Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you’ve reached the level of working the harder problems then you will probably already understand the basics fairly well and won’t need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven’t been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

**Review : Solving Trig Equations with Calculators, Part I**

1. Find all the solutions to \( 7 \cos(4x) + 11 = 10 \). Use at least 4 decimal places in your work.

   **Hint 1 :** Isolate the cosine (with a coefficient of one) on one side of the equation.

   **Step 1**
   Isolating the cosine (with a coefficient of one) on one side of the equation gives,
   \[
   \cos(4x) = -\frac{1}{7}
   \]

   **Hint 2 :** Using a calculator and your knowledge of the unit circle to determine all the angles in the range \([0, 2\pi]\) for which cosine will have this value.

   **Step 2**
   First, using our calculator we can see that,
   \[
   4x = \cos^{-1}\left(-\frac{1}{7}\right) = 1.7141
   \]
Now we’re dealing with cosine in this problem and we know that the $x$-axis represents cosine on a unit circle and so we’re looking for angles that will have a $x$ coordinate of $-\frac{1}{7}$. This means that we’ll have angles in the second (this is the one our calculator gave us) and third quadrant. Here is a unit circle for this situation.

From the symmetry of the unit circle we can see that we can either use $-1.7141$ or $2\pi - 1.7141 = 4.5691$ for the second angle. Each will give the same set of solutions. However, because it is easy to lose track of minus signs we will use the positive angle for our second solution.

Hint 3 : Using the two angles above write down all the angles for which cosine will have this value and use these to write down all the solutions to the equation.

Step 3

From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding “$+2\pi n$ for $n = 0, \pm 1, \pm 2, \ldots$” onto each of these.

This then means that we must have,

$$4x = 1.7141 + 2\pi n \quad \text{OR} \quad 4x = 4.5691 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

Finally, to get all the solutions to the equation all we need to do is divide both sides by 4.

$$x = 0.4285 + \frac{\pi n}{2} \quad \text{OR} \quad x = 1.1423 + \frac{\pi n}{2} \quad n = 0, \pm 1, \pm 2, \ldots$$
Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4\textsuperscript{th} decimal place or so however.

2. Find the solution(s) to $6 + 5 \cos \left(\frac{x}{3}\right) = 10$ that are in $[0, 3\pi]$. Use at least 4 decimal places in your work.

Hint 1: Find all the solutions to the equation without regard to the given interval. The first step in this process is to isolate the cosine (with a coefficient of one) on one side of the equation.

Step 1
Isolating the cosine (with a coefficient of one) on one side of the equation gives,

$$\cos \left(\frac{x}{3}\right) = \frac{4}{5}$$

Hint 2: Using a calculator and your knowledge of the unit circle to determine all the angles in the range $[0, 2\pi]$ for which cosine will have this value.

Step 2
First, using our calculator we can see that,

$$\frac{x}{3} = \cos^{-1} \left(\frac{4}{5}\right) = 0.6435$$

Now we’re dealing with cosine in this problem and we know that the $x$-axis represents cosine on a unit circle and so we’re looking for angles that will have a $x$ coordinate of $\frac{4}{5}$. This means that we’ll have angles in the first (this is the one our calculator gave us) and fourth quadrant. Here is a unit circle for this situation.
From the symmetry of the unit circle we can see that we can either use \(-0.6435\) or \(2\pi - 0.6435 = 5.6397\) for the second angle. Each will give the same set of solutions. However, because it is easy to lose track of minus signs we will use the positive angle for our second solution.

**Hint 3:** Using the two angles above write down all the angles for which cosine will have this value and use these to write down all the solutions to the equation.

**Step 3**
From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding \(\pm 2\pi n\) for \(n = 0, \pm 1, \pm 2, \ldots\)” onto each of these.

This then means that we must have,

\[
\frac{x}{3} = 0.6435 + 2\pi n \quad \text{OR} \quad \frac{x}{3} = 5.6397 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots
\]

Finally, to get all the solutions to the equation all we need to do is multiply both sides by 3 and we’ll convert everything to decimals to help with the final step.

\[
x = 1.9305 + 6\pi n \quad \text{OR} \quad x = 16.9191 + 6\pi n \quad n = 0, \pm 1, \pm 2, \ldots
\]

\[
= 1.9305 + 18.8496n \quad \text{OR} \quad = 16.9191 + 18.8496n \quad n = 0, \pm 1, \pm 2, \ldots
\]

**Hint 4:** Now all we need to do is plug in values of \(n\) to determine which solutions will actually fall in the given interval.

