Preface

Here are my online notes for my Calculus III course that I teach here at Lamar University. Despite the fact that these are my “class notes” they should be accessible to anyone wanting to learn Calculus III or needing a refresher in some of the topics from the class.

These notes do assume that the reader has a good working knowledge of Calculus I topics including limits, derivatives and integration. It also assumes that the reader has a good knowledge of several Calculus II topics including some integration techniques, parametric equations, vectors, and knowledge of three dimensional space.

Here are a couple of warnings to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn calculus I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn’t covered in class.

2. In general I try to work problems in class that are different from my notes. However, with Calculus III many of the problems are difficult to make up on the spur of the moment and so in this class my class work will follow these notes fairly close as far as worked problems go. With that being said I will, on occasion, work problems off the top of my head when I can to provide more examples than just those in my notes. Also, I often don’t have time in class to work all of the problems in the notes and so you will find that some sections contain problems that weren’t worked in class due to time restrictions.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible in writing these up, but the reality is that I can’t anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I’ve not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.
Differentials

This is a very short section and is here simply to acknowledge that just like we had differentials for functions of one variable we also have them for functions of more than one variable. Also, as we’ve already seen in previous sections, when we move up to more than one variable things work pretty much the same, but there are some small differences.

Given the function \( z = f(x, y) \) the differential \( dz \) or \( df \) is given by,

\[
dz = f_x \, dx + f_y \, dy \quad \text{or} \quad df = f_x \, dx + f_y \, dy
\]

There is a natural extension to functions of three or more variables. For instance, given the function \( w = g(x, y, z) \) the differential is given by,

\[
dw = g_x \, dx + g_y \, dy + g_z \, dz
\]

Let’s do a couple of quick examples.

Example 1  Compute the differentials for each of the following functions.

(a) \( z = e^{x^2+y^2} \tan(2x) \)

(b) \( u = \frac{t^3 \, r^6}{s^2} \)

Solution

(a) \( z = e^{x^2+y^2} \tan(2x) \)

There really isn’t a whole lot to these outside of some quick differentiation. Here is the differential for the function.

\[
dz = \left( 2xe^{x^2+y^2} \tan(2x) + 2e^{x^2+y^2} \sec^2(2x) \right)dx + 2ye^{x^2+y^2} \tan(2x)dy
\]

(b) \( u = \frac{t^3 \, r^6}{s^2} \)

Here is the differential for this function.

\[
du = \frac{3t^2 \, r^6}{s^2} \, dt + \frac{6t^3 \, r^5}{s^2} \, dr - \frac{2t^3 \, r^6}{s^3} \, ds
\]

Note that sometimes these differentials are called the total differentials.