Derivatives

Basic Properties/Formulas/Rules

$\frac{d}{dx}\left(cf(x)\right) = cf'(x)$, c is any constant.

$\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$, n is any number.

$\frac{d}{dx}\left(c\right) = 0$, c is any constant.

$\left(f(x)\,g(x)\right)' = f'(x)\,g(x) + f(x)\,g'(x)$ – **Product Rule**

$\frac{d}{dx}\left(\mathbf{e}^{g(x)}\right) = g'(x)\mathbf{e}^{g(x)}$

$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{\left(g(x)\right)^2}$ – **Quotient Rule**

$\frac{d}{dx}\left[\ln(g(x))\right] = \frac{g'(x)}{g(x)}$

$\frac{d}{dx}\left[f\left(g(x)\right)\right] = f'\left(g(x)\right)g'(x)$ – **Chain Rule**

Common Derivatives

Polynomials

$\frac{d}{dx}\left(c\right) = 0$ $\qquad \frac{d}{dx}(x) = 1$ $\qquad \frac{d}{dx}\left(cx\right) = c$ $\qquad \frac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \frac{d}{dx}\left(cx^n\right) = ncx^{n-1}$

Trig Functions

$\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$ $\qquad \frac{d}{dx}\left[\cos(x)\right] = -\sin(x)$ $\qquad \frac{d}{dx}\left[\tan(x)\right] = \sec^2(x)$

$\frac{d}{dx}\left[\csc(x)\right] = -\csc(x)\cot(x)$ $\qquad \frac{d}{dx}\left[\sec(x)\right] = \sec(x)\tan(x)$ $\qquad \frac{d}{dx}\left[\cot(x)\right] = -\csc^2(x)$

Inverse Trig Functions

$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}\left[\cos^{-1}(x)\right] = -\frac{1}{\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2}$

$\frac{d}{dx}\left[\csc^{-1}(x)\right] = -\frac{1}{|x|\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}\left[\sec^{-1}(x)\right] = \frac{1}{|x|\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}\left[\cot^{-1}(x)\right] = -\frac{1}{1+x^2}$

Exponential & Logarithm Functions

$\frac{d}{dx}\left[a^x\right] = a^x\ln(a)$ $\qquad \frac{d}{dx}\left[\mathbf{e}^x\right] = \mathbf{e}^x$

$\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x},\ x > 0$ $\qquad \frac{d}{dx}\left[\ln|x|\right] = \frac{1}{x},\ x \neq 0$ $\qquad \frac{d}{dx}\left[\log_a(x)\right] = \frac{1}{x\ln(a)},\ x > 0$

Hyperbolic Functions

$\frac{d}{dx}\left[\sinh(x)\right] = \cosh(x)$ $\qquad \frac{d}{dx}\left[\cosh(x)\right] = \sinh(x)$ $\qquad \frac{d}{dx}\left[\tanh(x)\right] = \text{sech}^2(x)$

$\frac{d}{dx}\left[\text{csch}(x)\right] = -\text{csch}(x)\coth(x)$ $\qquad \frac{d}{dx}\left[\text{sech}(x)\right] = -\text{sech}(x)\tanh(x)$ $\qquad \frac{d}{dx}\left[\coth(x)\right] = -\text{csch}^2(x)$

Integrals

Basic Properties/Formulas/Rules

$\int cf(x)\,dx = c\int f(x)\,dx$, c is a constant. $\int f(x)\pm g(x)\,dx = \int f(x)dx \pm \int g(x)\,dx$

$\int_a^b f(x)\,dx = f(x)\Big|_a^b = F(b)-F(a)$ where $f(x) = \int f(x)\,dx$

$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, c is a constant. $\int_a^b f(x)\pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

$\int_a^a f(x)\,dx = 0$ $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ $\int_a^b c\,dx = c(b-a)$, c is a constant.