**Step 4**
Now let’s find all the solutions. First notice that, in this case, if we plug in negative values of \( n \) we will get negative solutions and these will not be in the interval and so there is no reason to even try these. So, let’s start at \( n = 0 \) and see what we get.

\[
\begin{align*}
  n = 0 : & \quad x = 1.9305 \quad \text{OR} \quad x = 16.9191 \\
  n = 1 : & \quad x = 20.7801 \quad \text{OR} \quad x = 35.7687
\end{align*}
\]

Notice that with each increase in \( n \) we were really just adding another 18.8496 onto the previous results and by doing this to the results from the \( n = 1 \) step we will get solutions that are outside of the interval and so there is no reason to even plug in \( n = 2 \).

So, it looks like we have the four solutions to this equation in the given interval.

\[
\{ x = 1.9305, 16.9191, 20.7801, 35.7687 \}
\]

Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4th decimal place or so however.

3. Find all the solutions to \( 3 = 6 - 11 \sin \left( \frac{t}{8} \right) \). Use at least 4 decimal places in your work.

Hint 1 : Isolate the sine (with a coefficient of one) on one side of the equation.

Step 1
Isolating the sine (with a coefficient of one) on one side of the equation gives,

\[
\sin \left( \frac{t}{8} \right) = \frac{3}{11}
\]

Hint 2 : Using a calculator and your knowledge of the unit circle to determine all the angles in the range \([0, 2\pi]\) for which sine will have this value.

Step 2
First, using our calculator we can see that,

\[
\frac{t}{8} = \sin^{-1} \left( \frac{3}{11} \right) = 0.2762
\]

Now we’re dealing with sine in this problem and we know that the \( y \)-axis represents sine on a unit circle and so we’re looking for angles that will have a \( y \) coordinate of \( \frac{3}{11} \). This means that we’ll
have angles in the first (this is the one our calculator gave us) and second quadrant. Here is a unit circle for this situation.

From the symmetry of the unit circle we can see that \( \pi - 0.2762 = 2.8654 \) is the second angle.

Hint 3 : Using the two angles above write down all the angles for which sine will have this value and use these to write down all the solutions to the equation.

Step 3
From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding “\( + 2\pi n \) for \( n = 0, \pm 1, \pm 2, \ldots \)” onto each of these.

This then means that we must have,

\[
\frac{t}{8} = 0.2762 + 2\pi n \quad \text{OR} \quad \frac{t}{8} = 2.8654 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots
\]

Finally, to get all the solutions to the equation all we need to do is multiply both sides by 8.

\[
\begin{align*}
t &= 2.2096 + 16\pi n \quad \text{OR} \quad t &= 22.9232 + 16\pi n \\
&n = 0, \pm 1, \pm 2, \ldots
\end{align*}
\]

Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4th decimal place or so however.
4. Find the solution(s) to \( 4 \sin(6z) + \frac{13}{10} = -\frac{3}{10} \) that are in \( [0, 2\pi] \). Use at least 4 decimal places in your work.

Hint 1: Find all the solutions to the equation without regard to the given interval. The first step in this process is to isolate the sine (with a coefficient of one) on one side of the equation.

Step 1
Isolating the sine (with a coefficient of one) on one side of the equation gives,

\[
\sin(6z) = -\frac{2}{5}
\]

Hint 2: Using a calculator and your knowledge of the unit circle to determine all the angles in the range \( [0, 2\pi] \) for which sine will have this value.

Step 2
First, using our calculator we can see that,

\[
6z = \sin^{-1}\left(-\frac{2}{5}\right) = -0.4115
\]

Now we’re dealing with sine in this problem and we know that the \( y \)-axis represents sine on a unit circle and so we’re looking for angles that will have a \( y \) coordinate of \(-\frac{2}{5}\). This means that we’ll have angles in the fourth (this is the one our calculator gave us) and third quadrant. Here is a unit circle for this situation.

![Unit Circle Diagram]
From the symmetry of the unit circle we can see that the second angle will make an angle of 0.4115 with the negative x-axis and so the second angle will be $\pi + 0.4115 = 3.5531$. Also, as noted on the unit circle above a positive angle that represents the first angle (i.e. the angle in the fourth quadrant) is $2\pi - 0.4115 = 5.8717$. We can use either the positive or the negative angle here and we’ll get the same solutions. However, because it is often easy to lose track of minus signs we will be using the positive angle in the fourth quadrant for our work here.

