Preface

Here are my online notes for my differential equations course that I teach here at Lamar University. Despite the fact that these are my “class notes” they should be accessible to anyone wanting to learn how to solve differential equations or needing a refresher on differential equations.

I’ve tried to make these notes as self contained as possible and so all the information needed to read through them is either from a Calculus or Algebra class or contained in other sections of the notes.

A couple of warnings to my students who may be here to get a copy of what happened on a day that you missed.

1. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn differential equations I have included some material that I do not usually have time to cover in class and because this changes from semester to semester it is not noted here. You will need to find one of your fellow class mates to see if there is something in these notes that wasn’t covered in class.

2. In general I try to work problems in class that are different from my notes. However, with Differential Equation many of the problems are difficult to make up on the spur of the moment and so in this class my class work will follow these notes fairly close as far as worked problems go. With that being said I will, on occasion, work problems off the top of my head when I can to provide more examples than just those in my notes. Also, I often don’t have time in class to work all of the problems in the notes and so you will find that some sections contain problems that weren’t worked in class due to time restrictions.

3. Sometimes questions in class will lead down paths that are not covered here. I try to anticipate as many of the questions as possible in writing these up, but the reality is that I can’t anticipate all the questions. Sometimes a very good question gets asked in class that leads to insights that I’ve not included here. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are.

4. This is somewhat related to the previous three items, but is important enough to merit its own item. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. As already noted not everything in these notes is covered in class and often material or insights not in these notes is covered in class.
Summary of Separation of Variables

Throughout this chapter we’ve been talking about and solving partial differential equations using the method of separation of variables. However, the one thing that we’ve not really done is completely work an example from start to finish showing each and every step.

Each partial differential equation that we solved made use somewhere of the fact that we’d done at least part of the problem in another section and so it makes some sense to have a quick summary of the method here.

Also note that each of the partial differential equations only involved two variables. The method can often be extended out to more than two variables, but the work in those problems can be quite involved and so we didn’t cover any of that here.

So with all of that out of the way here is a quick summary of the method of separation of variables for partial differential equations in two variables.

1. Verify that the partial differential equation is linear and homogeneous.

2. Verify that the boundary conditions are in proper form. Note that this will often depend on what is in the problem. So,
   a. If you have initial conditions verify that all the boundary conditions are linear and homogeneous.
   b. If there are no initial conditions (such as Laplace’s equation) the verify that all but one of the boundary conditions are linear and homogeneous.
   c. In some cases (such as we saw with Laplace’s equation on a disk) a boundary condition will take the form of requiring that the solution stay finite and in these cases we just need to make sure the boundary condition is met.

3. Assume that solutions will be a product of two functions each a function in only one of the variables in the problem. This is called a product solution.

4. Plug the product solution into the partial differential equation, separate variables and introduce a separation constant. This will produce two ordinary differential equations.

5. Plug the product solution into the homogeneous boundary conditions. Note that often it will be better to do this prior to doing the differential equation so we can use these to help us chose the separation constant.

6. One of the ordinary differential equations will be a boundary value problem. Solve this to determine the eigenvalues and eigenfunctions for the problem.

   Note that this is often very difficult to do and in some cases it will be impossible to completely find all eigenvalues and eigenfunctions for the problem. These cases can be dealt with to get at least an approximation of the solution, but that is beyond the scope of this quick introduction.

7. Solve the second ordinary differential equation using any remaining homogeneous boundary conditions to simplify the solution if possible.
8. Use the Principle of Superposition and the product solutions to write down a solution to the partial differential equation that will satisfy the partial differential equation and homogeneous boundary conditions.

9. Apply the remaining conditions (these may be initial condition(s) or a single nonhomogeneous boundary condition) and use the orthogonality of the eigenfunctions to find the coefficients.

Note that in all of our examples the eigenfunctions were sines and/or cosines however they won’t always be sines and cosines. If the boundary value problem is sufficiently nice (and that’s beyond the scope of this quick introduction to the method) we can always guarantee that the eigenfunctions will be orthogonal regardless of what they are.