Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Review: Inverse Functions

1. Find the inverse for \( f(x) = 6x + 15 \). Verify your inverse by computing one or both of the composition as discussed in this section.

Hint: Remember the process described in this section. Replace the \( f(x) \), interchange the x’s and y’s, solve for y and the finally replace the y with \( f^{-1}(x) \).

Step 1
\[
y = 6x + 15
\]

Step 2
\[
x = 6y + 15
\]

Step 3
\[
x - 15 = 6y
\]
\[
y = \frac{1}{6}(x - 15) \quad \rightarrow \quad f^{-1}(x) = \frac{1}{6}(x - 15)
\]
Finally, compute either \( (f \circ f^{-1})(x) \) or \( (f^{-1} \circ f)(x) \) to verify our work.

Step 4
Either composition can be done so let’s do \( (f \circ f^{-1})(x) \) in this case.

\[
(f \circ f^{-1})(x) = f\left[f^{-1}(x)\right]
= 6\left[\frac{1}{6}(x-15)\right] + 15
= x - 15 + 15
= x
\]

So, we got \( x \) out of the composition and so we know we’ve done our work correctly.

2. Find the inverse for \( h(x) = 3 - 29x \). Verify your inverse by computing one or both of the composition as discussed in this section.

Hint : Remember the process described in this section. Replace the \( h(x) \), interchange the \( x \)'s and \( y \)'s, solve for \( y \) and the finally replace the \( y \) with \( h^{-1}(x) \).

Step 1
\[ y = 3 - 29x \]

Step 2
\[ x = 3 - 29y \]

Step 3
\[ x - 3 = -29y \]
\[ y = -\frac{1}{29}(x - 3) \quad \rightarrow \quad h^{-1}(x) = \frac{1}{29}(3 - x) \]

Notice that we multiplied the minus sign into the parenthesis. We did this in order to avoid potentially losing the minus sign if it had stayed out in front. This does not need to be done in order to get the inverse.

Finally, compute either \( (h \circ h^{-1})(x) \) or \( (h^{-1} \circ h)(x) \) to verify our work.

Step 4
Either composition can be done so let’s do \((h \circ h^{-1})(x)\) in this case.

\[
(h \circ h^{-1})(x) = h\left[h^{-1}(x)\right]
\]

\[
= 3 - 29\left[\frac{1}{29}(3 - x)\right]
\]

\[
= 3 - (3 - x)
\]

\[
= x
\]

So, we got \(x\) out of the composition and so we know we’ve done our work correctly.

3. Find the inverse for \(R(x) = x^3 + 6\). Verify your inverse by computing one or both of the composition as discussed in this section.

Hint : Remember the process described in this section. Replace the \(R(x)\), interchange the \(x\)'s and \(y\)'s, solve for \(y\) and the finally replace the \(y\) with \(R^{-1}(x)\).

Step 1

\[
y = x^3 + 6
\]

Step 2

\[
x = y^3 + 6
\]

Step 3

\[
x - 6 = y^3
\]

\[
y = \sqrt[3]{x - 6}
\]

\[
R^{-1}(x) = \sqrt[3]{x - 6}
\]

Finally, compute either \((R \circ R^{-1})(x)\) or \((R^{-1} \circ R)(x)\) to verify our work.

Step 4

Either composition can be done so let’s do \((R^{-1} \circ R)(x)\) in this case.

\[
(R^{-1} \circ R)(x) = R^{-1}\left[R(x)\right]
\]

\[
= \sqrt[3]{(x^3 + 6) - 6}
\]

\[
= \sqrt[3]{x^3}
\]

\[
= x
\]

So, we got \(x\) out of the composition and so we know we’ve done our work correctly.
4. Find the inverse for \( g(x) = 4(x - 3)^5 + 21 \). Verify your inverse by computing one or both of the composition as discussed in this section.

Hint: Remember the process described in this section. Replace the \( g(x) \), interchange the \( x \)'s and \( y \)'s, solve for \( y \) and the finally replace the \( y \) with \( g^{-1}(x) \).