Hint 3 : Using the two angles above write down all the angles for which sine will have this value and use these to write down all the solutions to the equation.

Step 3
From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding “$+2\pi n$ for $n = 0, \pm 1, \pm 2, \ldots$” onto each of these.

This then means that we must have,

$$6z = 3.5531 + 2\pi n \quad \text{OR} \quad 6z = 5.8717 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

Finally, to get all the solutions to the equation all we need to do is divide both sides by 6 and we’ll convert everything to decimals to help with the final step.

$$z = 0.5922 + \frac{\pi n}{3} \quad \text{OR} \quad z = 0.9786 + \frac{\pi n}{3} \quad n = 0, \pm 1, \pm 2, \ldots$$

$$= 0.5922 + 1.0472n \quad \text{OR} \quad = 0.9786 + 1.0472n \quad n = 0, \pm 1, \pm 2, \ldots$$

Hint 4 : Now all we need to do is plug in values of $n$ to determine which solutions will actually fall in the given interval.

Step 4
Now let’s find all the solutions. First notice that, in this case, if we plug in negative values of $n$ we will get negative solutions and these will not be in the interval and so there is no reason to even try these. So, let’s start at $n = 0$ and see what we get.

$$n = 0: \quad z = 0.5922 \quad \text{OR} \quad z = 0.9786$$

$$n = 1: \quad z = 1.6394 \quad \text{OR} \quad z = 2.1025 > 2$$

Notice that with each increase in $n$ we were really just adding another 1.0472 onto the previous results and by doing this to the results from the $n = 1$ step we will get solutions that are outside of the interval and so there is no reason to even plug in $n = 2$. Also, as we’ve seen in this problem it is completely possible for only one of the solutions from a given interval to be in the given interval so don’t worry about that when it happens.

So, it looks like we have the three solutions to this equation in the given interval.

Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4th decimal place or so however.

5. Find the solution(s) to \( 9 \cos \left( \frac{4z}{9} \right) + 21 \sin \left( \frac{4z}{9} \right) = 0 \) that are in \([-10, 10]\). Use at least 4 decimal places in your work.

Hint 1 : Find all the solutions to the equation without regard to the given interval. The first step in this process is to reduce the equation down to a single trig function (with a coefficient of one) on one side of the equation.

Step 1
Because we’ve got both a sine and a cosine here it makes some sense to reduce this down to tangent. So, reducing to a tangent (with a coefficient of one) on one side of the equation gives,

\[ \tan \left( \frac{4z}{9} \right) = -\frac{3}{7} \]

Hint 2 : Using a calculator and your knowledge of solving trig equations involving tangents to determine all the angles in the range \([0, 2\pi]\) for which tangent will have this value.

Step 2
First, using our calculator we can see that,

\[ \frac{4z}{9} = \tan^{-1} \left( -\frac{3}{7} \right) = -0.4049 \]

As we discussed in Example 5 of this section the second angle for equations involving tangent will always be the \( \pi \) plus the first angle. Therefore, \( \pi + (-0.4049) = 2.7367 \) will be the second angle.

Also, because it is very easy to lose track of minus signs we’ll use the fact that we know that any angle plus \( 2\pi \) will give another angle whose terminal line is identical to the original angle to eliminate the minus sign on the first angle. So, another angle that will work for the first angle is \( 2\pi + (-0.4049) = 5.8783 \). Note that there is nothing wrong with using the negative angle and if you chose to work with that you will get the same solutions. We are using the positive angle only to make sure we don’t accidentally lose the minus sign on the angle we received from our calculator.
Hint 3: Using the two angles above write down all the angles for which tangent will have this value and use these to write down all the solutions to the equation.

Step 3
From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding \( +2\pi n \) for \( n = 0, \pm 1, \pm 2, \ldots \) onto each of these.

