \\
0 & 0 & 1 & | -2
\end{bmatrix}
\]

Step 7
Next we need to turn the -\( \frac{2}{19} \) and the 2 in the third column into zeroes. The following elementary row operations will do that for us.

\[
\begin{bmatrix}
1 & -7 & 2 & | 55 \\
0 & 1 & -\frac{2}{19} & | -\frac{148}{19} \\
0 & 0 & 1 & | -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -7 & 0 & | 59 \\
0 & 1 & 0 & | -8 \\
0 & 0 & 1 & | -2
\end{bmatrix}
\]

Note that the fractions are now completely gone! This won’t always happen but it also will happen fairly regularly that fractions get introduced in intermediate steps and then go away in later steps.

Step 8
For the final operation we need to turn the -7 in the second column into a zero and we can do that as follows,
Step 9
From the final augmented matrix we found in Step 8 we get the solution to the system is:
\[ x = 3, \quad y = -8, \quad z = -2. \]

8. For the following system of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

\[
\begin{align*}
3x - 9z &= 33 \\
7x - 4y - z &= -15 \\
4x + 6y + 5z &= -6
\end{align*}
\]

Step 1
The first step is to write down the augmented matrix for the system of equations.
\[
\begin{bmatrix}
3 & 0 & -9 & | & 33 \\
7 & -4 & -1 & | & -15 \\
4 & 6 & 5 & | & -6
\end{bmatrix}
\]

Note the zero in the second column of the first row. Recall that the second column corresponds to the coefficients of the \( y \)'s in each equation and because there is no \( y \) in the first equation that coefficient must be zero.

Step 2
We need to make the number in the upper left corner a one. We can easily do that with the following elementary row operation.
\[
\begin{bmatrix}
3 & 0 & -9 & | & 33 \\
7 & -4 & -1 & | & -15 \\
4 & 6 & 5 & | & -6
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 & | & 11 \\
7 & -4 & -1 & | & -15 \\
4 & 6 & 5 & | & -6
\end{bmatrix}
\]

Step 3
Next, we need to convert the 7 and the 4 below the 1 in the first column into zeroes and we can do that with the following elementary row operations.
Step 4
We now need to turn the -4 in the second row into a one and that can be done with the following elementary row operation.

\[
\begin{bmatrix}
1 & 0 & -3 & \mid & 11 \\
0 & -4 & 20 & \mid & -92 \\
0 & 6 & 17 & \mid & -50 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 & \mid & 11 \\
0 & 1 & -5 & \mid & 23 \\
0 & 6 & 17 & \mid & -50 \\
\end{bmatrix}
\]

Step 5
Next we need to turn the 6 in the third row into a zero. The following elementary row operation will do that for us.

\[
\begin{bmatrix}
1 & 0 & -3 & \mid & 11 \\
0 & 1 & -5 & \mid & 23 \\
0 & 6 & 17 & \mid & -50 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 & \mid & 11 \\
0 & 1 & -5 & \mid & 23 \\
0 & 0 & 47 & \mid & -188 \\
\end{bmatrix}
\]

Step 6
Okay, we need to turn the 47 in the third row into a one and we can do that as follows,

\[
\begin{bmatrix}
1 & 0 & -3 & \mid & 11 \\
0 & 1 & -5 & \mid & 23 \\
0 & 0 & 47 & \mid & -188 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 & \mid & 11 \\
0 & 1 & -5 & \mid & 23 \\
0 & 0 & 1 & \mid & -4 \\
\end{bmatrix}
\]

Step 7
Next we need to turn the -5 and the -3 in the third column into zeroes. The following elementary row operations will do that for us.

\[
\begin{bmatrix}
1 & 0 & -3 & \mid & 11 \\
0 & 1 & -5 & \mid & 23 \\
0 & 0 & 1 & \mid & -4 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & \mid & -1 \\
0 & 1 & 0 & \mid & 3 \\
0 & 0 & 1 & \mid & -4 \\
\end{bmatrix}
\]

Step 8
Normally we would have another step to do. We would need to turn the number in the first row and second column into a zero. However, in this case there is already a zero there and so there is no work to do in this step.

The final form of the augmented matrix is then,
As this step has shown we occasionally will get a number “for free”. In other words, the work we put into an intermediate step will give us not only the number we were looking for in that step but will also put in a number that we need in a later step. Or, as in this case, the number we needed was actually there from the start.

Step 9
From the final augmented matrix we found in Step 8 we get the solution to the system is:

\[ x = -1, \quad y = 3, \quad z = -4. \]