Step 1
\[ y = 4(x - 3)^5 + 21 \]

Step 2
\[ x = 4(y - 3)^5 + 21 \]

Step 3
\[ x - 21 = 4(y - 3)^5 \]
\[ \frac{1}{4}(x - 21) = (y - 3)^5 \]
\[ \sqrt[5]{\frac{1}{4}(x - 21)} = y - 3 \]
\[ y = 3 + \sqrt[5]{\frac{1}{4}(x - 21)} \]

\[ g^{-1}(x) = 3 + \sqrt[5]{\frac{1}{4}(x - 21)} \]

Finally, compute either \( (g \circ g^{-1})(x) \) or \( (g^{-1} \circ g)(x) \) to verify our work.

Step 4
Either composition can be done so let’s do \( (g \circ g^{-1})(x) \) in this case.
So, we got \( x \) out of the composition and so we know we’ve done our work correctly.

5. Find the inverse for \( W(x) = \sqrt[3]{9 - 11x} \). Verify your inverse by computing one or both of the composition as discussed in this section.

Hint: Remember the process described in this section. Replace the \( W(x) \), interchange the \( x \)'s and \( y \)'s, solve for \( y \) and the finally replace the \( y \) with \( W^{-1}(x) \).

Step 1
\[ y = \sqrt[3]{9 - 11x} \]

Step 2
\[ x = \sqrt[3]{9 - 11y} \]

Step 3
\[ x = \sqrt[3]{9 - 11y} \]
\[ x^3 = 9 - 11y \]
\[ x^3 - 9 = -11y \]
\[ y = -\frac{1}{11}(x^3 - 9) \quad \rightarrow \quad W^{-1}(x) = \frac{1}{11}(9 - x^3) \]

Notice that we multiplied the minus sign into the parenthesis. We did this in order to avoid potentially losing the minus sign if it had stayed out in front. This does not need to be done in order to get the inverse.

Finally, compute either \( (W \circ W^{-1})(x) \) or \( (W^{-1} \circ W)(x) \) to verify our work.
Step 4
Either composition can be done so let’s do \((W^{-1} \circ W)(x)\) in this case.

\[
(W^{-1} \circ W)(x) = W^{-1}[W(x)]
\]

\[
= \frac{1}{11}\left(9 - \left[\sqrt[5]{9 - 11x}\right]^5\right)
\]

\[
= \frac{1}{11}(9 - [9 - 11x])
\]

\[
= \frac{1}{11}(11x)
\]

\[
= x
\]

So, we got \(x\) out of the composition and so we know we’ve done our work correctly.

6. Find the inverse for \(f(x) = \sqrt[5]{5x + 8}\). Verify your inverse by computing one or both of the composition as discussed in this section.

Hint: Remember the process described in this section. Replace the \(f(x)\), interchange the \(x\)’s and \(y\)’s, solve for \(y\) and the finally replace the \(y\) with \(f^{-1}(x)\).

Step 1
\[ y = \sqrt[5]{5x + 8} \]

Step 2
\[ x = \sqrt[5]{5y + 8} \]

Step 3
\[ x = \sqrt[5]{5y + 8} \]
\[ x^5 = 5y + 8 \]
\[ x^5 - 8 = 5y \]

\[ y = \frac{1}{5}(x^5 - 8) \quad \rightarrow \quad f^{-1}(x) = \frac{1}{5}(x^5 - 8) \]

Finally, compute either \((f \circ f^{-1})(x)\) or \((f^{-1} \circ f)(x)\) to verify our work.
Either composition can be done so let’s do \((f \circ f^{-1})(x)\) in this case.

\[
(f \circ f^{-1})(x) = f \left[ f^{-1}(x) \right]
\]

\[
= \sqrt[5]{\frac{1}{5}(x^7 - 8)} + 8
\]

\[
= \sqrt[5]{x^7 - 8} + 8
\]

\[
= \sqrt[5]{x^7}
\]

\[
= x
\]

So, we got \(x\) out of the composition and so we know we’ve done our work correctly.

7. Find the inverse for \(h(x) = \frac{1+9x}{4-x}\). Verify your inverse by computing one or both of the composition as discussed in this section.