This then means that we must have,

\[
\frac{4\pi}{9} = 2.7367 + 2\pi n \quad \text{OR} \quad \frac{4\pi}{9} = 5.8783 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots
\]

Finally, to get all the solutions to the equation all we need to do is multiply both sides by \( \frac{9}{4} \) and we’ll convert everything to decimals to help with the final step.

\[
z = 6.1576 \pm \frac{9\pi n}{2} \quad \text{OR} \quad z = 13.2262 \pm \frac{9\pi n}{2} \quad n = 0, \pm 1, \pm 2, \ldots
\]

Hint 4: Now all we need to do is plug in values of \( n \) to determine which solutions will actually fall in the given interval.

Step 4
Now let’s find all the solutions.

\[
\begin{align*}
n = -1: \quad z & = -7.9796 \quad \text{OR} \quad z = -0.9110 \\
n = 0: \quad z & = 6.1576 \quad \text{OR} \quad z = 13.2262 > 10
\end{align*}
\]

Notice that with each increase in \( n \) we were really just adding/subtracting (depending on the sign of \( n \)) another 14.1372 onto the previous results. A quick inspection of the results above will quickly show us that we don’t need to go any farther and we won’t bother with any other values of \( n \). Also, as we’ve seen in this problem it is completely possible for only one of the solutions from a given interval to be in the given interval so don’t worry about that when it happens.

So, it looks like we have the three solutions to this equation in the given interval.

\[
z = -7.9796, -0.9110, 6.1576
\]

Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4\textsuperscript{th} decimal place or so however.
6. Find the solution(s) to $3 \tan \left( \frac{w}{4} \right) - 1 = 11 - 2 \tan \left( \frac{w}{4} \right)$ that are in $[0, 50]$. Use at least 4 decimal places in your work.

Hint 1: Find all the solutions to the equation without regard to the given interval. The first step in this process is to isolate the tangent (with a coefficient of one) on one side of the equation.

Step 1
Isolating the tangent (with a coefficient of one) on one side of the equation gives,

$$\tan \left( \frac{w}{4} \right) = \frac{12}{5}$$

Hint 2: Using a calculator and your knowledge of solving trig equations involving tangents to determine all the angles in the range $[0, 2\pi]$ for which tangent will have this value.

Step 2
First, using our calculator we can see that,

$$\frac{w}{4} = \tan^{-1} \left( \frac{12}{5} \right) = 1.1760$$

As we discussed in Example 5 of this section the second angle for equations involving tangent will always be the $\pi$ plus the first angle. Therefore, $\pi + 1.1760 = 4.3176$ will be the second angle.

Hint 3: Using the two angles above write down all the angles for which tangent will have this value and use these to write down all the solutions to the equation.

Step 3
From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding “$+ 2\pi n$ for $n = 0, \pm 1, \pm 2, \ldots$” onto each of these.

This then means that we must have,

$$\frac{w}{4} = 1.1760 + 2\pi n \quad \text{OR} \quad \frac{w}{4} = 4.3176 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

Finally, to get all the solutions to the equation all we need to do is multiply both sides by 4 and we’ll convert everything to decimals to help with the final step.

$$w = 4.7040 + 8\pi n \quad \text{OR} \quad w = 17.2704 + 8\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

$$= 4.7040 + 25.1327n \quad \text{OR} \quad = 17.2704 + 25.1327n \quad n = 0, \pm 1, \pm 2, \ldots$$
Calculus I

Hint 4: Now all we need to do is plug in values of \( n \) to determine which solutions will actually fall in the given interval.

Step 4
Now let’s find all the solutions. First, notice that if we plug in positive \( n \) or \( n = 0 \) we will have positive solutions and these solutions will be out of the interval. Therefore, we’ll start with \( n = -1 \).

\[
\begin{align*}
  n = -1: & \quad w = -20.4287 \quad \text{OR} \quad w = -7.8623 \\
  n = -2: & \quad w = -45.5614 \quad \text{OR} \quad w = -32.9950
\end{align*}
\]

Notice that with each increase in \( n \) we were really just subtracting another 25.1327 from the previous results. A quick inspection of the results above will quickly show us that we don’t need to go any farther and we won’t bother with any other values of \( n \).

So, it looks like we have the four solutions to this equation in the given interval.

\[ w = -45.5614, -32.9950, -20.4287, -7.8623 \]

Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4th decimal place or so however.

7. Find the solution(s) to \( 17 - 3\sec \left( \frac{z}{2} \right) = 2 \) that are in \( [20, 45] \). Use at least 4 decimal places in your work.

Hint 1: Find all the solutions to the equation without regard to the given interval. The first step in this process is to isolate the secant (with a coefficient of one) on one side of the equation.