If $f(x)\geq 0$ on $a\leq x\leq b$ then $\int_a^b f(x)\,dx \geq 0$

If $f(x)\geq g(x)$ on $a\leq x\leq b$ then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

Common Integrals

Polynomials

$\int dx = x+c$ $\int k\,dx = kx+c$ $\int x^n dx = \frac{1}{n+1}x^{n+1}+c,\ n\neq -1$

$\int \frac{1}{x}\,dx = \ln|x|+c$ $\int x^{-1}\,dx = \ln|x|+c$ $\int x^{-n}dx = \frac{1}{-n+1}x^{-n+1}+c,\ n\neq 1$

$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b|+c$ $\int x^{\frac{p}{q}}dx = \frac{1}{\frac{p}{q}+1}x^{\frac{p}{q}+1}+c = \frac{q}{p+q}x^{\frac{p+q}{q}}+c$

Trig Functions

$\int \cos(u)\,du = \sin(u)+c$ $\int \sin(u)\,du = -\cos(u)+c$ $\int \sec^2 u\,du = \tan(u)+c$

$\int \sec(u)\tan(u)\,du = \sec(u)+c$ $\int \csc(u)\cot(u)\,du = -\csc(u)+c$ $\int \csc^2 u\,du = -\cot(u)+c$

$\int \tan(u)\,du = -\ln|\cos(u)|+c = \ln|\sec(u)|+c$ $\int \cot(u)\,du = \ln|\sin(u)|+c = -\ln|\csc(u)|+c$

$\int \sec(u)\,du = \ln|\sec(u)+\tan(u)|+c$ $\int \sec^3(u)\,du = \frac{1}{2}\Big(\sec(u)\tan(u)+\ln|\sec(u)+\tan(u)|\Big)+c$

$\int \csc(u)\,du = \ln|\csc(u)-\cot(u)|+c$ $\int \csc^3(u)\,du = \frac{1}{2}\Big(-\csc(u)\cot(u)+\ln|\csc(u)-\cot(u)|\Big)+c$

Exponential & Logarithm Functions

$$\int \mathbf{e}^u\,du = \mathbf{e}^u + c \qquad \int a^u\,du = \frac{a^u}{\ln(a)} + c \qquad \int \ln(u)\,du = u\ln(u) - u + c$$

$$\int \mathbf{e}^{au}\sin(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\Big(a\sin(bu) - b\cos(bu)\Big) + c \qquad \int u\mathbf{e}^u du = (u-1)\mathbf{e}^u + c$$

$$\int \mathbf{e}^{au}\cos(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\Big(a\cos(bu) + b\sin(bu)\Big) + c \qquad \int \frac{1}{u\ln(u)}\,du = \ln\big|\ln(u)\big| + c$$

Inverse Trig Functions

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + c \qquad \int \sin^{-1}(u)\,du = u\sin^{-1}(u) + \sqrt{1-u^2} + c$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c \qquad \int \tan^{-1}(u)\,du = u\tan^{-1}(u) - \frac{1}{2}\ln\left(1+u^2\right) + c$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + c \qquad \int \cos^{-1}(u)\,du = u\cos^{-1}(u) - \sqrt{1-u^2} + c$$

Hyperbolic Functions

$$\int \sinh(u)du = \cosh(u) + c \qquad \int \operatorname{sech}(u)\tanh(u)du = -\operatorname{sech}(u) + c \qquad \int \operatorname{sech}^2(u)du = \tanh(u) + c$$

$$\int \cosh(u)du = \sinh(u) + c \qquad \int \operatorname{csch}(u)\coth(u)du = -\operatorname{csch}(u) + c \qquad \int \operatorname{csch}^2(u)du = -\coth(u) + c$$

$$\int \tanh(u)du = \ln\left(\cosh(u)\right) + c \qquad \int \operatorname{sech}(u)du = \tan^{-1}\left|\sinh(u)\right| + c$$

Miscellaneous

$$\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + c \qquad \int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + c$$

$$\int \frac{1}{u^2-a^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + c \qquad \int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + c$$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + c$$

Standard Integration Techniques

u Substitution : $\int_a^b f\big(g(x)\big)\,g'(x)\,dx$ will convert the integral into $\int_a^b f\big(g(x)\big)\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$ using the substitution $u = g(x)$ where $du = g'(x)dx$. For indefinite integrals drop the limits of integration.

