Preface

Here are my online notes for my Calculus III course that I teach here at Lamar University. Despite the fact that these are my “class notes”, they should be accessible to anyone wanting to learn Calculus III or needing a refresher in some of the topics from the class.

These notes do assume that the reader has a good working knowledge of Calculus I topics including limits, derivatives and integration. It also assumes that the reader has a good knowledge of several Calculus II topics including some integration techniques, parametric equations, vectors, and knowledge of three dimensional space.

Here are a couple of warnings to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn calculus I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn’t covered in class.

2. In general I try to work problems in class that are different from my notes. However, with Calculus III many of the problems are difficult to make up on the spur of the moment and so in this class my class work will follow these notes fairly close as far as worked problems go. With that being said I will, on occasion, work problems off the top of my head when I can to provide more examples than just those in my notes. Also, I often don’t have time in class to work all of the problems in the notes and so you will find that some sections contain problems that weren’t worked in class due to time restrictions.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible in writing these up, but the reality is that I can’t anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I’ve not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.
Cylindrical Coordinates

As with two dimensional space the standard \((x, y, z)\) coordinate system is called the Cartesian coordinate system. In the last two sections of this chapter we’ll be looking at some alternate coordinate systems for three dimensional space.

We’ll start off with the cylindrical coordinate system. This one is fairly simple as it is nothing more than an extension of polar coordinates into three dimensions. Not only is it an extension of polar coordinates, but we extend it into the third dimension just as we extend Cartesian coordinates into the third dimension. All that we do is add a \(z\) on as the third coordinate. The \(r\) and \(\theta\) are the same as with polar coordinates.

Here is a sketch of a point in \(\mathbb{R}^3\).

The conversions for \(x\) and \(y\) are the same conversions that we used back when we were looking at polar coordinates. So, if we have a point in cylindrical coordinates the Cartesian coordinates can be found by using the following conversions.

\[
x = r \cos \theta \\
y = r \sin \theta \\
z = z
\]

The third equation is just an acknowledgement that the \(z\)-coordinate of a point in Cartesian and polar coordinates is the same.

Likewise, if we have a point in Cartesian coordinates the cylindrical coordinates can be found by using the following conversions.
Let’s take a quick look at some surfaces in cylindrical coordinates.

**Example 1** Identify the surface for each of the following equations.

(a) \( r = 5 \)

(b) \( r^2 + z^2 = 100 \)

(c) \( z = r \)

**Solution**

(a) In two dimensions we know that this is a circle of radius 5. Since we are now in three dimensions and there is no \( z \) in equation this means it is allowed to vary freely. So, for any given \( z \) we will have a circle of radius 5 centered on the \( z \)-axis.

In other words, we will have a cylinder of radius 5 centered on the \( z \)-axis.

(b) This equation will be easy to identify once we convert back to Cartesian coordinates.

\[
\begin{align*}
  r^2 + z^2 &= 100 \\
  x^2 + y^2 + z^2 &= 100
\end{align*}
\]

So, this is a sphere centered at the origin with radius 10.

(c) Again, this one won’t be too bad if we convert back to Cartesian. For reasons that will be apparent eventually, we’ll first square both sides, then convert.

\[
\begin{align*}
  z^2 &= r^2 \\
  z^2 &= x^2 + y^2
\end{align*}
\]

From the section on [quadric surfaces](http://tutorial.math.lamar.edu/terms.aspx) we know that this is the equation of a cone.