Step 1
Isolating the secant (with a coefficient of one) on one side of the equation gives,

\[ \sec \left( \frac{z}{2} \right) = 5 \]

Hint 2: Using a calculator and your knowledge of the unit circle to determine all the angles in the range \( [0, 2\pi] \) for which secant will have this value. The best way to do this is to rewrite the equation into one in terms of a different trig function that we can more easily deal with.

Step 2
In order to get the solutions it will be much easier to recall the definition of secant in terms of cosine and rewrite the equation into one involving cosine. Doing this gives,
The solution(s) to the equation involving the cosine are the same as the solution(s) to the equation involving the secant and so working with that will be easier. Using our calculator we can see that,

\[ \frac{z}{2} = \cos^{-1}\left(\frac{1}{5}\right) = 1.3694 \]

Now we’re dealing with cosine in this problem and we know that the -axis represents cosine on a unit circle and so we’re looking for angles that will have a - coordinate of \(\frac{1}{5}\). This means that we’ll have angles in the first (this is the one our calculator gave us) and fourth quadrant. Here is a unit circle for this situation.

From the symmetry of the unit circle we can see that we can either use \(-1.3694\) or \(2\pi - 1.3694 = 4.9138\) for the second angle. Each will give the same set of solutions. However, because it is easy to lose track of minus signs we will use the positive angle for our second solution.

Hint 3: Using the two angles above write down all the angles for which cosine/secant will have this value and use these to write down all the solutions to the equation.

Step 3
From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding “$+2\pi n$ for $n = 0, \pm 1, \pm 2, \ldots$” onto each of these.

This then means that we must have,

$$\frac{z}{2} = 1.3694 + 2\pi n \quad \text{OR} \quad \frac{z}{2} = 4.9138 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

Finally, to get all the solutions to the equation all we need to do is multiply both sides by 2 and we’ll convert everything to decimals to help with the final step.

$$z = 2.7388 + 4\pi n \quad \text{OR} \quad z = 9.8276 + 4\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

$$= 2.7388 + 12.5664n \quad \text{OR} \quad = 9.8276 + 12.5664n \quad n = 0, \pm 1, \pm 2, \ldots$$

Hint 4 : Now all we need to do is plug in values of $n$ to determine which solutions will actually fall in the given interval.

Step 4
Now let’s find all the solutions. First notice that, in this case, if we plug in negative values of $n$ we will get negative solutions and these will not be in the interval and so there is no reason to even try these. Also note that if we use $n = 0$ we will still not be in the interval and so let’s start things off at $n = 1$.

$$n = 1: \quad \frac{z}{2} = 15.3052 < 20 \quad \text{OR} \quad z = 22.3940$$

$$n = 2: \quad z = 27.8716 \quad \text{OR} \quad z = 34.9604$$

$$n = 3: \quad z = 40.4380 \quad \text{OR} \quad z = 47.5268 > 45$$

Notice that with each increase in $n$ we were really just adding another 12.5664 onto the previous results and by a quick inspection of the results above we can see that we don’t need to go any farther. Also, as we’ve seen in this problem it is completely possible for only one of the solutions from a given interval to be in the given interval so don’t worry about that when it happens.

So, it looks like we have the four solutions to this equation in the given interval.

$$z = 22.3940, 27.8716, 34.9604, 40.4380$$

Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4th decimal place or so however.
8. Find the solution(s) to \(12 \sin(7y) + 11 = 3 + 4 \sin(7y)\) that are in \([-2, -\frac{1}{2}]\). Use at least 4 decimal places in your work.

Hint 1 : Find all the solutions to the equation without regard to the given interval. The first step in this process is to isolate the sine (with a coefficient of one) on one side of the equation.

Step 1
Isolating the sine (with a coefficient of one) on one side of the equation gives,
\[
\sin(7y) = -1
\]

Hint 2 : Use your knowledge of the unit circle to determine all the angles in the range \([0, 2\pi]\) for which sine will have this value.

Step 2
If you need to use a calculator to get the solution for this that is fine, but this is also one of the standard angles as we can see from the unit circle below.

Because we’re dealing with sine in this problem and we know that the \(y\)-axis represents sine on a unit circle we’re looking for angle(s) that will have a \(y\) coordinate of \(-1\). The only angle that will have this \(y\) coordinate will be \(\frac{3\pi}{2} = 4.7124\). 