Integration by Parts : $\int u\,dv = uv - \int v\,du$ and $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$. Choose u and dv from integral and compute du by differentiating u and compute v using $v = \int dv$.

Trig Substitutions : If the integral contains the following root use the given substitution and formula.

$$\sqrt{a^2 - b^2x^2} \quad \Rightarrow \quad x = \frac{a}{b}\sin(\theta) \quad \text{and} \quad \cos^2(\theta) = 1 - \sin^2(\theta)$$
$$\sqrt{b^2x^2 - a^2} \quad \Rightarrow \quad x = \frac{a}{b}\sec(\theta) \quad \text{and} \quad \tan^2(\theta) = \sec^2(\theta) - 1$$
$$\sqrt{a^2 + b^2x^2} \quad \Rightarrow \quad x = \frac{a}{b}\tan(\theta) \quad \text{and} \quad \sec^2(\theta) = 1 + \tan^2(\theta)$$

Partial Fractions : If integrating a rational expression involving polynomials, $\int \frac{P(x)}{Q(x)}\,dx$, where the degree (largest exponent) of $P(x)$ is smaller than the degree of $Q(x)$ then factor the denominator as completely as possible and find the partial fraction decomposition of the rational expression. Integrate the partial fraction decomposition (P.F.D.). For each factor in the denominator we get term(s) in the decomposition according to the following table.

Factor of $Q(x)$	Term in P.F.D	Factor is $Q(x)$	Term in P.F.D
$ax + b$	$\frac{A}{ax+b}$	$(ax+b)^k$	$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_k}{(ax+b)^k}$
$ax^2 + bx + c$	$\frac{Ax+B}{ax^2+bx+c}$	$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{ax^2+bx+c} + \cdots + \frac{A_kx+B_k}{(ax^2+bx+c)^k}$

Products and (some) Quotients of Trig Functions :

For $\int \sin^n(x)\cos^m(x)\,dx$ we have the following :

1. $\boldsymbol{n}$ **odd.** Strip 1 sine out and convert rest to cosines using $\sin^2(x) = 1 - \cos^2(x)$, then use the substitution $u = \cos(x)$.
2. $\boldsymbol{m}$ **odd.** Strip 1 cosine out and convert rest to sines using $\cos^2(x) = 1 - \sin^2(x)$, then use the substitution $u = \sin(x)$.
3. $\boldsymbol{n}$ **and** $\boldsymbol{m}$ **both odd.** Use either 1. or 2.
4. $\boldsymbol{n}$ **and** $\boldsymbol{m}$ **both even.** Use double angle and/or half angle formulas to reduce the integral into a form that can be integrated.

For $\int \tan^n(x)\sec^m(x)\,dx$ we have the following :

1. $\boldsymbol{n}$ **odd.** Strip 1 tangent and 1 secant out and convert the rest to secants using $\tan^2(x) = \sec^2(x) - 1$, then use the substitution $u = \sec(x)$.
2. $\boldsymbol{m}$ **even.** Strip 2 secants out and convert rest to tangents using $\sec^2(x) = 1 + \tan^2(x)$, then use the substitution $u = \tan(x)$.
3. $\boldsymbol{n}$ **odd and** $\boldsymbol{m}$ **even.** Use either 1. or 2.
4. $\boldsymbol{n}$ **even and** $\boldsymbol{m}$ **odd**. Each integral will be dealt with differently.

Convert Example : $\cos^6(x) = \left(\cos^2(x)\right)^3 = \left(1 - \sin^2(x)\right)^3$