Hint : Remember the process described in this section. Replace the \(h(x)\), interchange the \(x\)'s and \(y\)'s, solve for \(y\) and the finally replace the \(y\) with \(h^{-1}(x)\).

Step 1

\[
y = \frac{1+9x}{4-x}
\]

Step 2

\[
x = \frac{1+9y}{4-y}
\]

Step 3

\[
x = \frac{1+9y}{4-y}
\]

\[
x(4-y) = 1+9y
\]

\[
4x-xy = 1+9y
\]

\[
4x-1 = 9y+xy
\]

\[
4x-1 = (9+x)y
\]

\[
y = \frac{4x-1}{9+x}
\]

\[
h^{-1}(x) = \frac{4x-1}{9+x}
\]
Note that the Algebra in these kinds of problems can often be fairly messy, but don’t let that make you decide that you can’t do these problems. Messy Algebra will be a fairly common occurrence in a Calculus class so you’ll need to get used to it!

Finally, compute either \((h \circ h^{-1})(x)\) or \((h^{-1} \circ h)(x)\) to verify our work.

Step 4
Either composition can be done so let’s do \((h^{-1} \circ h)(x)\) in this case. As with the previous step, the Algebra here is going to be messy and in fact will probably be messier.

\[
(h^{-1} \circ h)(x) = h^{-1}\left[h(x)\right]
\]

\[
= \frac{4\left[\frac{1+9x}{4-x}\right] - 1}{4-x}
\]

\[
= 9 + \frac{1+9x}{4-x}
\]

\[
= \frac{4(1+9x) - (4-x)}{9(4-x)+1+9x}
\]

\[
= \frac{4+36x-4+x}{36-9x+1+9x}
\]

\[
= \frac{37x}{37}
\]

\[
= x
\]

In order to do the simplification we multiplied the numerator and denominator of the initial fraction by \(4-x\) in order to clear out some of the denominators. This in turn allowed a fair amount of simplification.

So, we got \(x\) out of the composition and so we know we’ve done our work correctly.

---

8. Find the inverse for \(f(x) = \frac{6-10x}{8x+7}\). Verify your inverse by computing one or both of the composition as discussed in this section.

Hint: Remember the process described in this section. Replace the \(f(x)\), interchange the \(x\)’s and \(y\)’s, solve for \(y\) and the finally replace the \(y\) with \(f^{-1}(x)\).

Step 1

\[
y = \frac{6-10x}{8x+7}
\]
Calculus I

Step 2

\[ x = \frac{6 - 10y}{8y + 7} \]

Step 3

\[ x = \frac{6 - 10y}{8y + 7} \]
\[ x(8y + 7) = 6 - 10y \]
\[ 8xy + 7x = 6 - 10y \]
\[ 8xy + 10y = 6 - 7x \]
\[ (8x + 10)y = 6 - 7x \]
\[ y = \frac{6 - 7x}{8x + 10} \]

Note that the Algebra in these kinds of problems can often be fairly messy, but don’t let that make you decide that you can’t do these problems. Messy Algebra will be a fairly common occurrence in a Calculus class so you’ll need to get used to it!

Finally, compute either \( (f \circ f^{-1})(x) \) or \( (f^{-1} \circ f)(x) \) to verify our work.

Step 4

Either composition can be done so let’s do \( (f \circ f^{-1})(x) \) in this case. As with the previous step, the Algebra here is going to be messy and in fact will probably be messier.

\[ (f \circ f^{-1})(x) = f[f^{-1}(x)] \]

\[ = \frac{6 - 10}{8} \left[ \frac{6 - 7x}{8x + 10} \right] + \frac{8x + 10}{8x + 10} \]
\[ = \frac{6(8x + 10) - 10(6 - 7x)}{8(6 - 7x) + 7(8x + 10)} \]
\[ = \frac{48x + 60 - 60 + 70x}{48 - 56x + 56x + 70} \]
\[ = \frac{118x}{118} \]
\[ = x \]

So, we got \( x \) out of the composition and so we know we’ve done our work correctly.