Note that unlike all the other problems that we’ve worked to this point this will be the only angle. There is simply not another angle in the range \([0, 2\pi]\) for which sine will have this value. Don’t
get so locked into the *usual* case where we get two possible angles in the \([0, 2\pi]\) that when these single solution cases roll around you decide you must have done something wrong. They happen on occasion and we need to be able to deal with them when they occur.

Hint 3 : Using the angle above write down all the angles for which sine will have this value and use these to write down all the solutions to the equation.

Step 3
From the discussion in the notes for this section we know that once we have the angle above we can get all possible angles by simply adding “\(+2\pi n\) for \(n = 0, \pm 1, \pm 2, \ldots\)” onto the angle.

This then means that we must have,

\[
y = 4.7124 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots
\]

Finally, to get all the solutions to the equation all we need to do is divide both sides by 7 and we’ll convert everything to decimals to help with the final step.

\[
y = 0.6732 + \frac{2\pi n}{7} \quad n = 0, \pm 1, \pm 2, \ldots
\]

Hint 4 : Now all we need to do is plug in values of \(n\) to determine which solutions will actually fall in the given interval.

Step 4
Now let’s find all the solutions. First notice that, in this case, if we plug in positive values of \(n\) or \(n = 0\) we will get positive solutions and these will not be in the interval and so there is no reason to even try these. So, let’s start at \(n = -1\) and see what we get.

\[
\begin{align*}
n = -1: & \quad y = -0.2244 < -0.5 \\
n = -2: & \quad y = -1.122 \\
n = -3: & \quad y = -2.0196 < -2
\end{align*}
\]

So, it looks like we have only a single solution to this equation in the given interval.

\[
y = -1.122
\]

Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4th decimal place or so however.
9. Find the solution(s) to \(5 - 14 \tan(8x) = 30\) that are in \([-1,1]\). Use at least 4 decimal places in your work.

Hint 1 : Find all the solutions to the equation without regard to the given interval. The first step in this process is to isolate the tangent (with a coefficient of one) on one side of the equation.

Step 1
Isolating the tangent (with a coefficient of one) on one side of the equation gives,

\[
\tan(8x) = -\frac{25}{14}
\]

Hint 2 : Using a calculator and your knowledge of solving trig equations involving tangents to determine all the angles in the range \([0, 2\pi]\) for which tangent will have this value.

Step 2
First, using our calculator we can see that,

\[
8x = \tan^{-1}\left(-\frac{25}{14}\right) = -1.0603
\]

As we discussed in Example 5 of this section the second angle for equations involving tangent will always be the \(\pi\) plus the first angle. Therefore, \(\pi + (-1.0603) = 2.0813\) will be the second angle.

Hint 3 : Using the two angles above write down all the angles for which tangent will have this value and use these to write down all the solutions to the equation.

Step 3
From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding \(\pm 2\pi n\) for \(n = 0, \pm 1, \pm 2, \ldots\) onto each of these.

This then means that we must have,

\[
8x = -1.0603 + 2\pi n \quad \text{OR} \quad 8x = 2.0813 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots
\]

Finally, to get all the solutions to the equation all we need to do is divide both sides by 8 and we’ll convert everything to decimals to help with the final step.

\[
x = -0.1325 + \frac{\pi n}{4} \quad \text{OR} \quad x = 0.2602 + \frac{\pi n}{4} \quad n = 0, \pm 1, \pm 2, \ldots
\]

\[
x = -0.1325 + 0.7854n \quad \text{OR} \quad x = 0.2602 + 0.7854n \quad n = 0, \pm 1, \pm 2, \ldots
\]
Hint 4 : Now all we need to do is plug in values of \( n \) to determine which solutions will actually fall in the given interval.

Step 4
Now let’s find all the solutions.

\[
\begin{align*}
  n = -1 : & \quad x = -0.9179 \quad \text{OR} \quad x = -0.5252 \\
  n = 0 : & \quad x = -0.1325 \quad \text{OR} \quad x = 0.2602 \\
  n = 1 : & \quad x = 0.6529 \quad \text{OR} \quad x = \frac{1}{0.456} > 1 
\end{align*}
\]

Notice that with each increase in \( n \) we were really just adding/subtracting another 0.7854 from the previous results. A quick inspection of the results above will quickly show us that we don’t need to go any farther and we won’t bother with any other values of \( n \). Also, as we’ve seen in this problem it is completely possible for only one of the solutions from a given interval to be in the given interval so don’t worry about that when it happens.

So, it looks like we have the five solutions to this equation in the given interval.

\[
x = -0.9179, -0.5252, -0.1325, 0.2602, 0.6529
\]

Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4\(^{th}\) decimal place or so however.

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10. Find the solution(s) to \( \frac{182}{3} \csc \left( \frac{t}{3} \right) + 9 = 0 \) that are in \([0, 5]\). Use at least 4 decimal places in your work.

Hint 1 : Find all the solutions to the equation without regard to the given interval. The first step in this process is to isolate the cosecant (with a coefficient of one) on one side of the equation.

Step 1
Isolating the cosecant (with a coefficient of one) on one side of the equation gives,

\[
\csc \left( \frac{t}{3} \right) = -9
\]

Hint 2 : Using a calculator and your knowledge of the unit circle to determine all the angles in the range \([0, 2\pi]\) for which cosecant will have this value. The best way to do this is to rewrite the equation into one in terms of a different trig function that we can more easily deal with.

Step 2
In order to get the solutions it will be much easier to recall the definition of cosecant in terms of sine and rewrite the equation into one involving sine. Doing this gives,

\[
\csc\left(\frac{t}{3}\right) = \frac{1}{\sin\left(\frac{t}{3}\right)} = -9 \quad \Rightarrow \quad \sin\left(\frac{t}{3}\right) = -\frac{1}{9}
\]

The solution(s) to the equation involving the sine are the same as the solution(s) to the equation involving the cosecant and so working with that will be easier. Using our calculator we can see that,

\[
\frac{t}{3} = \sin^{-1}\left(-\frac{1}{9}\right) = -0.1113
\]

Now we’re dealing with sine in this problem and we know that the \(y\)-axis represents sine on a unit circle and so we’re looking for angles that will have a \(y\) coordinate of \(-\frac{1}{9}\). This means that we’ll have angles in the fourth (this is the one our calculator gave us) and third quadrant. Here is a unit circle for this situation.

From the symmetry of the unit circle we can see that the second angle will make an angle of 0.1113 with the negative \(x\)-axis and so the second angle will be \(\pi + 0.1113 = 3.2529\). Also, as noted on the unit circle above a positive angle that represents the first angle (\(i.e.\) the angle in the fourth quadrant) is \(2\pi - 0.1113 = 6.1719\). We can use either the positive or the negative angle here and we’ll get the same solutions. However, because it is often easy to lose track of minus signs we will be using the positive angle in the fourth quadrant for our work here.

Hint 3 : Using the two angles above write down all the angles for which sine/cosecant will have this value and use these to write down all the solutions to the equation.
Step 3
From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding “$+ 2\pi n$ for $n = 0, \pm 1, \pm 2, \ldots$” onto each of these.

This then means that we must have,

$$\frac{t}{3} = 3.2529 + 2\pi n \quad \text{OR} \quad \frac{t}{3} = 6.1719 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

Finally, to get all the solutions to the equation all we need to do is multiply both sides by 3 and we’ll convert everything to decimals to help with the final step.

$$t = 9.7587 + 6\pi n \quad \text{OR} \quad t = 18.5157 + 6\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

$$= 9.7587 + 18.8496n \quad \text{OR} \quad = 18.5157 + 18.8496n \quad n = 0, \pm 1, \pm 2, \ldots$$

Hint 4 : Now all we need to do is plug in values of $n$ to determine which solutions will actually fall in the given interval.

Step 4
Now let’s find all the solutions. First notice that, in this case, if we plug in negative values of $n$ we will get negative solutions and these will not be in the interval and so there is no reason to even try these. Also note that even if we start off with $n = 0$ we will get solutions that are already out of the given interval.

So, despite the fact that there are solutions to this equation none of them fall in the given interval and so there are no solutions to this equation. Do not get excited about the answer here. This kind of situation will happen on occasion and so we need to be aware of it and able to deal with it.

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11 Find the solution(s) to

$$\frac{1}{2} \cos \left( \frac{x}{8} \right) + \frac{1}{4} = \frac{2}{3}$$

that are in $[0, 100]$. Use at least 4 decimal places in your work.

Hint 1 : Find all the solutions to the equation without regard to the given interval. The first step in this process is to isolate the cosine (with a coefficient of one) on one side of the equation.

Step 1
Isolating the cosine (with a coefficient of one) on one side of the equation gives,

$$\cos \left( \frac{x}{8} \right) = \frac{5}{6}$$
Hint 2 : Using a calculator and your knowledge of the unit circle to determine all the angles in the range \([0, 2\pi]\) for which cosine will have this value.

Step 2
First, using our calculator we can see that,

\[
\frac{x}{8} = \cos^{-1}\left(\frac{5}{6}\right) = 0.5859
\]

Now we’re dealing with cosine in this problem and we know that the \(x\)-axis represents cosine on a unit circle and so we’re looking for angles that will have a \(x\) coordinate of \(\frac{5}{6}\). This means that we’ll have angles in the first (this is the one our calculator gave us) and fourth quadrant. Here is a unit circle for this situation.

From the symmetry of the unit circle we can see that we can either use \(-0.5859\) or \(2\pi - 0.5859 = 5.6973\) for the second angle. Each will give the same set of solutions. However, because it is easy to lose track of minus signs we will use the positive angle for our second solution.

Hint 3 : Using the two angles above write down all the angles for which cosine will have this value and use these to write down all the solutions to the equation.

Step 3
From the discussion in the notes for this section we know that once we have these two angles we can get all possible angles by simply adding \(+2\pi n\) for \(n = 0, \pm 1, \pm 2, \ldots\) onto each of these.
This then means that we must have,
\[
\frac{x}{8} = 0.5859 + 2\pi n \quad \text{OR} \quad \frac{x}{8} = 5.6973 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots
\]

Finally, to get all the solutions to the equation all we need to do is multiply both sides by 8 and we’ll convert everything to decimals to help with the final step.
\[
x = 4.6872 + 16\pi n \quad \text{OR} \quad x = 45.5784 + 16\pi n \quad n = 0, \pm 1, \pm 2, \ldots
\]
\[
= 4.6872 + 50.2655n \quad \text{OR} \quad = 45.5784 + 50.2655n \quad n = 0, \pm 1, \pm 2, \ldots
\]

Hint 4 : Now all we need to do is plug in values of \(n\) to determine which solutions will actually fall in the given interval.

Step 4
Now let’s find all the solutions. First notice that, in this case, if we plug in negative values of \(n\) we will get negative solutions and these will not be in the interval and so there is no reason to even try these. So, let’s start at \(n = 0\) and see what we get.

\[
\begin{align*}
  n &= 0 : \quad x = 4.6872 & \text{OR} & \quad x = 45.5784 \\
  n &= 1 : \quad x = 54.9527 & \text{OR} & \quad x = 95.8439 
\end{align*}
\]

Notice that with each increase in \(n\) we were really just adding another 50.2655 onto the previous results and by doing this to the results from the \(n = 1\) step we will get solutions that are outside of the interval and so there is no reason to even plug in \(n = 2\).

So, it looks like we have the four solutions to this equation in the given interval.
\[x = 4.6872, 45.5784, 54.9527, 95.8439\]

Note that depending upon the amount of decimals you used here your answers may vary slightly from these due to round off error. Any differences should be slight and only appear around the 4th decimal place or so however.

12. Find the solution(s) to \(\frac{4}{3} = 1 + 3\sec(2t)\) that are in \([-4, 6]\). Use at least 4 decimal places in your work.

Hint 1 : Find all the solutions to the equation without regard to the given interval. The first step in this process is to isolate the secant (with a coefficient of one) on one side of the equation.

Step 1
Isolating the secant (with a coefficient of one) on one side of the equation gives,
Calculus I

\[ \sec(2t) = \frac{1}{9} \]

At this point we can stop. We know that
\[ \sec \theta \leq -1 \quad \text{or} \quad \sec \theta \leq 1 \]

This means that it is impossible for secant to ever be \( \frac{1}{9} \) and so there will be no solution to this equation.

Note that if you didn’t recall the restrictions on secant the next step would have been to convert this to cosine so let’s do that.

\[ \sec(2t) = \frac{1}{\cos(2t)} = \frac{1}{9} \quad \Rightarrow \quad \cos(2t) = 9 \]

At this point we can note that \(-1 \leq \cos \theta \leq 1\) and so again there is no way for cosine to be 9 and again we get that there will be no solution to this equation.